Method of Lines to Hyperbolic Integro-Differential Equations in $\mathbb{R}^n$

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Abstract: In this work we consider a hyperbolic integro-differential equation in $\mathbb{R}^n$. We reformulate it into an evolution equation in a suitable Hilbert space and establish the existence and uniqueness of a strong solution using the method of lines and the theory of semigroups of contractions in a Hilbert space.

Keywords: Wave equation; semigroups; contractions; method of lines; strong solution.

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1 Introduction

In this paper we are concerned with the following perturbed wave equation in $\mathbb{R}^n$,

$$\begin{cases} \frac{\partial^2 w(x,t)}{\partial t^2}(x,t) - \Delta w(x,t) = f(x,t) + \int_0^t k(t-s) \Delta w(x,s) \, ds, \; (x,t) \in \mathbb{R}^n \times (0,T], \\ w(x,0) = f_1(x), \quad \frac{\partial w}{\partial t}(x,0) = f_2(x), \quad x \in \mathbb{R}^n, \end{cases}$$

(1)

where $\Delta$ denotes the $n$-dimensional Laplacian, the unknown real valued function $w$ is to be defined on $\mathbb{R}^n \times [0,T]$, $0 < T < \infty$, $k$ is a real valued function defined on $[0,T]$, the real valued function $f$ is defined on $\mathbb{R}^n \times [0,T]$, the real valued functions $f_i$ are defined on $\mathbb{R}^n$, $i = 1, 2$.

The problem (1) with $k \equiv 0$ has been extensively treated by many authors, see, for instance, Yosida [21, 22] and Pazy [18]. Our aim is to reformulate (1) as a first order