A Comparison in the Theory of Calculus of Variations on Time Scales with an Application to the Ramsey Model

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Abstract: The purpose of this paper is to provide a comparison between the existing results on the calculus of variations with the ∆ and ∇ operators on time scales. We will also prove the theorems pertaining to free boundary conditions, for dynamic models missing one or both end points conditions. To illustrate our results we shall use a well known Ramsey model and an adjustment model in economics.

Keywords: time scales; calculus of variations; nabla derivative; delta derivative; dynamic model.


1 Introduction

The theory of calculus of variations on time scales has been developed in two directions, one with the Δ operator and one with the ∇ operator. It is possible to write one derivative in terms of the other derivative operator on time scales under certain continuity assumptions [4, Theorem 8.49]. On the other hand, it is not always possible to optimize the dynamic model on time scales by using the deterministic optimization method, namely calculus of variations, since the theory has been developed, to the authors’ knowledge, for functionals of the form

\[ \int_{[a,b] \cap T} L(t, y(\sigma(t)), y(\Delta t)) \Delta t \quad \text{and} \quad \int_{[a,b] \cap T} N(t, y(\rho(t)), y(\nabla t)) \nabla t. \]

As a result of this, here are some questions which need to be answered.

- Which is more advantageous using the ∆ or ∇ derivative in dynamic modelling?