Some Linear and Nonlinear Integral Inequalities on Time Scales in Two Independent Variables

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Abstract: We establish some linear and nonlinear integral inequalities of Gronwall–Bellman–Bihari type for functions with two independent variables on general time scales. The results are illustrated with examples, obtained by fixing the time scales to concrete ones. An estimation result for the solution of a partial delta dynamic equation is given as an application.

Keywords: integral inequalities; Gronwall–Bellman–Bihari inequalities; time scales; two independent variables.

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1 Introduction

Inequalities have always been of great importance for the development of several branches of mathematics. For instance, in approximation theory and numerical analysis, linear and nonlinear inequalities, in one and more than one variable, play an important role in the estimation of approximation errors [12].

Time scales, which are defined as nonempty closed subsets of the real numbers, are the basic but fundamental ingredient that permits to define a rich calculus that encompasses both differential and difference tools [8, 9]. At the same time one gains more (cf., e.g., Corollary 3.1). For an introduction to the calculus on time scales we refer the reader to [6] and [4, 5], respectively for functions of one and more than one independent variables.

Integral inequalities of Gronwall–Bellman–Bihari type for functions of a single variable on a time scale can be found in [2, 3, 7, 11, 14]. To the best of the authors knowledge, no such results exist in the literature of time scales when functions of two independent variables are considered. It is our aim to obtain here a first insight on this type of inequalities.

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