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Aquifer Parameter Identification with Hybrid Ant Colony System

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Abstract: A new approach to parameter estimation in groundwater hydrology is developed using hybrid ant colony system with simulated annealing. Based on the information from the observed water heads and calculated water heads, an objective function for inverse problem is proposed. The inverse problem of parameter identification is formulated as an optimization problem. Simulated annealing has the ability of probabilistic hill-climbing and is combined with ant colony system to produce an adaptive algorithm. A hybrid ant colony optimization is presented to identify the transmissivity and storage coefficient for a two-dimensional, unsteady state groundwater flow model. The ill-posedness of the inverse problem as characterized by instability and non-uniqueness is overcome by using computational intelligence. As compared with the gradient-based optimization methods, hybrid ant colony system is a global search algorithm which can find parameter set in a stable manner. A numerical example is used to demonstrate the efficiency of hybrid ant colony system.

Keywords: *Ant colony system; parameter identification; inverse problem; simulated annealing.*

Mathematics Subject Classification (2000): 65N21.

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1 Introduction

Parameter identification, or model calibration, is a critical step in the application of mathematical models in hydrologic sciences. Unfortunately, parameter identification is an inherently difficult process and, as an inverse problem, it is plagued by the well-documented problems of nonumiquenes, nonidentificability and instability [1]. Numerous optimization techniques have been used to solve groundwater remediation design and parameter identification problems. In a parameter identification problem, the objective function can be the weighted difference between the observed and calculated values at certain observation points in the aquifer. The identified parameters can be hydraulic conductivity or other aquifer parameters, such as storage coefficient [2]. In recent years, global optimization methods are being increasingly used to solve groundwater remediation design and parameter identification problems. These methods include simulated annealing, genetic algorithm, tabu search and ant colony system. As compared with gradient based local search methods, global optimization methods do not require the objective function to be continuous, convex, or differentiable. They have also shown other attractive features such as robustness, ease of implementation, and the ability to solve many types of highly complex, nonlinear problems. One common drawback of these global optimization methods is that many objective function evaluations are typically required to obtain optimal or near-optimal solutions [3]. Karimi investigated the problem of robust dynamic parameter-dependent output feedback (RDP-DOF) stabilization under H1 performance index for a class of linear time invariant parameter-dependent (LTIPD) systems with multi-time delays in the state vector and in the presence of norm-bounded non-linear uncertainties [4]. Li proposed a new interpretation to solve the inverse heat conduction problem using hybrid genetic algorithm. In order to identify parameters of non-linear heat transfer efficiently and in a robust manner, the hybrid genetic algorithm, which combines genetic algorithm with simulated annealing and the elitist strategy, is presented for the identification of the material thermal parameters [5]. Lou studied the robust stability of nonlinear uncertain neural networks with constant or time-varying delays. An approach combining the Lyapunov-Krasovskii functional with the linear matrix inequality is taken to study the problems [6]. Yu proposed an online learning algorithm for feedforward neural networks (FNN) based on the optimized learning rate and adaptive forgetting factor for online financial time series prediction [7]. The ant colony system is a kind of natural algorithm inspired by behavior or processes presented in nature. Ant colony system has been widely used in the traveling salesman problem, job-shop scheduling problem and quadratic assignment problem. When compared with traditional first-order methods, the ant colony system is recognized to have a better capability to find the global optimum solution. The objective of this paper is to present a new method based on hybrid ant colony system for obtaining the parameters of a linear groundwater flow model.

2 Calculation of Groundwater Flow Models

The partial differential equation for groundwater flow, assuming constant fluid density and viscosity, can be expressed as follows

$$\nabla \cdot (K \nabla h) + w = S_s \partial h / \partial t, \tag{1}$$

where h is the hydraulic head, K is the hydraulic conductivity tensor, w is the fluid sink/source term, and S_s is the specific storage coefficient of the aquifer. The first kind boundary condition is expressed as follows:

$$h(x,y,z)|_{\Gamma_1} = h_0(x,y,z), \tag{2}$$

where $h_0(x,y,z)$ is the already known head. The second kind boundary condition is written as follows:

$$Q_0(x,y,z)|_{\Gamma_2} = k_x \frac{\partial h}{\partial x} l_x + k_y \frac{\partial h}{\partial y} l_y + k_z \frac{\partial h}{\partial z} l_z, \tag{3}$$

where $Q_0(x,y,z)$ is the drainage already known, l_x, l_y and l_z are the direction cosines of the exterior normal of the boundary in x, y and z direction, respectively. And k_x, k_y and k_z are permeability coefficient in x, y and z direction, respectively. When the boundary condition and the predicted permeability coefficient are determined, the finite element equation is adopted to compute the distribution of the water head and the drainage in the whole seepage field, which provides modal data to the analysis of the inversion problem of the permeability coefficient. The numerical methods are based on spatial and temporal discretization which divides the continuous space and time domains into a network of discrete nodal points and a series of finite time intervals. When the various aquifer parameters are known, the hydraulic heads at any nodal points and time intervals can be obtained using finite-element code.

3 Classical Ant Colony System

Ant colonies have always fascinated human beings. Social insects, such as ants, bees, termites and wasps, often exhibit a collective problem-solving ability [8].The ant colony system is first applied to the traveling salesman problem. In Ant System, the traveling salesman problem is expressed as a graph (N, E), where N is the set of towns and E is the set of edges between towns. The objective of the traveling salesman problem is to find the minimal length closed tour that visits each town once. Each ant is a simple agent to fulfill the task. It obeys the following rules: 1) It chooses the next town with a probability which is a function of the town distance and of the amount of trail present on the connecting edge; 2) before a tour is completed, it can not choose the already visited towns; 3) when it completes a tour, it lays a substance called trail on each edge (i,j) visited; 4) it lives in an environment where time is discrete.

It must choose the next town at time t, and be there at time $t+1$. Let m, n be the total number of ants and towns. An iteration of the Ant system is called, as the m ants all carry their next moves during time interval $(t,\ t+1)$. The n iterations constitute a cycle. In one cycle, each ant has completed a tour. Let $\tau_{ij}(t)$ denote the intensity of trail on edge (i,j). After a cycle, the trail intensity is updated as [9]:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \Delta\tau_{ij}, \tag{4}$$

where ρ is a coefficient, and $1-\rho$ represents the evaporation of trail between times t and $t+1$

$$\Delta\tau_{ij} = \sum_{k=1}^{Z} \Delta\tau_{ij}^k, \tag{5}$$

where $\Delta\tau_{ij}$ is the quantity per unit of length of trail substance placed on path(i,j) by the k-th ant between times t and $t+1$ [10]

$$\Delta\tau_{ij}^k = \begin{cases} Q/J_k, & path(i,j) \quad is \quad selected \\ 0, & otherwise \end{cases}, \tag{6}$$

where Q is a constant related to the quantity of trail laid by ants. Ants build solutions using a probabilistic transition rule. The probability $p_{ij}^k(t)$ with which ant k in town i at iteration t chooses the next town j to move to is a function of the heuristic function of the desirability η_{ij} and the artificial pheromone trail $\tau_{ij}(t)$:

$$p_{ij}^k(t) = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum\limits_{l=1}^{L} [\tau_{ij}]^\alpha [\eta_{ij}]^\beta}, \tag{7}$$

where α, β are adjustable constants, which can weigh the relative importance of pheromone trail and of objective function. A reasonable heuristic function is written as follows:

$$\eta_{ij} = \frac{1}{J_k}, \tag{8}$$

where J_k is the objective function of k-th ant path. Ant colony system could not perform well without pheromone evaporation. From Eq. (7), it is obvious that the transition probability is proportional to the visibility and the trail intensity at time t. The visibility shows that the closer towns have a higher probability of being chosen. The mechanism behind this is a greedy constructive heuristic. While the trail intensity shows that the more trail on edge (i, j), the more attractive it is. The process can be characterized by a positive feedback loop, in which an ant chooses a path thus reinforces it. In order to constrain the ants not to visit a previous visited town, a data structure called the tabu list is associated with each ant. All the visited towns are saved in it. When an ant finishes a cycle, the tabu list is then emptied and the ant is free again to choose. Let tabu $_k$ denote the tabu list of the k-th ant.

4 Parameter Estimation Approach in Groundwater Hydrology Using Hybrid Ant Colony System

4.1 Solution definition of inverse problem and its ill-posedness

The parameter identification problem can be formulated to find the model parameters by adjusting m until the measured data match the corresponding data computed from the parameter set in a least-squares fashion. The objective function is defined as follows [11]

$$J(m) = [h_m - h_c(m)]^T w [h_m - h_c(m)], \tag{9}$$

where h_m is the measured displacement vector; h_c is the computing displacement vector, which is related to the identified parameter vector m. w is weighting matrix in order to take into account the different observed equipments for the water head measurements. This objective function clearly depends on the measured data and the parameters of model.

Figure 4.1 shows the groundwater flow model, in which the four observing points for water heads are set in order to get measurement data and to identify the aquifer parameters. The objective function can become complex as shown in Figure 4.1, such as non-convex, or even multi-modal if errors contained in the model equation or /and errors in the measurement data are large. The multi local minima can be found from Figure 4.1. In such a case, the solution may vibrate or diverge when conventional gradient-based optimization methods are used, which gives rise to the necessary for a robust optimization method such that a stable convergence is always achieved.

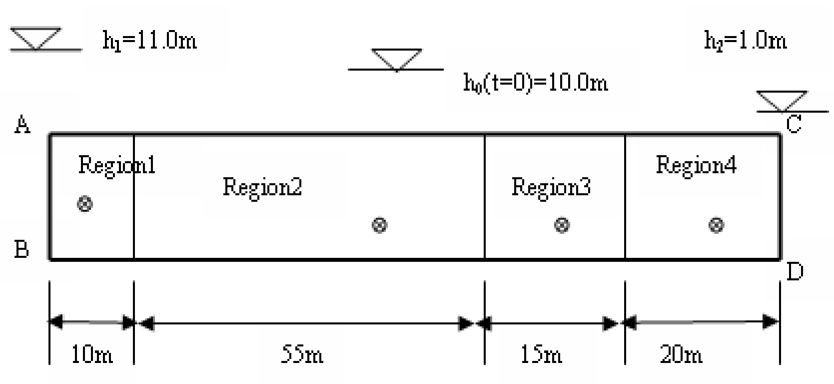

Figure 4.1: Configuration of the two-dimensional groundwater flow model.

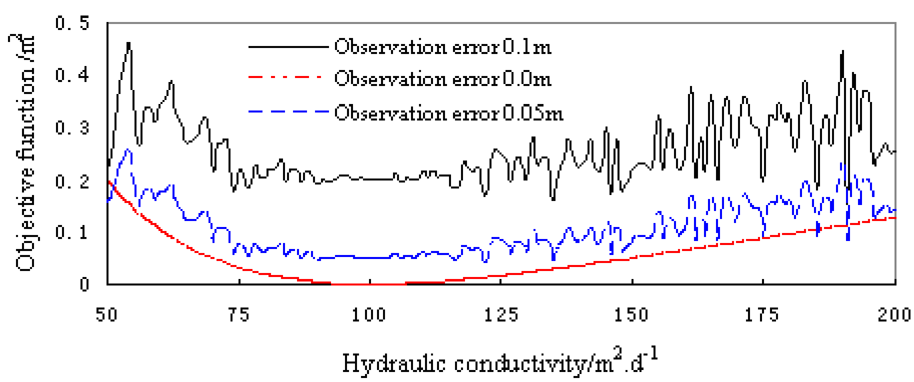

Figure 4.2: Configuration of the two-dimensional groundwater flow model.

The solution of the inverse problem consists in obtaining a minimum of an objective function which is defined taking into account the mathematical structure of the material model and asset of experimental data. This generally results in a non-linear programming constrained problem of the form [12]:

$$\min\{J(m, h_m)\ ,\quad m \in R^{\ P}, h_m \in R^M;\ g^j < 0\}, \tag{10}$$

where m represents the variable vector, which belongs to the space of admissible parameters R^P, h_m the vector of measured data, which belongs to the space R^M. g^j are inequality constraints, which define the feasible domain S:

$$S = \{m \in R^P, g^j < 0\}. \tag{11}$$

The constraints can represent physical links between the primary physical variables and the model parameters, information concerning the values of parameters and conditions to guarantee that all mathematical functions involved can be defined and calculated. In the optimization process, the difference between the experimental result and the theoretical prediction is measured by a norm value, here referred to as the individual norm. The individual norms of the tests form an objective function $F(x)$ which then gives a scalar measure of the error between the experimental observations and the model predictions. From mathematical point of view, the optimization problem involves the minimization of the objective function [13]:

$$J(m) \to \min, \tag{12}$$

where m is a vector containing the optimization variables (here model parameters) with the bound constraints:

$$m_l < m < m_u, \tag{13}$$

where m_l and m_u are the lower and upper bounds of m respectively.

4.2 Simulated annealing algorithm for neighborhood search

Simulated annealing is another important algorithm which is powerful in optimization and high-order problems. It uses random processes to help guide the form of its search for minimal energy states. Simulated annealing is a generalization of a Monte Carlo method for examining the equations of state and frozen states of n-body systems. The concept is based on the manner in which liquids freeze or metals recrystalize in the process of annealing [14]. In an annealing process a melt, initially at high temperature and disordered, is slowly cooled so that the system at any time is approximately in thermodynamic equilibrium. As cooling proceeds, the system becomes more ordered and approaches a "frozen" ground state at T=0. Hence the process can be thought of as an adiabatic approach to the lowest energy state. If the initial temperature of the system is too low or cooling is done insufficiently slowly the system may become quenched forming defects or freezing out in meta-stable states, that is, trapped in a local minimum energy state. Simulated annealing is a very general optimization method which stochastically simulates the slow cooling of a physical system. The idea is that there is a objective function F, which associates an objective function with a state of the system, a temperature T, and various ways to change the state of the system. The algorithm works by iteratively proposing change and either accepting or rejecting each change. Having proposed a change we may evaluate the change δ in F. The proposed change may be accepted or rejected by the Metropolis criterion; if the objective function decreases ($\delta J < 0$), the change is accepted unconditionally; otherwise it is accepted but only with probability $exp(-\delta J/T)$. For given old solution, a new solution can be created as follows [15]:

$$m_{new} = m_{old} + \Delta m, \tag{14}$$

where Δm is a random perturbation of solution. The accepted probability of the new solution, p_{new}, will be expressed:

$$p_{new} = \begin{cases} 1, & J_{old} \geq J_{new} \\ \exp[-\delta J/T_k], & J_{old} < J_{new} \end{cases}, \tag{15}$$

where $\delta J = J_{new} - J_{old}$. Three parameters essential for implementation of the simulated annealing algorithm are as follows: 1) initial value of the control parameter T_i, 2) the number of perturbations generated at each T, and 3) the decrement of the control parameter T. These parameters affect the speed of the algorithm and the quality of the final solution. A simple approach is to choose a value for T_i, that allows a large percentage of non-improving solutions to be accepted [14]. The number of solutions generated at each T is selected to allow equilibrium to take place before decreasing T. The decrement of T is chosen such that it allows only small changes in the value of T. The equation used for decreasing T is expressed as follows:

$$T_{k+1} = \xi T_k, \tag{16}$$

where $\xi = 0.9$ is a typical selection. A crucial requirement for the proposed changes is reachability or ergodicity that there be a sufficient variety of possible changes that one can always find a sequence of changes so that any system state may be reached from any other. When the temperature is zero, changes are accepted only if F decreases, an algorithm also known as hill-climbing, or more generally, the greedy algorithm. The initial temperature can be determined as [16]

$$T_i = -\frac{1}{\ln p_i}, \tag{17}$$

where p_i is the desired initial acceptable probability. It is usually between 0.7 and 0.9. similarly, the final temperature can be determined as

$$T_f = -\frac{1}{\ln p_f}, \tag{18}$$

where p_f is the desired final acceptable probability. It is usually very close to zero. The system soon reaches a state in which none of the proposed changes can decrease the objective function, but this is usually a poor optimum. In real life, we might be trying to achieve the highest point of a mountain range by simply walking upwards; we soon arrive at the peak of a small foothill and can go no further. On the contrary, if the temperature is very large, all changes are accepted, and we simply move at random ignoring the cost function. Because of the reachability property of the set of changes, we explore all states of the system, including the global optimum. The system evolves until a stop criterion is reached.

Very fast simulated annealing scheme proposed by Ingber is applied to produce new solution [17]

$$\Delta m_{\text{i}} = \eta_{\text{i}}(m_{\text{i}max} - m_{\text{i}min}), \tag{19}$$

$$\eta_i = sign(u_i - 0.5)T_i[(1 + \frac{1}{T_i})^{|2u_i-1|} - 1], \tag{20}$$

where m_{max} and m_{min} represent up and down bounds for parameters respectively; u_i is a random value in [-1,1) domain, the value of η_i is just located in [-1,1). The whole process for parameter identification using simulated annealing is shown as follows: Step 1: Initial parameters(initial T and temperature descent rate α) are fixed; Step 2: Initial solution is generated and the corresponding J is calculated; Step 3: The system solution is updated according to the mechanism designed; Step 4: Parameter T is modified according to the descent rate established; Step 5: One comes back to the step 3 to calculate the next solution from the current one up to T or ε reaches a value fixed beforehand; Step 6: The best solution visited is written as last solution of inverse problem.

4.3 Hybrid ant colony system for parameter identification

Ant colony system is global search techniques for optimization. However, it is poor at hill-climbing. Simulated annealing has the ability of probabilistic hill-climbing. Therefore, the two techniques are combined here to produce a new algorithm that has the merits of both ant colony system and simulated annealing, by introducing a local search. A new hybridization of ant colony system with simulated annealing is proposed. The main concept of inverse problem of parameter identification with the hybrid ant colony system can be summarized in the following steps: Step 1: depict each unknown parameter by an interval based on available prior information; Step 2: discretize each interval into a number of strata, let the middle of each stratum represent that stratum; Step 3: run the simulation model of choice for all, or a randomly selected subset of all the possible parameter combinations; Step 4: evaluate each stratum on the basis of the smallest value of the objective function in such a way that small values of the objective function receive higher scores; Step 5: Produce new individual based on SA neighborhood structures; Step 6: Accept new individual based on SA accepted probability; Step 7: on the basis of the value of the objective function, place a certain amount of trail(pheromone in the case of real ants) on each stratum visited along its pathway; Step 8: Decrease temperature of SA according to decreasing scheme; Step 9: repeat step 4 to step 8 until some convergence criterion is satisfied.

In order to study the performance of the proposed strategy, extensive numerical experimentations have been performed with Schaffer's function. The Schaffer's function is expressed as follows

$$f(x,y) = 0.5 - \frac{\sin^2(x^2+y^2)^{0.5} - 0.5}{(1+0.001(x^2+y^2))^2}, \qquad -4 < x, y < 4. \tag{21}$$

Figure 4.3 shows the fundamental structure of hybrid ant colony system with simulated annealing.

The Schaffer's function shape is shown in Figure 4.3 and searching values with HANC is listed in Table 4.1. The convergence processes of objective function with different algorithms are shown in Figure 4.3.

To test the applicability and efficiency of the hybrid ant colony system, a two dimensional flow problem is considered, as shown in Figure 4.1. The aquifer is bound by two constant-head boundaries with the initial heads both at 11m. at $t > 0$, the head at right boundary is instantaneously lowered to 10m. the transient head distribution is simulated

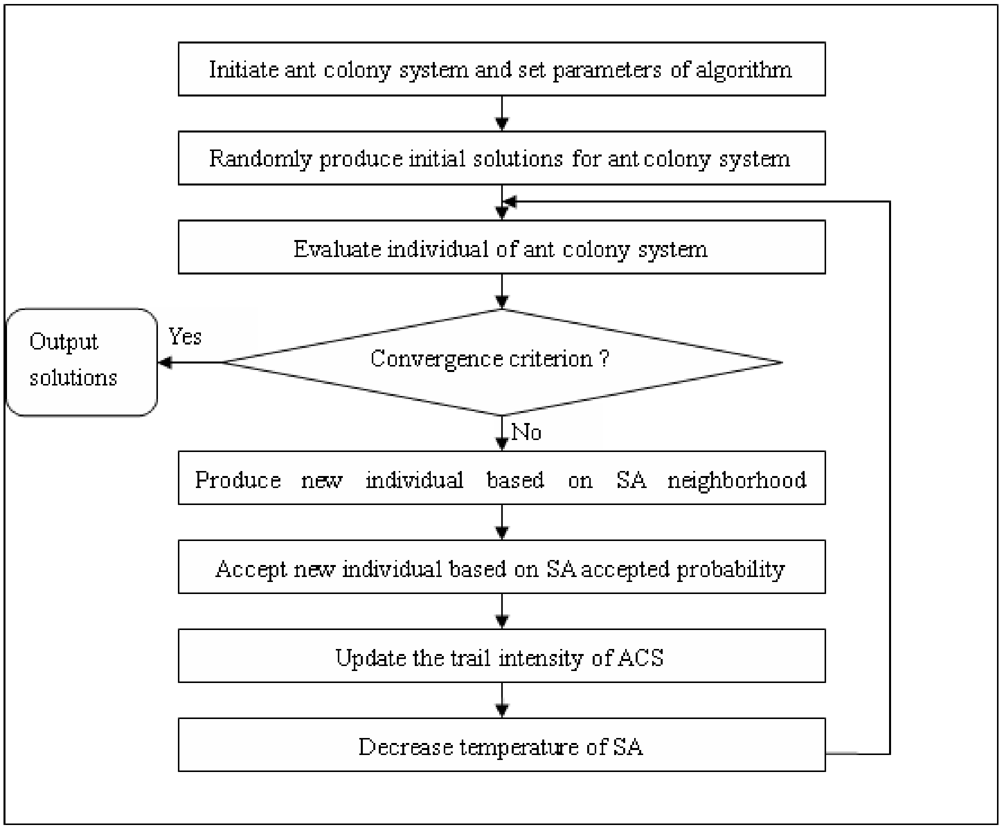

Figure 4.3: Fundamental structure of hybrid ant colony system with simulated annealing.

Theoretical values			Searching values with HANC		
x	y	f	x	y	f
0.00	0.00	1.00	0.00	-0.00	0.9999

Table 4.1: Computational values of Schaffer's function with hybrid ant colony system.

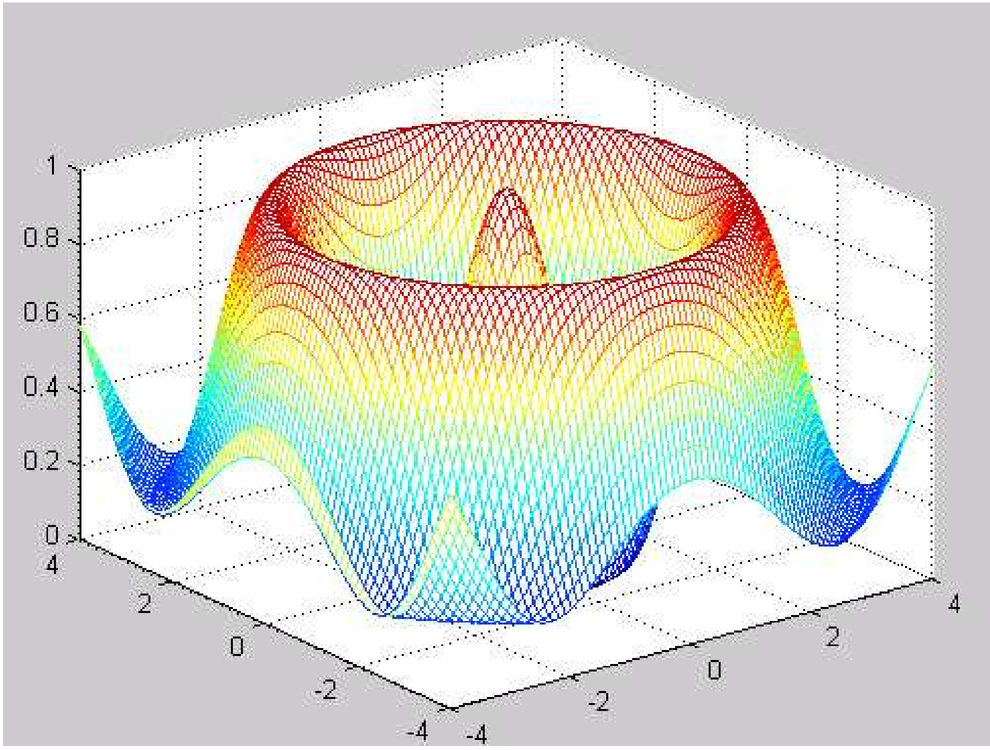

Figure 4.4: Schaffer's function shape.

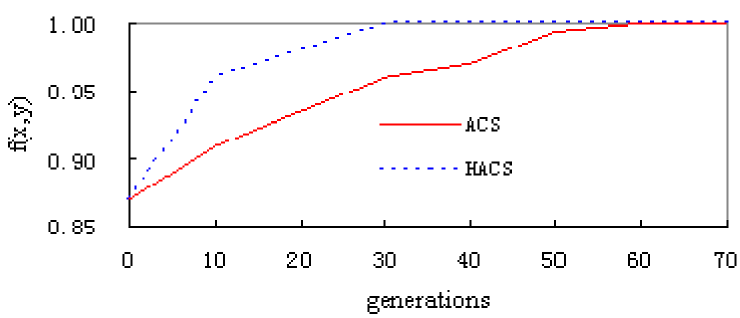

Figure 4.5: Search processes of Schaffer's function.

using finite element method with a time step of 0.1 day. Hydraulic conductivity and specific storage coefficient are listed in Table 4.2. Table 4.3 records measured water head data at different points at different times.

Parameters	k_1 / m^2·d^{-1}	k_2 / m^2·d^{-1}	k_3 / m^2·d^{-1}	k_4 / m^2·d^{-1}	S_s
Values	2000.0	100.0	10.0	1000.0	0.02

Table 4.2: Hydraulic conductivity and specific storage coefficient.

Observation time/d	Observation point 1# /m	Observation point 2#/m	Observation point 3#/m	Observation point 4#/m
0.1	10.687	9.9798	9.1706	3.640
0.2	10.728	9.8074	8.396	3.374
0.3	10.697	9.5376	7.756	3.121
0.4	10.635	9.198	7.028	1.584
0.5	10.560	8.835	6.398	1.423

Table 4.3: Measured water head data at different points at different times.

According to the flow mathematical model with finite element method and measured water-head data, the aquifer parameters are identified with hybrid ant colony system with simulated annealing. Figure 4.3 shows the influence of initial temperature of simulated annealing algorithm on convergence process.

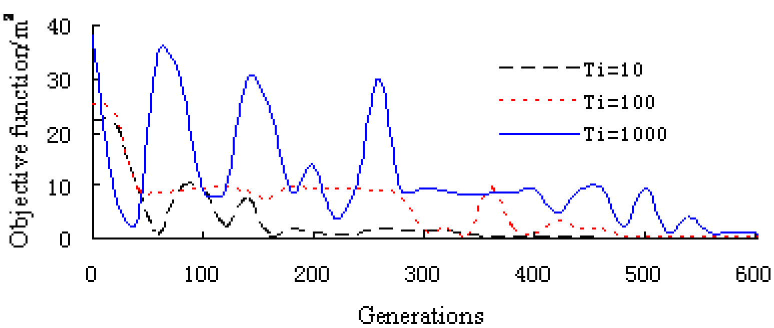

Figure 4.6: Influence of initial temperatures on convergence process.

In order to simulate observation errors, the measured water heads can be simulated by adding a random error to the theoretical values

$$h_m^* = h_m + sign(R - 0.5) \times \Delta h, \tag{22}$$

where h_m^* are the measured data with observation errors, h_m are the measured data without observation errors. $Sign$ is the sign function, and R is a random variable in the interval [0,1], Δh is an observation error. Comparison of identified hydraulic conductivity and specific storage coefficient with theoretical values is listed in Table 4.4. Figure 4.3 shows the convergence process of objective function with classical ant colony system and hybrid ant colony system.

Model parameters	$k_1/m^2 \cdot d^{-1}$	$k_2/m^2 \cdot d^{-1}$	$k_3/m^2 \cdot d^{-1}$	$k_4/m^2 \cdot d^{-1}$	S_s
Theoretical values	2000.0	100.0	10.0	1000.0	0.02
Identified values by HACS	2010.3	99.7	10.2	1009.0	0.018
Identified values by ACS	2022.7	110.2	9.95	1012.5	0.025

Table 4.4: Comparison of identified hydraulic conductivity and specific storage coefficient with theoretical values.

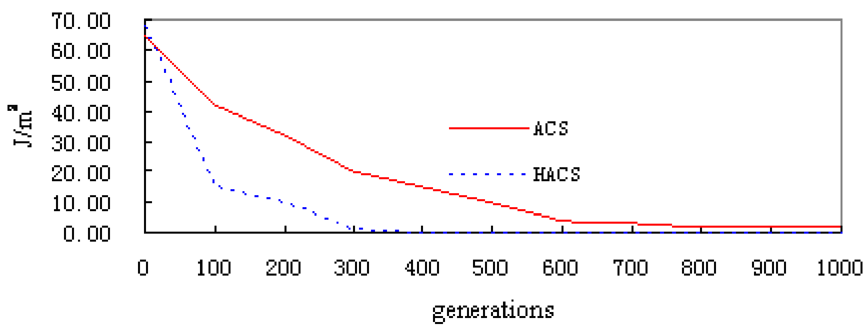

Figure 4.7: Convergence process of objective function with different searching method.

5 Practical Applications of Inversion Algorithm

Baishan Hydropower Station, as shown in Figure 5.1, is located in the Second Songhuajiang River in Jilin province, China. It consists of a 149.5-meter-high concrete heavy-pressure dam, a weir with four 12×13 meter tunnels on top of the 404-meter-high spillway dam, three 6×7 meter tunnels for discharging water are 350 meters high, an underground powerhouse with an installed generating capacity of 900,000 KW and another powerhouse on the surface with an installed generating capacity of 600,000 KW. The dam is 423.5 meters high and the reservoir has a storage capacity of 6.812 billion cubic meters. Its highest normal storage water level is 413 meters. The capacity for water control storage is 3.54 billion cubic meters while the flood control storage capacity is 950 million cubic meters. Cross-section of Baishan dam at block 18 is shown in Figure 5.2. Figure 5.3 shows the disposition of observation holes for dam uplift pressure at block 18.

In order to identify the permeability coefficients of rock foundation, the three-dimensional finite element model for seepage calculation is carried out. The seepage fields of the dam and its rock foundation at different load cases are computed. According to the prior information of pumping water test in field, the domains of identification parameters are determined. The training sample pairs are got basing on finite element

Figure 5.1: Baishan Hydropower Station.

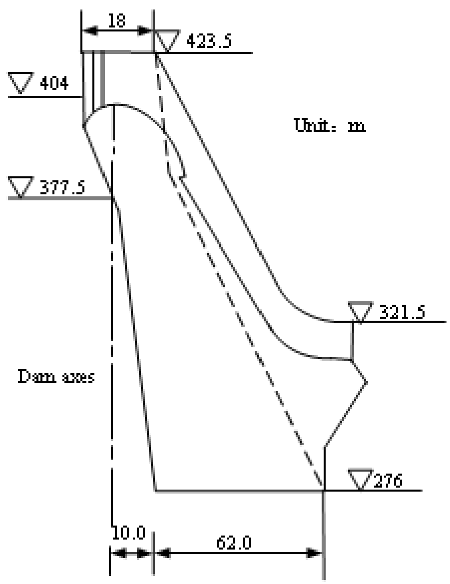

Figure 5.2: Cross-section of Baishan dam at block 18.

Measuring date	Upstream water elevation	Downstream water elevation	water head h_1	water head h_2	water head h_3	water headh_4
19980910	413.00	290.80	291.82	283.59	284.74	281.72

Table 5.1: Measured data of water heads in the observation holes.

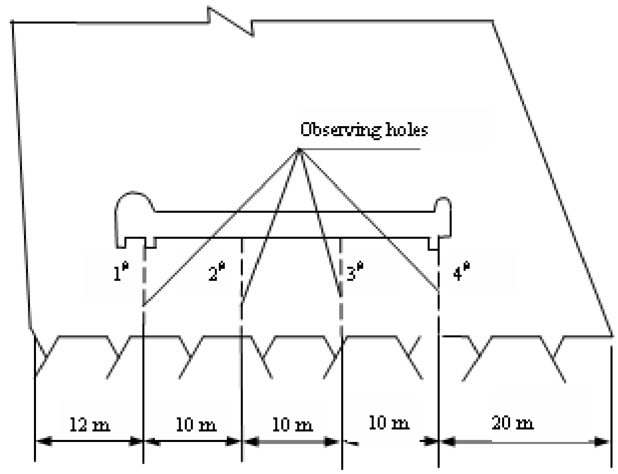

Figure 5.3: Disposition of observation holes for dam uplift pressure at block 18.

Rock foundation (I) $k_1/10^{-9}m \cdot s^{-1}$	Concrete certain $k_2/10^{-9}m \cdot s^{-1}$	Rock foundation (II) $k_3/10^{-9}m \cdot s^{-1}$
44.05	6.16	52.80

Table 5.2: Identification results of permeability coefficients.

Measuring date	No.1 measured point	No.2 measured point	No.3 measured point	No.4 measured point
19971015	291.82/291.78	283.54/283.49	284.23/283.19	281.72/281.13
19980108	291.31/291.29	283.59/283.44	282.19/283.15	281.82/282.14
19980901	291.82/291.78	283.59/283.49	284.74/283.19	281.72/282.12
19981012	291.82/291.78	283.49/283.49	284.74/283.19	281.72/282.12

Table 5.3: Comparison between measured and forecasted water heads.
Note: measured water heads/computed water heads.

analysis. The rock foundation is divided into 3 sub-regions, rock base before concrete certain, concrete certain and rock base after concrete certain.

The measured water heads in four holes are recorded in Table 5.1. Based on the measured water heads and finite element model for dam seepage calculation, the permissibility coefficients are identified and listed in Table 5.2. Table 5.3 shows the comparison between measured and forecasted water heads with finite element method according to estimated permissibility coefficients.

6 Conclusion

Hybrid ant colony system for solving the parameter identification problem is proposed. The three characteristics of the ant colony system, such as positive feedback process, greedy constructive heuristic and distributed computation, work together to find the solution to the inverse problems fast and efficiently. However, classical ant colony system is poor at hill-climbing. Simulated annealing has the ability of probabilistic hill-climbing. Therefore, the two techniques are combined here to produce a new algorithm that has the merits of both ant colony system and simulated annealing, by introducing a local search. A new hybridization of ant colony system with simulated annealing is proposed.. Modern heuristic search techniques, such as genetic algorithm, simulated annealing and ant colony system, are well suited for solving the parameter identification problem in groundwater flow model. The gradient based methods are not applicable for this type of inverse problem because of the difficulty in evaluating the function derivatives and the presence of many local minimum points in the objective function. One of the advantages of ant colony system over other optimization methods is that it is easy to implement complex inverse problem.

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