PERSONAGE IN SCIENCE

Professor N.V. Azbelev

A.I. Bulgakov [1], V.P. Maksimov [2*], A.A. Martynyuk [3], and E.L. Tonkov [4]

[1] *Tambov State University, International'naya Str., 33, Tambov, 392000, Russia*
[2] *Perm State University, Bukirev Str. 15, Perm, 614990, Russia*
[3] *Institute of Mechanics National Academy of Sciences of Ukraine,*
Nesterov Str. 3, Kiev, 03057, Ukraine
[4] *Udmurt State University, Universitetskaya Str. 1, Izhevsk, 426034, Russia*

Professor Nikolay Viktorovich Azbelev, a well-known Russian mathematician, has been a leading figure in the differential and integral equations profession for about five decades. To commemorate Professor Azbelev's valuable contribution to nonlinear dynamics, the Editorial Board of the Journal presents a biographical sketch of his life and academic activities. The main stages of Azbelev's life and activity are also presented in the special papers in "Differential'nye Uravneniya" 18, No.4, 1982; 33, No.4, 1997; 38, No.4, 2002; 43, No.5, 2007, "Mem. Differential Equations Math. Phys.", 26, 2002; 41, 2007 and "Functional Differential Equations", 9, No.3-4, 2002.

1 N.V. Azbelev's life

Nikolay Viktorovich Azbelev was born on April 15, 1922 in selo (small village) Bazlovo, Pskov Region, Russia, in a physician's family. His mother, Antonina Fedorovna Khlebnikova, was a scholar and collaborator of a famous botanist V.L. Komarov, later the President of the Academy of Sciences of the USSR. His father, Viktor Nikolaevich, graduated from Military Medical Academy in St.Petersburg in 1905, further attended lectures on microbiology at Robert Koch's Institute, Berlin, Germany, and was a physician in a field hospital during the World War I. Later he was the director of Polar Institute of Bacteriology in Arkhangelsk, Russia.

In 1941 Nikolay Viktorovich enrolled at Moscow State University (MSU). His studies at the Faculty of Mechanics and Mathematics of MSU were interrupted due to his military service in the Soviet Army during the World War II. In 1945 he entered Moscow Aviation Institute, from which he graduated with the degree in engineering in 1949. The same year he started to work at the Design Bureau headed by Prof. A.A. Mikulin, a member of the Academy of Sciences. At this place Nikolay Viktorovich gained an experience in several areas of applied mathematics and solved several important problems. For example, he

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* Corresponding author: maksimov@econ.psu.ru

proposed an original computational method in the study of strength properties of a kind of ball bearing. In 1947 he was among the first to make use of the so-called method of electrical analogy as applied to turbine dynamics. He also designed an analog computer to find the frequencies of the shift vibration in turbo-jet engines.

In 1951-54 he was a post-graduate student at the Department of Higher Mathematics of Moscow Machine and Instruments Institute under supervision of Prof. B.I. Segal. In 1954 Nikolay Viktorovich defended his Candidate of Sciences (the Soviet equivalent of Ph.D. degree) thesis "On the boundaries of feasibility of Chaplygin's theorem on differential inequalities" at Moscow State University (in his report, V.V. Nemytskyi, the official reader of the thesis, emphasized a very high mathematical level of the work). The same year he left Moscow for Izhevsk, a city situated in the vicinity of the Ural Mountains, to become the head of the Higher Mathematics Department at Izhevsk Mechanical Institute (IMI). One of the first things Nikolay Viktorovich did upon his arrival at IMI was to found the Izhevsk Mathematical Seminar. It soon became the central meeting point for mathematicians and engineers. Azbelev's warmth and sensitiveness were tremendously important for the creation of the mathematical community around IMI. The works of the participants of this seminar concerning the theory of integral, differential and difference inequalities allowed to solve a number of problems on existence, uniqueness, and asymptotic behavior of solutions to differential equations. Other works of the Izhevsk Mathematical Seminar are devoted to the search of effective conditions and criteria for unique solvability of boundary value problems for ordinary differential equations and investigation of the properties of Green's function for those problems. Since 1961 the major attention of N.V. Azbelev and his seminar was focused on the problems of the general theory of equations with discontinuous operators. In Izhevsk he wrote his Doctoral thesis "On the Chaplygin problem", which was defended in 1962 at Kazan State University. In 1964 he was granted the title of professor.

In 1966 Professor Azbelev was elected to be the Head of the Higher Mathematics Department in Tambov Institute for Chemical Engineering. Nikolay Viktorovich moved to Tambov together with his wife Lina Fazylovna Rakhmatullina, a brilliant mathematician and the closest collaborator. A large group of postgraduate students and colleagues of Prof. Azbelev from Izhevsk joined them in Tambov. Soon after that the Tambov Seminar under the leadership of N.V. Azbelev and L.F. Rakhmatullina started its work. It dealt with equations with deviating argument. The activity of the Tambov Seminar implied the creation of an effective theory of differential equations with deviating argument. This theory became a basis of the contemporary Theory of Functional Differential Equations.

In 1975 Professor Azbelev accepted invitation of the Rector of Perm Polytechnic Institute (PPI), Prof. M.N. Dedyukin, and moved to Perm, where he founded the Department of Mathematical Analysis. Azbelev's scientific expertise and leadership contributed immensely to the development of this department. As a result, it had become one of the well known mathematical centers and the core of the Perm Seminar on Functional Differential Equations. Since 1994 to his last day N. Azbelev has been the head of the Research Center on Functional Differential Equations at Perm State Technical University (former PPI).

Nikolay Viktorovich was a true representative of the Russian intelligentsia. He was a connoisseur of the Russian poetry and a great admirer of the classical music. As a passionate traveller he together with Lina Fazylovna travelled across the Caucasus, Middle Asia, and Central part of Russia first by his motorbike and later by his car.

2 Main Direction of His Research

Nikolay Azbelev's research covers integral, differential and functional differential equations and inequalities, numerical methods, stability theory, boundary value problems and calculus of variations. He is one of the founders of the Russian scientific school of differential and integral inequalities. In his first papers N. Azbelev gave a solution to the Chaplygin problem on the boundaries of feasibility of the differential inequality theorem. His works essentially expanded the area of applications of differential inequalities. The activity of Azbelev and his Tambov Seminar implied the creation of the theory of differential equations with deviating argument. This theory became a basis of the contemporary Theory of Functional Differential Equations which was worked out by the members of Perm Seminar under the leadership of Azbelev. In 1991 the Publishing House Nauka, Moscow, published N. Azbelev's book (with V. Maksimov and L. Rakhmatullina) "Introduction to the Theory of Functional Differential Equations". It can be said that up to now this monograph remains a reference book for specialists in the theory of FDE. The further development of the FDE theory was treated thoroughly in eight books, four of them in English. On his last day N. Azbelev dealt with the galley proof of a new book. Now this theory covers many classes of equations containing the ordinary derivatives of the solution function. Of special importance are the contributions of N. Azbelev to creation and development (jointly with L. Rakhmatullina) of the theory of Abstract FDE, further generalization of the equations with ordinary derivatives, covering wide classes of n-th order FDEs, systems with impulses, singular equations. It is worth noting that this theory has become a very useful tool for solving some variational problems, especially in the cases when the problem of minimization of a functional is unsolvable within the framework of the classical calculus of variations, as well as for the study of boundary value problems with arbitrary finite number of boundary conditions in the form of equalities and inequalities.

3 General Education and Science Activity

N.Azbelev's influence was not limited to the original and fundamental contributions to the theory of integral and functional differential equations. A characteristic feature of N. Azbelev's activity was his ability to unite around himself colleagues and all those who where enthusiastic about Science. He significantly contributed to the education of young mathematicians, supervised over 60 Candidates and 10 Doctors of Sciences. In the 60's he became a founder of mathematical schools for gifted children in Izhevsk, Russia. N. Azbelev was a member of editorial boards of "Differentsial'nye Uravneniya" (for more than 25 years), "Nonlinear Dynamics and System Theory", "Memoirs on Differential Equations and Mathematical Physics","Functional Differential Equations" and the editor-in-chief of periodical interuniversity proceedings of scientific works "Functional-differential equations" and "Boundary Value Problems" (Perm, Russia).

Professor Azbelev received many honors and awards in the course of his career. He was awarded orders and medals, recognized as a Meritorious Science Worker of the Russian Federation, awarded the Grant of the Russian Federation President for Leading Scientist, selected as a Georg Soros Emeritus Professor, conferred the title of Honored member of the Academy of Nonlinear Sciences.

4   List of Monographs by N.V. Azbelev

[1] Introduction to the Theory of Functional Differential Equations, Moscow: Nauka, 1991, 280 p. (in Russian, with V.P. Maksimov, L.F. Rakhmatullina)

[2] Introduction to the Theory of Linear Functional Differential Equations, Atlanta: World Federat. Publ. Company, 1995, 213 p. (with V.P. Maksimov, L.F. Rakhmatullina).

[3] Theory of Linear Abstract Functional Differential Equations and Applications// Memoirs on Differential Equations and Mathematical Physics, 8 (1996), pp. 1-102. Tbilisi: Publishing House GCI, 1996 (with L.F. Rakhmatullina).

[4] Methods of the Contemporary Theory of Linear Functional Differential Equations, Moscow-Izhevsk: Regular and Chaotic Dynamics, 2000, 300 p. (in Russian, with V.P. Maksimov, L.F. Rakhmatullina)

[5] Stability of Solutions to Equations with Ordinary Derivatives, Perm: Perm State University, 2001, 230 p. (in Russian, with P.M. Simonov)

[6] Elements of the Contemporary Theory of Functional Differential Equations. Methods and Applications, Moscow: Institute of Computer-Assisted Study, 2002, 384 c. (in Russian, with V.P. Maksimov, L.F. Rakhmatullina)

[7] Stability of Differential Equations with Aftereffect, London: Taylor and Francis Publishing Group, 2002, 222 p. (with P.M. Simonov)

[8] Functional Differential Equations and Variational Problems, Moscow-Izhevsk: Regular and Chaotic Dynamics, Institute of Computer-Assisted Study, 2006, 122 p. (with S.Yu. Kultyshev, V.Z. Tsalyuk)

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5   Selected Articles

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