Nonlinear Dynamics and Systems Theory, 8 (4) (2008) 349–358

A Robust Detector for a Class of Uncertain Systems

Maher Hammami [1*] and Ines Ellouze [2]

[1] *Département de Physique, Faculté des Sciences de Sfax*
Rte de Soukra Km 3.5, B.1171 - 3000 Sfax-Tunisie
Fax: (216) 74 274 437
[2] *Département de Mathématiques, Faculté des Sciences de Sfax*

Received: October 30, 2007; Revised: September 22, 2008

Abstract: This paper studies the problem of output feedback stabilization of a class of uncertain systems. We construct a robust detector which provides an approximation of the state of the system. The state trajectory control by state observation for a class of uncertain systems based on output feedback is treated, where the nominal system is linear and the uncertainties are bounded. This work is based on Lyapunov techniques. Furthermore, a numerical example is given to illustrate the applicability of our main result.

Keywords: *Uncertain systems; state observation; output-controller; detector.*

Mathematics Subject Classification (2000): 93D09, 34D20.

1 Introduction

The problem of state trajectory control for nonlinear systems by output feedback is addressed by several authors ([1]–[10]) using several basic methods of studying the stability and constructing stabilizing output controllers.

In this paper, we treat this problem for a class of uncertain systems. The perturbation term could result from errors in modeling the nonlinear system, aging of parameters or uncertainties. In a typical situation, we do not know the uncertainties but we know some information about it. We can no longer study stability of the origin as an equilibrium point, nor should we expect the solution of the uncertain systems to approach the origin as t tends to infinity. The best we can hope that, if the uncertainties are bounded by a small term in some sense, then the solution will be ultimately bounded by a small bound

* Corresponding author: hammami_maher@yahoo.fr

for sufficiently large t. Under some conditions we construct a robust detector (dynamical system which is expected to produce an estimation of the state on the hole space except on a small neighborhood of the origin) as the one introduced by Vidyasager [11]. We study the state trajectory control for non-linear system by output feedback. We obtain Global Uniform Ultimate Boundedness (GUUB) trajectory (see [12]) for the state of the error equation.

Consider the state space model

$$\begin{cases} \dot{x} &= A(\cdot)x + B(\cdot)u, \\ y &= C(\cdot)x, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$, $y \in \mathbb{R}^p$, $A(\cdot)$, $B(\cdot)$ and $C(\cdot)$ depend on some parameter and $(n \times n)$, $(n \times q)$ and $(p \times n)$ matrices respectively. We shall assume that the dimension of the state model is finite. We consider throughout this paper specifically perturbations of the state space from of the plant dynamics (i.e., perturbations of the $A(\cdot)$, $B(\cdot)$ and $C(\cdot)$ matrices). Let A_0, B_0 and C_0 be the linearized nominal model of the plant. The matrices $A(\cdot)$, $B(\cdot)$ and $C(\cdot)$ can be factored as follows

$$A(\cdot) = A_0 + \Delta A,$$

$$B(\cdot) = B_0 + \Delta B,$$

$$C(\cdot) = C_0 + \Delta C.$$

We suppose here the exact knowledge of the state space matrices $(A_0,\ B_0,\ C_0)$. The elements of the matrix A_0 are $\{a_{ij}\}$ while the elements of the matrix $A(\cdot)$ are $\{a_{ij} + \delta_{ij}\}$. In the absence of nonlinearities, the problem is reduced to the linear one.

$$\begin{cases} \dot{x} &= A_0x + B_0u, \\ y &= C_0x. \end{cases} \tag{2}$$

Uncertain systems are an important class of nonlinear systems, several authors are interested to study this kind of systems. In [13] and [14], the authors studied this class of system when the nonlinear part is of the form $Ew + \sigma$, the noise w is described in a general state space form and it also includes the case of state dependent noise. The Ew factor may represent a stochastic parameter variation of the system matrix A while σ represents an external additive perturbation.

2 System and Definitions

Let consider the system (1) which can be described by the following state equations

$$\begin{cases} \dot{x} &= A_0x + B_0u + \Delta Ax + \Delta Bu, \\ y &= C_0x + \Delta Cx, \end{cases} \tag{3}$$

and the following detector

$$\dot{\hat{x}} = A_0\hat{x} + B_0u - L(C_0\hat{x} - y) + \Delta A\hat{x} + \Delta Bu.$$

Let $e = \hat{x} - x$. The error equation is given locally by

$$\dot{e} = (A_0 - LC_0)e + o(e).$$

Since the terms $\|\Delta Ax\|$, $\|\Delta Bu\|$ and $\|\Delta Cx\|$ are locally bounded, then these dynamics are locally exponentially stable provided that the pair (A_0, C_0) is detectable. In [15], [16] the authors studied this class of systems and constructed a global detector in the presence of nonlinear perturbation.

In the general case, we consider the perturbed system (3) which can be described by the following state equations

$$\begin{cases} \dot{x} = & F(t,x,u) & = A_0x + B_0u + \Delta f(t,x,u), \\ y = & h(t,x) & = C_0x + \Delta h(t,x), \end{cases} \tag{4}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^q$, $y \in \mathbb{R}^p$, A_0, B_0 and C_0 are known $(n \times n)$, $(n \times q)$, $(p \times n)$ constant matrices respectively and $\Delta f(t,x,u)$, $\Delta h(t,x)$ are locally Lipschitz continuous represents of the uncertainties in the plant. For our case, in the presence of uncertainties, we give a definition of detectability, where we introduce the notion of a global detector and we will study the state observation law for a class of uncertain systems in the GUUB trajectory sense.

Consider the system

$$\begin{cases} \dot{x} & = & f(t,x), \\ y & = & h(t,x), \end{cases} \tag{5}$$

where $t \in \mathbb{R}^+$, $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^q$ is the control and $y \in \mathbb{R}^p$ is the output of the system. The functions $f : [0,+\infty[\times\mathbb{R}^n \to \mathbb{R}^n$ respectively $h : [0,+\infty[\times\mathbb{R}^n \to \mathbb{R}^p$ are pieceswise continuous in t and globally Lipschitz in x on $[0,+\infty[\times\mathbb{R}^n$.

We now introduce the notions of uniform boundedness and uniform ultimate boundedness of a trajectory of (5) (see [12]).

Definition 2.1 The system (5) is uniformly bounded when

- for all $R_1 > 0$, there exists a $R_2 = R_2(R_1) > 0$ such that for all $x_0 \in \mathbb{R}^n$, for all t_0 and for all $t \geq t_0$
$$\|x_0\| \leq R_1 \Longrightarrow \|x(t)\| \leq R_2.$$

Definition 2.2 The system (5) is uniformly ultimately bounded when

- there exists a $R > 0$ such that for all $R_1 > 0$, there exists a $T = T(R_1) > 0$ such that for all $x_0 \in \Re^n$, for all t_0 and for all $t \geq t_0 + T$
$$\|x_0\| \leq R_1 \Longrightarrow \|x(t)\| \leq R.$$

The above definition means that we have the ultimately bound of the trajectory uniformly on t_0. The classical theorem of Lyapunov proves uniform asymptotic stability of the equilibrium point $x = 0$ of a dynamical system $\dot{x}(t) = f(x(t), t)$ when there exists a positive definite and decrescent Lyapunov function $V(x,t)$ whose derivative $\dot{V}(x,t)$ along the solutions of the system is negative definite. When there exists a $R_V > 0$ such that the derivative $\dot{V}(x,t)$ along the solutions of the system is negative for x with $\|x\| > R_V > 0$.

Definition 2.3 The system (5) is GUUB solution if $\dot{V}$ satisfies the following estimation:

$$\dot{V}(x(t)) \leq -\eta V(x(t)) + r \tag{6}$$

with $\eta > 0$ and $r > 0$.

Remark 2.1 If equation (6) holds then the state of (3) satisfies:

$$\| x(t) \| \quad \leq \quad \| x(t_0) \| \, e^{-\eta(t-t_0)} + \frac{r}{\eta}, \ \forall \ \ t \geq t_0.$$

The problem is to design a continuous detector with input $y(t)$ such that the estimates denoted by $\hat{x}(t)$ converge to $x(t)$ in the ultimate bounded sense (as in the Definition 2.3).

Definition 2.4 (Robust detector). A system

$$\dot{\hat{x}} = G\,(t, \hat{x}, y, u)$$

is called a robust detector for (3) if for all input signals u,

$$\forall \ \ \|\hat{x}(t_0) - x(t_0)\| \ \ \in \ \ \Re^n \backslash B(0, \frac{r}{\eta})$$

one has

$$\|\hat{x}(t) - x(t)\| \quad \leq \quad \lambda_1 \|\hat{x}(t_0) - x(t_0)\| e^{-\eta(t-t_0)} + \frac{r}{\eta}, \ \ \forall \ \ t \geq t_0.$$

$B(0, \frac{r}{\eta})$ denotes the ball of radius $\frac{r}{\eta} > 0$ with $\lambda_1 > 0, r > 0$ and $\eta > 0$. Note that the state of the error equation converges to the ball $B(0, \frac{r}{\eta})$ when t goes to infinity.

3 Robust Detector

We now highlight the major assumptions, with regard to the system given by (3) that are used in the observer stability proof.

$(\mathcal{A}_1)$ The pair (A_0, C_0) is observable, then there exists a matrix L such that the eigenvalues of $(A_0 - LC_0)$ are in the open left-half plane [17]. For all definite positive symmetric matrix Q there exists a definite positive symmetric matrix P such that:

$$(A_0 - LC_0)^T P + P(A_0 - LC_0) = -Q.$$

$(\mathcal{A}_2)$ There exists a function ϕ where $\phi(.,.,.) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^q \rightarrow \mathbb{R}^p$, such that

$$P\Delta f(t, x, u) = C_0^T \phi(t, x, u),$$

where P is the unique positive definite solution to the Lyapunov equation which is given in $(\mathcal{A}_1)$.

$(\mathcal{A}_3)$ There exits a positive scalar function $\delta_1(t)$ such that

$$\|\phi(t, x, u)\| \leq \delta_1(t),$$

where $\|.\|$ denotes the Euclidean norm on $\mathbb{R}^n$.

$(\mathcal{A}_4)$ There exists a function γ where $\gamma(.,.) : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^p$, such that

$$PL\Delta h(t, x) = C_0^T \gamma(t, x),$$

where P is the unique positive definite solution to the Lyapunov equation which is given in $(\mathcal{A}_1)$.

$(\mathcal{A}_5)$ There exists a positive scalar function $\delta_2(t)$ such that

$$\|\gamma(t, x)\| \leq \delta_2(t),$$

where $\|.\|$ denotes the Euclidean norm on $\mathbb{R}^n$.

Theorem 3.1 *If the assumption* $(\mathcal{A}_1)$, $(\mathcal{A}_2)$, $(\mathcal{A}_3)$, $(\mathcal{A}_4)$ *and* $(\mathcal{A}_5)$ *hold, then the system*

$$\dot{\hat{x}} = A_0\hat{x} + B_0u + \varphi(t,\hat{x},y,u) - L(C_0\hat{x} - y),$$

where

$$\varphi(t,\hat{x},y,u) = -\frac{P^{-1}C_0^T(C_0\hat{x} - y)\delta(t)^2}{\|C_0e\|\delta(t) + r_0} \tag{7}$$

with $r_0 > 0$ *and* $\delta(t) = \delta_1(t) + \delta_2(t)$, *is a robust detector for the system (4).*

Proof Let consider the following Lyapunov function $V(e) = e^TPe$ as in the $(\mathcal{A}_1)$. The derivative of this function along the trajectory of the closed-loop system by the output feedback y, or just along the error equation and using equation (4) and (7)

$$\dot{e} = (A_0 - LC_0)e - \Delta f(t,x,u) + \varphi(t,\hat{x},y,u) + L\Delta h(t,x)$$

is given by

$$\dot{V}(e) = -e^TQe - 2e^TP\Delta f(t,x,u) + 2e^TP\varphi(t,\hat{x},y,u) + 2e^TPL\Delta h(t,x)$$

and using equation (7), $(\mathcal{A}_2)$ and $(\mathcal{A}_4)$

$$\begin{aligned}
\dot{V}(e) &= -e^TQe - 2e^TC_0^T\phi(t,x,u) - 2e^TP\frac{P^{-1}C_0^T(C_0\hat{x} - y)\delta(t)^2}{\|C_0e\|\delta(t) + r_0} \\
&\quad +2e^TC_0^T\gamma(t,x) \\
&= -e^TQe - 2e^TC_0^T\phi(t,x,u) - 2\frac{e^TC_0^TC_0e\delta(t)^2}{\| C_0e \| \delta(t) + r_0} + 2\frac{e^TC_0^T\Delta h(t,x)\delta(t)^2}{\| C_0e \| \delta(t) + r_0} \\
&\quad +2e^TC_0^T\gamma(t,x).
\end{aligned}$$

Since

$$\lambda_{min}(Q) \| e \|^2 \leq \ e^TQe \ \leq \ \lambda_{max}(Q) \| e \|^2, \tag{8}$$

where $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote the minimum and maximum eigenvalues of the matrix A and using $(\mathcal{A}_3)$ and $(\mathcal{A}_5)$, one gets

$$\begin{aligned}
\dot{V}(e) &\leq -\lambda_{min}(Q)\|e\|^2 + 2\|C_0e\|\delta_1(t) - 2\frac{\|C_0e\|^2\delta(t)^2}{\|C_0e\|\delta(t) + r_0} + 2\frac{\|C_0e\|\|\Delta h(t,x)\|\delta(t)^2}{\|C_0e\|\delta(t) + r_0} \\
&\quad +2\|C_0e\|\delta_2(t)
\end{aligned}$$

with $\delta(t) = \delta_1(t) + \delta_2(t)$,

$$\begin{aligned}
\dot{V}(e) &\leq -\lambda_{min}(Q)\|e\|^2 + 2\|C_0e\|\delta(t) - 2\frac{\|C_0e\|^2\delta(t)^2}{\|C_0e\|\delta(t) + r_0} + 2\frac{\|C_0e\|\|\Delta h(t,x)\|\delta(t)^2}{\|C_0e\|\delta(t) + r_0} \\
&\leq -\lambda_{min}(Q)\|e\|^2 + 2r_0\frac{\|C_0e\|\delta(t)}{\|C_0e\|\delta(t) + r_0} + 2\delta(t)\|\Delta h(t,x)\|\frac{\|C_0e\|\delta(t)}{\|C_0e\|\delta(t) + r_0} \\
&\leq -\lambda_{min}(Q)\|e\|^2 + (2r_0 + 2\delta(t)\|\Delta h(t,x)\|)\ \frac{\|C_0e\|\delta(t)}{\|C_0e\|\delta(t) + r_0}.
\end{aligned}$$

Since,

$$\frac{\|C_0 e\|\delta(t)}{\|C_0 e\|\delta(t)+r_0} \ < \ 1, \quad \|\Delta h(t,x)\| < \frac{\|C_0\|\delta_2(t)}{\|PL\|},$$

we obtain

$$\begin{aligned} \dot{V}(e) &\leq -\lambda_{min}(Q)\|e\|^2 + 2r_0 + 2\frac{\|C_0\|\delta_2(t)}{\|PL\|}\delta(t) \\ \dot{V}(e) &\leq -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}V(e) + r \end{aligned}$$

with

$$r = 2r_0 + 2\frac{\|C_0\|\delta_2(t)}{\|PL\|}\delta(t).$$

So

$$\dot{V}(e) \leq -\eta V(e) + r$$

with

$$\eta = \frac{\lambda_{min}(Q)}{\lambda_{max}(P)}. \tag{9}$$

From the above Remark 2.1 one obtains the following estimation

$$\| V(e(t)) \| \quad \leq \quad \| V(e(t_0)) \| \, e^{-\eta(t-t_0)} + \frac{r}{\eta}$$

so,

$$\lambda_{min}(P)\|e(t)\|^2 \quad \leq \quad \lambda_{max}(P)\|e(t_0)\|^2 e^{-\eta(t-t_0)} + \frac{r}{\eta}.$$

Hence,

$$\|e(t)\| \quad \leq \quad \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}\|e(t_0)\|e^{-\frac{\eta}{2}(t-t_0)} + \sqrt{\frac{r}{\lambda_{min}(P)\eta}}.$$

Therefore, $e(t)$ converges to the ball $B(0, \sqrt{\frac{r}{\lambda_{min}(P)\eta}})$ in the ultimate bounded sense.□

Remark 3.1 Note that, if we take $r_0 = r_0(t)$ with $r_0(t)$ going to zero when t tends to infinity and $\delta \longrightarrow 0$ when $t \to +\infty$, the trajectory tends to the origin exponentially when $t \to +\infty$.

Next, we consider the system (4) under the condition that the uncertainties are bounded. When the states are not available the usual technique is to build an observer which gives an approximation of the state. Many authors studied the problem of the conception of the observer. For the concept of observer, we aim at simplifying the design of this system by exploiting the linear form of the nominal system. We first introduce the following definition as in [18] and [19].

Definition 3.1 A practical exponential observer for (4) is a dynamical system which has the following form:

$$\dot{\hat{x}} = F(t, \hat{x}, u) - L(C\hat{x} - y), \tag{10}$$

where L is the gain matrix and the origin of the error equation with $e = \hat{x} - x$, which is given by

$$\dot{e} = F(t, \hat{x}, u) - F(t, x, u) - LCe + L\Delta h(t, x) \tag{11}$$

is globally practically exponentially stable, it means that it is globally uniformly practically asymptotically stable and the following estimation holds:

$$\|e(t)\| \leq \lambda_1(\|e(t_0)\|)\, e^{-\lambda_2(t-t_0)} + r, \quad \forall\, t \geq t_0$$

with $\lambda_1,\ \lambda_2,\ r > 0$.

Note that, the origin $x = 0$ may not be an equilibrium point of the system (4). We can no longer study stability of the origin as an equilibrium point nor should we expect the solution of the system to approach the origin as $t \longrightarrow \infty$. The inequality given in Remark 2.1 implies that $x(t)$ will be ultimately bounded by a small bound $r > 0$, that is, $\|x(t)\|$ will be small for sufficiently large t. If r can be replaced by a smooth map $r(t)$ as a function of t which tends to zero as t tends to $+\infty$, the ultimate bound approaches zero. This can be viewed as a robustness property of convergence to the origin provided that F satisfies $F(t, 0, 0) = 0,\ \forall\ \ t \geq 0$, which is supposed in such a way the origin becomes an equilibrium point.

We consider the system (4) satisfying the assumption $(\mathcal{A}_1)$ and the following one

$(\mathcal{A}_6)$ There exist positive constants M_1 and M_2, such that for all $t \geq 0$

$$\begin{aligned} \|\Delta f(t, x)\| &\leq M_1, \\ \text{and} \qquad \|\Delta h(t, x)\| &\leq M_2. \end{aligned} \tag{12}$$

To design an observer, we shall consider the dynamical system

$$\dot{\hat{x}} = A\hat{x} + Bu + \Delta f(t, \hat{x}) - L(C\hat{x} - y), \tag{13}$$

where L is the gain matrix, $\hat{x} \in \mathbb{R}^n$ is the state estimate of $x(t)$ in the sense that $e(t) = \hat{x}(t) - x(t)$ satisfies the following estimation,

$$\|e(t)\| \leq \|e(t_0)\| e^{-\lambda(t-t_0)} + r,\ \forall\ \ t \geq t_0.$$

Proposition. *Under assumptions $(\mathcal{A}_1)$ and $(\mathcal{A}_6)$ the system* (13) *is a practical exponential observer for the system* (4).

Indeed, we consider the error equation with $e = \hat{x} - x$

$$\dot{e} = \dot{\hat{x}} - \dot{x} = (A_0 - LC_0)e + \Delta f(t, \hat{x}) - \Delta f(t, x) + L\Delta h(t, x) \tag{14}$$

and the quadratic Lyapunov function candidate $V(e) = e^T Pe$ as in the proof of the theorem. Taking into account $(\mathcal{A}_6)$, the derivative of W along the trajectories of system

(2) is given by

$$\begin{aligned}
\dot{W}(t,e) &= \dot{e}^T P e + e^T P \dot{e} \\
&= e^T\left[(A_0 - LC_0)^T P + P(A_0 - LC_0)\right] e + 2e^T P(\Delta f(t,\hat{x}) - \Delta f(t,x)) \\
&\quad + 2e^T P L \Delta h(t,x) \\
&= -e^T Q e + 2e^T P(\Delta f(t,\hat{x}) - \Delta f(t,x)) + 2e^T P L \Delta h(t,x) \\
&\le -e^T Q e + 2\|e^T P\| \cdot \|\Delta f(t,\hat{x}) - \Delta f(t,x)\| + 2\|e^T P L\| \cdot \|\Delta h(t,x)\| \\
&\le -e^T Q e + 2\|P\| \left(\|\Delta f(t,\hat{x})\| + \|\Delta f(t,x)\|\right) \|e\| + 2\|P\| \cdot \|L\| \cdot \|\Delta h(t,x)\| \cdot \|e\| \\
&\le -e^T Q e + 4\|P\| \cdot M_1 \cdot \|e\| + 2\|P\| \cdot \|L\| \cdot M_2 \cdot \|e\| \\
&\le -e^T Q e + \left(4\|P\| M_1 + 2\|P\| \cdot \|L\| \cdot M_2\right) \|e\| \\
&\le -\lambda_{min}(Q)\|e\|^2 + M\|e\|
\end{aligned}$$

with

$$M = \left(4\|P\| M_1 + 2\|P\| \cdot \|L\| \cdot M_2\right).$$

Using (8), we get

$$\dot{V}(e) \quad \le \quad -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)} V(e) + M\|e\|$$

and using (9)

$$\dot{V}(e) \quad \le \quad -\eta V(e) + M\|e\|.$$

Therefore,

$$\dot{V}(e) \ \le \ -\eta V(e) + \frac{M}{\sqrt{\lambda_{min}(P)}} \sqrt{V(e)}. \tag{15}$$

Let $W(t) = \sqrt{V(t)}$. The derivative with respect time yields

$$\dot{W}(t) = \frac{\dot{V}(t)}{2W(t)}.$$

So,

$$\dot{W}(t) \ \le \ -\frac{1}{2}\eta W(t) + \frac{1}{2}\frac{M}{\sqrt{\lambda_{min}(P)}}.$$

Using remark 2.1, one gets

$$\|W(t)\| \ \le \ \|W(t_0)\| e^{-\frac{1}{2}\eta(t-t_0)} + \frac{M}{\eta\sqrt{\lambda_{min}(P)}}.$$

Since $W(t) = \sqrt{e^T(t) P e(t)}$, it follows that

$$\|e(t)\| \ \le \ \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} \cdot \|e(t_0)\| e^{-\frac{\eta}{2}(t-t_0)} + \frac{M}{\eta \lambda_{min}(P)}.$$

We get an estimation as in (7). The origin of (14) satisfies an estimation as in Definition 3.1. Hence, we conclude that, the origin of system (10) is a practical exponential observer for the system (4). The solution converges to the ball $B(0, \frac{M}{\eta \lambda_{min}(P)})$. □

Note that $\|e(t)\|$ can be small for sufficiently large t, if we take $M = M(t)$ such that

$$\lim_{t \to \infty} M(t) = 0.$$

4 Numerical Example

Consider the system

$$\begin{cases} \dot{x}_1 &= x_1 + x_2 + e^{-t} \sin x_1, \quad t \geq 0, \\ \dot{x}_2 &= -3x_2 + 2u, \\ y &= x_1 + x_2, \end{cases} \tag{16}$$

with $x = (x_1, x_2)^T \in \mathbb{R}^2$,

$$\begin{array}{lll} A_0 = \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix}, & B_0 = \begin{pmatrix} 0, \\ 2 \end{pmatrix}, \\ C_0 = \begin{pmatrix} 1 & 1 \end{pmatrix}, & \Delta f(t,x) = e^{-t} \sin x_1. \end{array} \tag{17}$$

This system is observable with

$$L = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \begin{pmatrix} 2401 \\ -2283 \end{pmatrix}.$$

We get the following system

$$\dot{\hat{x}} = \begin{pmatrix} -2400 & -2400 \\ 2283 & 2280 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u + \begin{pmatrix} 2401 \\ -2283 \end{pmatrix} y + e^{-t} \sin \hat{x}_1 \tag{18}$$

with

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \lambda_{min}(Q) = \lambda_{max}(Q) = 1,$$

and P is given in $(\mathcal{A}_1)$:

$$P = \begin{bmatrix} 6,3458 & -6,3456 \\ -6,3456 & 6,3538 \end{bmatrix}, \quad \lambda_{min}(P) = 0,0042, \quad \lambda_{max}(P) = 12,6954.$$

Let $\|P\| = \lambda_{max}(P) = 12,6954$. Hence $\eta = 0,0788$ and $\Delta f(t,x) \leq 1, \ \forall \ t, x$. Here $M_1 = 1$, $M_2 = 0$ and $M = 4 \cdot \|P\| \cdot M_1 = 50,7816$.

Therefore, system (18) is an observer which can be considered as a robust detector as the definition 2.4 and the trajectory of the error equation converges to the ball $B(0, r)$ with $r \simeq 153498$.

Conclusion

This paper deals with the problem of the output stabilisation for a class of uncertains systems. It is shown that an output controller can be constructed under some sufficient conditions and a robust detector can be designed which provides an estimation of the state. A numerical example in the plane is given to illustrate our main result.

References

[1] Walcott, B. Nonlinear Output Stabilisation of Uncertain Systems. In: *Proc. of American Controls Conference*, 1988, 2253–2258.

[2] Steinberg, A. and Corless, H. Output Feedback Stabilisation of Uncertain Dynamical Systems. *IEEE Transactions on Automatic Control* **AC-30** (1985) 1025–1027.

[3] Walcott, B. and Zak, S. Combined Observer-Controller Synthesis for Uncertain Dynamical Systems with Applications. *IEEE Transactions on Systems, Man. and Cybernetics* **18** (1) (1988) 40–48.

[4] Yostricov, A. Utkin, V. and Frantsuzova, G. Systems with state Vector Derivative in the Control. *Automation and Remote Control* (6) (1986) 283–286.

[5] Marino, R. High-Gain Feedback Control in Non-Linear Systems. *International Journal of Control* **42** (6) (1985) 1369–1385.

[6] Baumann, V. and Rugh, V. Feedback Control of Nonlinear Systems by Extended Linearization. *IEEE Transactions on Automatic Control* **AC-31** (1) (1986) 40–46.

[7] Dawson, D.M., Qu, Z. and Carroll, J. On the state observation and output feedback problems for nonlinear uncertain dynamic systems. *Systems & Control Letters* **18** (2) (1992) 217–222.

[8] Qu, Z. *Robust Control of Nonlinear Uncertain Systems.* New York: Wiley, 1998.

[9] Xu, S., Shi, P., Feng, C., Guo, Y. and Zou, Y. Robust Observers for a Class of Uncertain Nonlinear Stochastic systems with State Delays. *Nonlinear Dynamics and Systems Theory* **4** (3) (2004) 369–380.

[10] Karimi, H.R. Robust Dynamic Parameter-Dependent Output Feedback Control of Uncertain Parameter-Dependent State-Delayed Systems. *Nonlinear Dynamics and Systems Theory* **6** (2) (2006) 143–158.

[11] Vidyasagar, M. *Nonlinear systems analysis*. New Jersey, Pratice Hall, Englewood Cliffs, NJ, 1993.

[12] Khalil, H. *Nonlinear Systems*. New Jersey, Prentice Hall, 3rd ed. Englewood Cliffs, NJ, 2002.

[13] Bensoussan, D. and Hammami, M. Robust feedforward control under plant dynamic variation. *IMACS-IFAC Symposium Lille.* France, 1991.

[14] Bensoussan, D. and Hammami, M. Achieving robustness to plant uncertainty using high gain state feedback. *IEEE Fourth International Multi-Conference on Systems, Signals & Devices* SSD-SAC-1073, 2007, 19–22.

[15] Hammami, M. On the detectability of a certain class of perturbed systems. *International Journal of Computational and Numerical Analysis and Applications* **3** (3) (2003) 239–248.

[16] Liu, B. and Liu, F. Robustly Global Exponential Stability of Time-varying Linear Impulsive Systems with Uncertainty *Nonlinear Dynamics and Systems Theory* **7** (2) (2007) 187–195.

[17] Kailath, T. *Linear Systems*. New Jersey, Pentice Hall, Englewood Cliffs, 1980.

[18] Ellouze, I. and Hammami, M.A. A Separation principle of time-varying dynamical systems: A pratical approach. *Mathematical Modelling and Analysis* **12** (2) (2007) 1–16.

[19] A. Ben Abdallah, Ellouze, I. and Hammami, M.A. Practical global uniform exponential stability of perturbed triagular systems: a separation principle (*to appear in International Journal of Control*).