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Oscillation Criteria for Half-Linear Delay Dynamic Equations on Time Scales

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Abstract: This paper is concerned with oscillation of the second-order half-linear delay dynamic equation

$$(r(t)(x^{\Delta})^{\gamma})^{\Delta} + p(t)x^{\gamma}(\tau(t)) = 0,$$

on a time scale $\mathbb{T}$, where $\gamma \geq 1$ is the quotient of odd positive integers, $p(t)$, and $\tau : \mathbb{T} \to \mathbb{T}$ are positive rd-continuous functions on $\mathbb{T}$, $r(t)$ is positive and (delta) differentiable, $\tau(t) \leq t$, and $\lim_{t\to\infty} \tau(t) = \infty$. We establish some new sufficient conditions which ensure that every solution oscillates or converges to zero. Our results in the special cases when $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{N}$ involve and improve some oscillation results for second-order differential and difference equations; and when $\mathbb{T} = h\mathbb{N}$, $\mathbb{T} = q^{\mathbb{N}_0}$ and $\mathbb{T} = \mathbb{N}^2$ our oscillation results are essentially new. Some examples illustrating the importance of our results are also included.

Keywords: *oscillation; delay half-linear dynamic equations; time scales.*

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