Dominant and Recessive Solutions of Self-Adjoint Matrix Systems on Time Scales

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Abstract: In this study, linear second-order self-adjoint delta-nabla matrix systems on time scales are considered with the motivation of extending the analysis of dominant and recessive solutions from the differential and discrete cases to any arbitrary dynamic equations on time scales. These results emphasize the case when the system is non-oscillatory.

Keywords: time scales; self-adjoint; matrix equations; second-order; non-oscillation; linear.


1 Introduction

To motivate this study of dominant and recessive solutions, consider the self-adjoint second-order scalar differential equation

\[(px')'(t) + q(t)x(t) = 0.\]

According to the classical formulation by Kelley and Peterson [1, Section 5.6], a solution \(u\) is recessive at \(\omega\) and a second, linearly-independent solution \(v\) is dominant at \(\omega\) if the conditions

\[
\lim_{t \to \omega^-} \frac{u(t)}{v(t)} = 0, \quad \int_{t_0}^{\omega} \frac{1}{p(t)u^2(t)} dt = \infty, \quad \int_{t_0}^{\omega} \frac{1}{p(t)v^2(t)} dt < \infty
\]

all hold; see also a related discussion for three-term difference equations in Ahlbrandt [2], Ahlbrandt and Peterson [3, Section 5.10], Ma [4], and scalar dynamic equations in Bohner

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