Oscillation of Solutions and Behavior of the Nonoscillatory Solutions of Second-order Nonlinear Functional Equations

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Received: July 15, 2008; Revised: June 5, 2009

Abstract: The aim of this study is to present new oscillation theorems for certain classes of second-order nonlinear functional differential equations of the type

\[ x''(t) + p(t)f(x(t), x(\tau(t))) = 0, \quad (*) \]
\[ x''(t) + p_1(t)f_1(t, x(t), x'(t))x'(t) + q(t)g_1(x(\tau(t))) = 0, \quad t \in [t_0, \infty), \quad t_0 > 0. \]

In the study of Eq. (\(*\)), no sign condition on \(p(t)\) is explicitly assumed. Also, we study the behavior of the nonoscillatory solution of Eq. (\(*\)).

Keywords: nonlinear; functional differential equations; oscillatory solution; nonoscillatory solution.

Mathematics Subject Classification (2000): 34K11, 34K12, 34C10.

1 Introduction

Over the last three decades, many studies have dealt with the oscillation theory for functional differential equations. For an excellent bibliography and later developments of this theory, we refer to the books by Agarwal, Bohner and Wan–Tong Li [1], Erbe, Kong and Zhang [3], Gopalsamy [4], Györi and Ladas [6], Ladde, Lakshmikantham and Zhang [10]. In this note, we consider the second-order nonlinear functional differential equations of the form

\[ x''(t) + p(t)f(x(t), x(\tau(t))) = 0, \quad (1.1) \]
\[ x''(t) + p_1(t)f_1(t, x(t), x'(t))x'(t) + q(t)g_1(x(\tau(t))) = 0, \quad t \in [t_0, \infty), \quad (1.2) \]

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