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Multimodel Approach using Neural Networks for Complex Systems Modeling and Identification

S. Talmoudi [1*], K. Abderrahim [2], R. Ben Abdennour [2] and M. Ksouri [3]

[1] *Superior Institute of Applicated Sciences and Technology of Mateur, 7000 Mateur, Tunisia*
[2] *National School of Engineers of Gabes- Street of Medenine, 6029 Gabes, Tunisia*
[3] *National School of Engineers of Tunis, BP 37, the Belvedere, 1002, Tunis, Tunisia*

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Abstract: This article presents a new approach of systematic determination of models base for the multimodel approach. The application of this approach requires, first, to classify a numeric data by exploiting the self-adapting artificial Kohonen neural-networks. The obtained data relative to the clusters are then exploited for both structural and parametric estimation of base models. To resolve the estimation problem of the validity of the elementary models, used we proposed a new technique, based on the minimization of a quadratic criterion. This criterion exploits the centers of clusters obtained in the determination of the models base step. A comparative study with the residues approach showed the contribution in precision of the proposed validities computation technique. The satisfactory results obtained in numerical simulation, incited us to validate experimentally the contributions already mentioned. Indeed, an on-line experimental validation of the new proposed multimodel representation was carried out on an olive oil esterification reactor. The obtained results are very satisfactory in terms of precision and robustness.

Keywords: *Modeling; multimodel approach; models base; Kohonen card; validities; on line experimental validation*

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* Corresponding author: samia.talmoudi@enig.rnu.tn

1 Introduction

The development of mathematical models is a major problem for the application of advanced techniques for analysis, prediction, control, optimization, automatic fault detection and diagnostic in the industrial processes. Hence, there is a potential for improved quality and flexibility of final product if the cost of the model development can be reduced. Consequently, a strong demand for advanced modeling and identification methods arises. The multimodel approach is an efficient and a powerful way to resolve problem of modeling and control of complex, non-linear and/or ill-defined processes. This approach is based on a "divide and conquer" strategy [23]. A complex modeling problem is divided into a number of smaller sub-problems, which are solved independently by identifying simple models (generally linear). The obtained group of models forms the so-called models base. Afterwards, it is necessary to compute coefficients called validities of models. The simple models are, thereafter, combined, according to their estimated validities, together to obtain the global model. The past few years have shown an increase in the use of the multimodel representation [16]. This concept includes a number of approaches such as: Takagi and Sugeno Fuzzy Inference Systems [29], local model networks [16], gain-scheduled control, statistical mixture models, Smooth Threshold Auto-Regressive (STAR) models of Tong [30] and the state dependent models of Priestley [20]. For the majority of these approaches, the model parameters are obtained from prior knowledge, linearization of physical model or identified from measured data [21]. In many cases, the local models can be quite simple, such as linear or affine models. Besides, the multimodel concept coincides with engineering design in which the division of problems into manageable parts is the major design methodology [23]. The multimodel approaches were succeeded in different domains such as academic, biomedical, process industries, etc. However, they remain so confronted with several difficulties such as the determination of the models base. To resolve this problem, a modeling framework based on an operating decomposition of the system's operating range has interested Johansen in [10]. Indeed, he has proposed an algorithm that able to identify decomposition into operating regimes and local models on the base of empirical data. However, this algorithm requires that the regime must be d-dimensional boxes with orthogonal edges. Besides, the introduction of this last complex description of the regime limits will increase the number of parameters necessary to represent theses boundaries or local model validity functions. This leads, consequently, to a more complex identification problem [11]. Murray-Smith in [16] proposes to use learning systems able to model unknown nonlinear dynamic processes from their observed input-output behaviour. Local model networks use a number of simple and locally accurate models to represent a globally complex process. A major difficulty with local model nets is the optimization of the model structure. Heikki [8] has proposed an evolutionary self-organizing map capable of creating an organized model bank from a data set. However, the proposed algorithm is very complex and requires a very large knowledge such as genetic algorithm, self-organizing card, etc. Besides, the computing of one map is relatively very long. In 1995, Gawthrop considered the approximation of the continuous-time non-linear system in the vicinity of the equilibrium operating points by a continuous-time local model network [7]. One global inconvenience of most of these last strategies, is that the determination of these local models needs to a certain extend a priori knowledge of the system and its structure [3, 5, 1, 6, 2, 21, 19, 23]. Besides, we cannot found a systematic method for local models determination; which supposes several preliminary tests before its choice. Recently, it is proposed in [13] an approach

of models base determination for the uncertain processes, which limit the number of base models to four or five models. This method is inspired from the algebric stability approach suggested by Kharitonov [13]. The models base is obtained by determining the four extreme models, and the average model, determined as an average of the boundary models. Mezghani in [17] proposed the extension of this last approach for discrete case using the d operator. These last approaches require the knowledge of the variation domains parameters of the uncertain process. But, this last information is always not still possible. Another inconvenience is that the last models base will contain models with the same structure. We propose, in this paper, a new systematic determination approach of a models base for the representation of uncertain discrete linear systems. This approach does not require the limits knowledge of the parameters. Besides, this method allows to generate automatically the number, the structures and the parameters of the elaborated models. Indeed, the proposed method requires three principle steps [25, 27, 28]. The first step consists in classifying numerical data by using a Self-adapting artificial Kohonen neural network. The second step is a structural and parametric estimations step in order to determine the base models. Also, to resolve the problem of validities computation we propose a new technique, based on the minimization of a quadratic criterion [26, 28]. This criterion exploits the centers of clusters obtained in the models base determination step. By comparison with the residues approaches, used by many researchers, we have demonstrated the efficiency and the precision of the suggested technique. In order to highlight the good performance in precision and the robustness under particularly severe conditions of the two suggested approaches, the theoretical study is, then, validated by numerical simulation and by experiments. This paper is organized as follows: in the section two, a principle of classification by using a Kohonen card is introduced. The new systematic approach determination of a models base is developed with details in the third section. The validities computation represents the subject of the fourth section. The principle of computation of multimodel'output is given in section five. A numerical example is presented in the section six. In section seven, an experimental validation, carried out on an olive oil esterification reactor, is considered. We finish the present work by a conclusion.

2 Classification of the Numerical Data by Using the Kohonen Card

The self-organizing Kohonen map is a well-known unsupervised algorithm used frequently for classification of data. The standard card can find the cluster centers and gives a visual interpretation of the distribution and clusters of the data. This classification strategy consists in applying the rule of Kohonen [18, 8, 22]. This rule is characterized by an unsupervised competitive learning. Where, a competition takes place before the modification of the network-weights. Only the neuron, which gained the competition, has the right to change their weight. The Kohonen rule has the property of self-adapting, which allows him to group together a set of data, presented to the corresponding network, around a certain number of representative centroides of these data clusters. The used neural network is formed by one input layer of p neurons and by one output layer of n neurons corresponding to the Kohonen card [18, 22]. The architecture of this network is given by (2.1) [25]. Each neuron of the Kohonen card receives p signals coming from the input layer. The weight w_{pn} is relative to the connection between the input neuron p and the card neuron n. The weight vector W_i associated to neuron i is then composed of p elements. The Kohonen rule works as follows [18, 22]:

1. The network receives a data set Y.

2. Each card neuron calculates the Euclidean distance between the weight vector W_i and the input vector Y.

3. The competition between card neurons starts. This competition is based on the winner-takes-all strategy. The neuron having the nearest weight vector Wi to the input Y wins the competition. The winner neuron output z_i is putting at 1 and the other ones are putting then at 0.

4. The different weights are modified according to the following relation:

$$W_i^{new} = W_i^{old} + \alpha\Big(Y - W_i^{old}\Big) z_i \tag{1}$$

where α is a constant such that $0 < \alpha < 1$.

At the end of the training, the Kohonen network generates the representative vectors of different clusters and their centers.

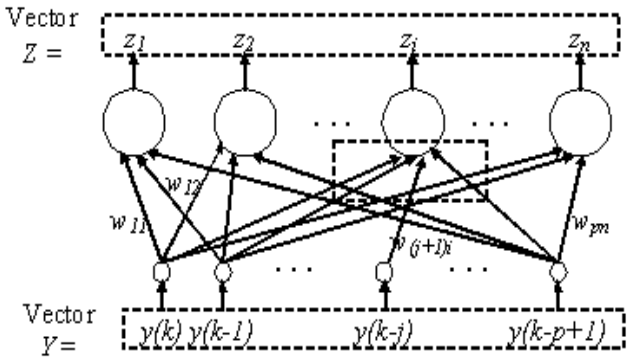

Figure 2.1: The retained architecture for the generation of different observations vectors for modeling.

3 A Systematic Determination Approach of a Models Base

The application of this approach requires firstly the determination of the clusters number. The classification of numerical data is the second stage. Then, there is a stage of structural and parametric estimation.

3.1 Determination of the clusters number

To classify the numerical data, it is necessary to pass throught the step of determination of the adequate clusters number and as consequence, the number of base models. To resolve the problem, we propose to consider a two-dimensional Kohonen card with a neurons number n in the output-layer which is relatively important. At the end of training, if the

network gives badly repartition clusters, it will remove the cluster i having an elements number N_{Ci} verifying:

$$N_{Ci} < \frac{1}{2}\frac{N_H}{n}, \tag{2}$$

where N_H is the observations'number. Also, we increase the network structure and we restart the training.

3.2 Classification of the numerical data by exploiting the Kohonen card

After determining the suitable number of classes, consequently the base models number, it is the question of classifying the measurements. These last are related to the output of an uncertain or ill-defined discrete linear system using the proposed method described in Section 2. Therefore, we exploit a Kohonen network, which has neurons number in the output-layer equal to the clusters number, determined by the method described in the last section. This network is able of looking into the output of a set of representative vectors of different clusters with their respective centers. These vectors are, then, exploited for the structural and parametric identification of the elaborated base models.

3.3 Structural and parametric estimation

The order estimation method of the retained models is called instrumental determinants' ratio-test [4, 25, 15]. This method consists in building an information matrix Q_m, containing the input-output measurements couples given by:

$$Q_m = \frac{1}{N_H}\sum_{k=1}^{N_H}\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k-m+1) \\ u(k+m) \end{bmatrix}\begin{bmatrix} y(k+1) \\ u(k+1) \\ \vdots \\ y(k+m) \\ u(k+m) \end{bmatrix}^T. \tag{3}$$

The instrumental determinants'ratio $RDI(m)$ is given by the following relation:

$$RDI(m) = \Big|\frac{det(Q_m)}{det(Q_{m+1}}\Big|. \tag{4}$$

For every value of m, the order determination procedure computes Qm and Q_{m+1} matrices and estimates the ratio RDI, the retained order m is the value for which the ratio $RDI(m)$ quickly increases for the first time. Indeed, Q_{m+1} matrix becomes singular when m becomes identified with the exact order.

The retained parametric estimation method is the Recursive Least Squares' method RLS [4].

4 A New Approach for Validities' Computation

Several validities computation methods was proposed in the literature [5, 6, 12, 13, 16, 17, 19]. All these methods are based on the residues computation and they are based on measuring the distance between the current state of the process and the considered model M_i. The geometric distance can be calculated by several methods; the simplest

one is the distance $r_i(k)$ between the process output $y(k)$ and the base models outputs $y_i(k)$:

$$r_i(k) = |y(k) - y_i(k)|. \tag{5}$$

Frequently, we choose the validities such as all the time their sum is equal to the unity. For example,

$$v_i(k) = \frac{|1 - r'_i(k)|}{C - 1}. \tag{6}$$

C represents the retained number of base models and is a normalized distance given by

$$r'_i(k) = \frac{r_i(k)}{\sum_{i=1}^{C} r_i(k)}. \tag{7}$$

The proposed method of validities computation is inspired from the fuzzy version of the "k-means" algorithm[18]. This method is based on the minimization of the following criterion:

$$J = \sum_{i=1}^{C} \sum_{k=1}^{N_H} v_i^2(k) \|y(k) - c_i\|^2 \tag{8}$$

with

$$\sum_{i=1}^{C} v_i(k) = 1, \tag{9}$$

where $v_i(k)$ represent the degree of validity of the model i at the instant k, c_i is the center of the class i.

It is a first order problem of optimization with equality constraint $g(v_i(k))$. The resolution of this type of problem requires the determination of the Lagrange's equation. In fact, so that $v_i(k)$ is a local extremum of the criterion J, it is necessary that there is a real λ such that the Lagrangian L of the problem can be written as follows:

$$L(v_i(k), \lambda) = J + \lambda g(v_i(k)) \tag{10}$$

is stationary with regard to $v_i(k)$ and λ. This leads to

$$\begin{cases} \frac{\partial(L(v_i(k),\lambda)}{\partial(v_i(k))} = 0, \\ \frac{\partial(L(v_i(k),\lambda))}{\partial(\lambda)} = 0, \end{cases} \tag{11}$$

where λ is the Lagrange's multiplier associated to the constraint. The relations (11) lead to the following system

$$\begin{cases} 2v_1(k) \|y(k) - c_1\|^2 + \lambda = 0, \\ 2v_2(k) \|y(k) - c_2\|^2 + \lambda = 0, \\ \vdots \\ 2v_i(k) \|y(k) - c_i\|^2 + \lambda = 0, \\ \vdots \\ 2v_C(k) \|y(k) - c_C\|^2 + \lambda = 0, \\ v_1(k) + v_2(k) + \ldots + v_i(k) + \cdots + v_C(k) = 1. \end{cases} \tag{12}$$

This problem becomes

$$\begin{cases} \left\{v_i(k), \|y(k)-c_i\|^2+\lambda=0, i\in[1,C]\right\}, \\ \sum_{l=1}^{C} v_l(k)=1. \end{cases} \tag{13}$$

The relations (13) give

$$v_i(k)=\frac{-\lambda}{2\|y(k)-c_i\|^2}. \tag{14}$$

This relation becomes

$$\sum_{l=1}^{C}\frac{-\lambda}{2\|y(k)-c_l\|^2}=1. \tag{15}$$

Then λ is given from the relation (15) and replaced in the equation (14). Finally, we can conclude that the expression of validity degree for a model M_i can be written as follows:

$$v_i(k)=\frac{1}{\sum_{l=1}^{C}(A_i^2(k)/A_l^2(k)}, \tag{16}$$

where $A_i^2(k)=\|y(k)-c_i\|^2$ (see (4.1)).

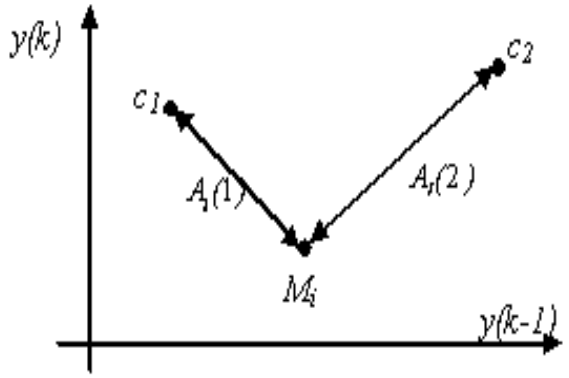

Figure 4.1: Euclidean distance illustrated by the new technique of validity computation.

5 Computation of Multimodel Output

The multimodel output is obtained by fusion of the local models pondered by their respective validities. The next relation (17) gives the expression of the final multimodel output:

$$y_{MM}(k)=\sum_{i=1}^{C} y_i(k)v_i(k). \tag{17}$$

6 Simulation Example in Stochastic Case

The object of this section is to demonstrate the interest and the robustness of both proposed methods: the multimodel representation and the validities computation technique. Let us consider a stochastic linear process with time varying parameters, described by the following equation [25, 15]:

$$y(k) = -a_1(k)y(k-1) - a_2(k)y(k-2) + b_1(k)u(k-1) + b_2(k)u(k-2) + e(k), \quad (18)$$

where $e(k)$ is a white noise $(0, \sigma^2)$ with covariance σ equal to 0.2. The variation laws of different parameters of the process are given by the Figure 6.1. The retained excitation signal $u(k)$ is a Pseudo Aleatory Binary Sequence.

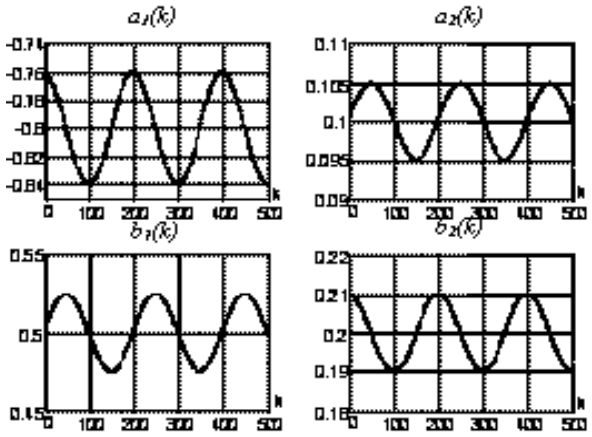

Figure 6.1: The variation laws of the considered process parameters.

6.1 Classification of the numerical data by exploiting the Kohonen card

The suggested approach for the systematic determination of the models base has been implemented. Indeed, the numerical noisy identification data obtained by exciting the system (18) by a Pseudo Aleatory Binary Sequence are presented to a Kohonen card formed by one input layer of two neurons and by one output layer of three neurons. The Figure 6.2 shows that three data sets relative to the various clusters are obtained at the end of learning of the neuronal network.

6.2 Structural and parametric estimation

From each of the data relative to the three clusters, we could determine the orders and the parameters of the transfer functions $H_1(q^{-1})$, $H_2(q^{-1})$ and $H_3(q^{-1})$ relative to the base models. Figure 6.3 shows the evolutions of the Instrumental Determinants' Ratio $RDI_i(m)(i = 1, 2 \text{ or } 3)$ for the three obtained clusters. We observe, clearly, that the orders of the three models are equal to 2.

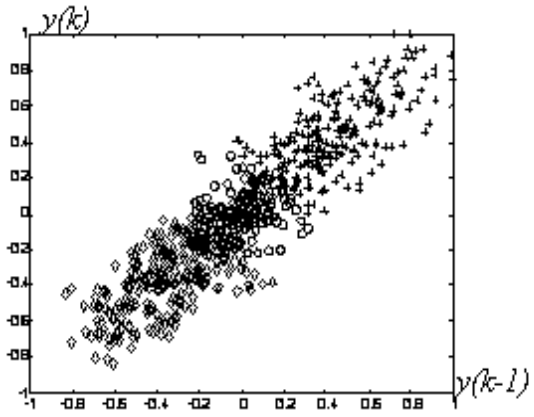

Figure 6.2: Three sets of numerical data relative to the different base models.

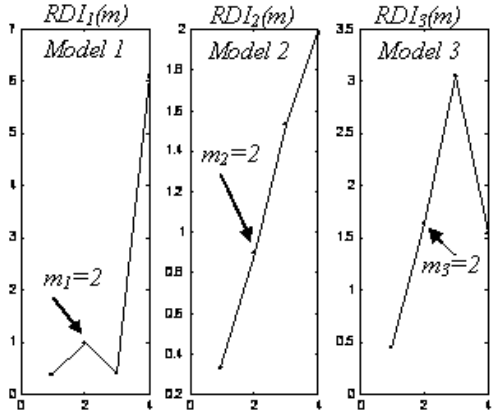

Figure 6.3: Evolutions of the RDI for the three obtained clusters.

After the parametric identification step, the obtained transfer functions $H_1(q^{-1}), H_2(q^{-1})$ and $H_3(q^{-1})$ can be written:

$$H_1(q^{-1}) = \frac{0.48765q^{-1} + 0.26243q^{-2}}{1 - 0.62912q^{-1} + 0.022475q^{-2}}, \tag{19}$$

$$H_2(q^{-1}) = \frac{0.49611q^{-1} + 0.22886q^{-2}}{1 - 0.70327q^{-1} + 0.019325q^{-2}}, \tag{20}$$

$$H_3(q^{-1}) = \frac{0.49987q^{-1} + 0.24861q^{-2}}{1 - 0.7443q^{-1} + 0.040774q^{-2}}. \tag{21}$$

6.3 Validation phase

The application of the following input sequence is the subject of validation step:

$$u(k) = 2 + \sin k/20. \tag{22}$$

The proposed approach for validities computation uses the clusters centers obtained in the stage of determination of a models base. The coordinates of the three obtained centers c_1, c_2, c_3 are: $c_1(-0.412; -0.4020); c_2(-0.0151; 0.0041); c_3(0.4738; 0.4687)$. The results of validation are given in the Figure 6.4. This figure shows that the multimodel output $y_{fn}(k)$ obtained by fusion of base models outputs pondered by the new technique validities, follows the real output $y_r(k)$ of the stochastic uncertain process with a relatively negligible error. In the case of modeling classical approach, we have exploited the same

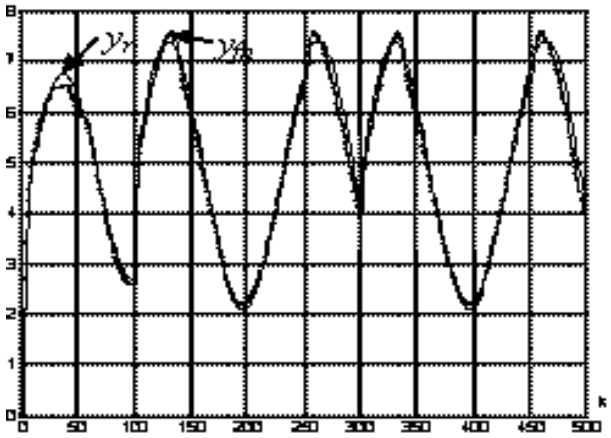

Figure 6.4: Evolutions of the real and multimodel outputs (New technique).

numerical noisy identification data used for the multimodel representation. By recourse to the instrumental determinants' ratio for the structural estimation and to the recursive least squares method for the parametric identification, the transfer function $H(q^{-1})$ of the global model "M" can be written as follows:

$$H(q^{-1}) = \frac{0.49457q^{-1} + 0.28186q^{-2}}{1 - 0.60115q^{-1} + 0.043232q^{-2}}. \tag{23}$$

The Figure 6.5 represents the evolutions of the relative errors between the real output and the global model "M" and the multimodel "MMn". This figure demonstrates that the multimodel representation offers a very satisfactory precision and robustness relatively to the case in which classical modeling, based on one global model "M", is considered. The evolutions of different validities of models are given by the Figure 6.6. This figure

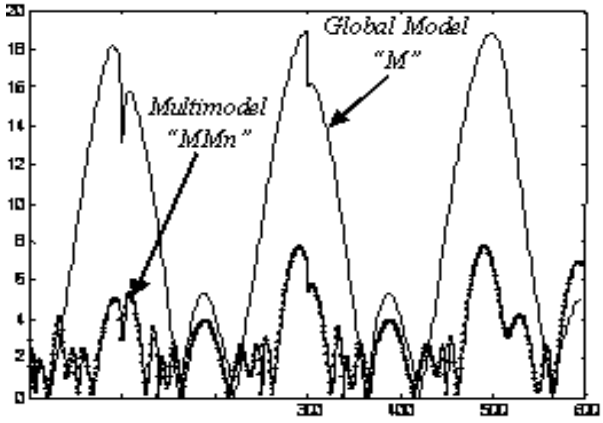

Figure 6.5: Evolutions of relative errors.

shows the complementarities of the different models in the different operation areas. It shows, also, that it is possible that one model can describe correctly the system (validity equal to the unity), the validities of the others models are equal to zero. This last result is not possible when the residues approach is applied. Indeed, in the Figure 6.7, we have presented the evolution of the three validities calculated by the residues approach in the same conditions. This figure shows that these validities cannot exceed $0,5$. This can be justified by the presence of term $'C-1'$ in the denominator of the validities expression (6). As consequent, the residues approach cannot evaluate correctly the contribution of every model of the base in the global behaviour of the system.

Figure 6.8 presents the evolutions of the prediction errors $er_1(k)$ and $er_2(k)$ of the two multimodel outputs respectively $y_{fc}(k)$ (residues approach) and $y_{fn}(k)$ (new technique) with regard to the real output. This figure shows the performance in precision and in robustness of the new technique of validities computation by comparison with the residues approach.

7 Experimental Validation: Olive Oil Esterification-Reactor

In order to show the contribution in precision and robustness of the suggested modeling strategy, we have implemented it practically in the case of modeling of an olive oil esterification-reactor. This discontinuous reactor carries out, by an alcohol, a chemical reaction of vegetable olive oil esterification. This type of reaction is given by the following scheme: $Acid + Alcohol \rightleftharpoons Ester + Water$. The obtained product is an ester with a very high benefit used mainly in the manufacture of cosmetic products. In previous work, the dynamic behaviour of this reactor has been modeled by a set of complex differential

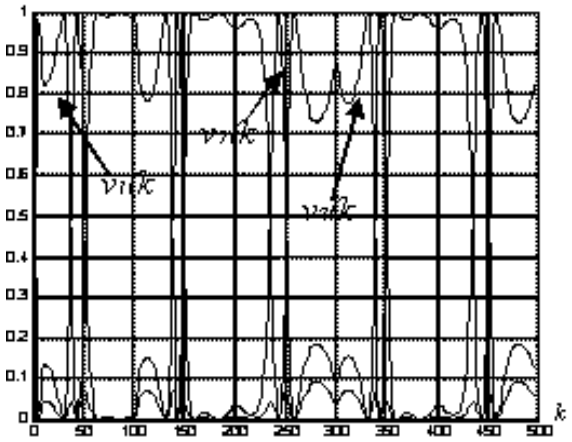

Figure 6.6: Evolutions of the validities (new technique).

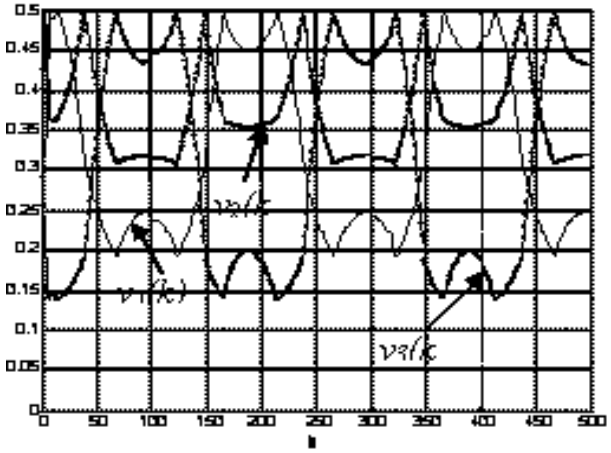

Figure 6.7: Evolutions of the validities (Residues approach).

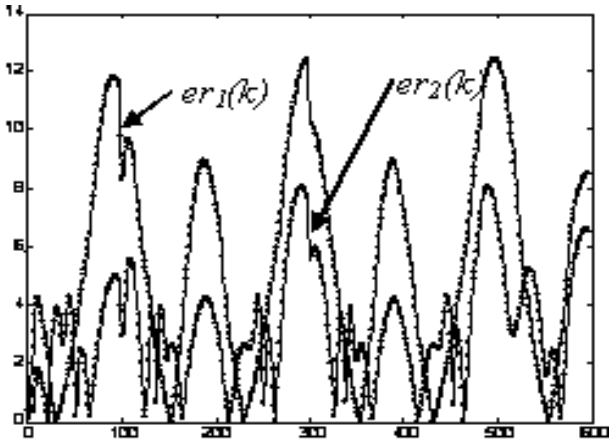

Figure 6.8: Evolution of the relative prediction errors.

equations. The static characteristic of the reactor is non-linear and, consequently, the classical modeling, based on one global model cannot lead to satisfactory results. To improve these results, we propose, in the next section, to use the suggested multimodel representation.

7.1 A modeling phase

In Figure 7.1, we have presented the input-output measurements picked out experimentally of the reactor for the identification step. By exploiting the last input-output

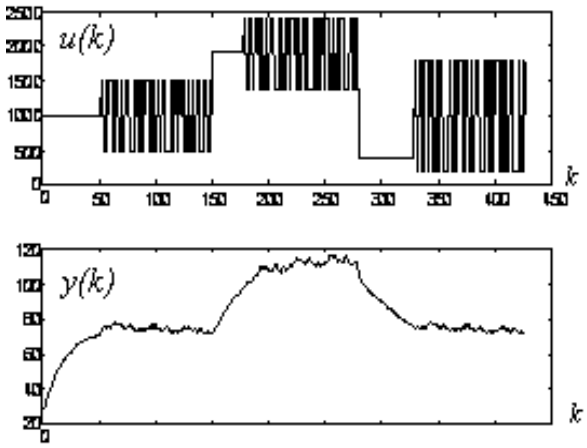

Figure 7.1: Evolutions of the input-output measurements u(k) and y(k).

measurements' file, the suggested approach for the determination of the models base has been implemented. Indeed, the experimental data are presented to a Kohonen network

having two inputs and one-dimensional card with 3 neurons in the output layer. Figure 7.2 shows that three sets of data relative to the different clusters are obtained at the end of the neural network training. From each of the data relative to a cluster $c(c = 1, .., 3)$,

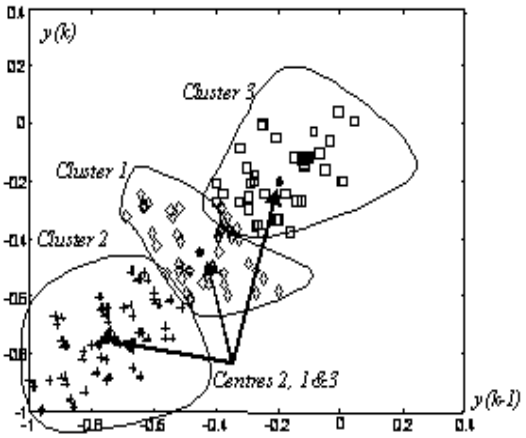

Figure 7.2: Three sets of the experimental data relative to the different base-models.

we could determine the transfer functions $(H_1(q^{-1}), H_2(q^{-1})$ and $H_3(q^{-1}))$ relative to the base-models. Figure 7.3 presents the evolutions of the Instrumental Determinants' Ratio $RDI_i(m)$ (i=1, 2 or 3) for the three obtained clusters. This figure shows that the adequate estimated orders of the three models are equal to 2. Finally, we have obtained

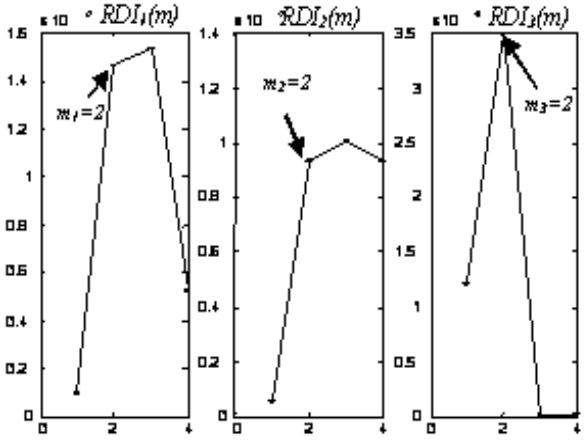

Figure 7.3: Evolutions of the RDI for the three obtained clusters.

the base formed by the models described by the following transfer functions:

$$H_1(q^{-1}) = \frac{0.0018269q^{-1} + 0.00043866q^{-2}}{1 - 1.3052q^{-1} + 0.32917q^{-2}}, \tag{24}$$

$$H_2(q^{-1}) = \frac{0.0018804q^{-1} + 4.2569.10^{-5}q^{-2}}{1 - 1.216q^{-1} + 0.24209q^{-2}}, \tag{25}$$

$$H_3(q^{-1}) = \frac{0.0011144q^{-1} + 0.00046594q^{-2}}{1 - 1.1185q^{-1} + 0.12743q^{-2}}. \tag{26}$$

In the case of modeling classical approach, the process is considered linear around an operation point. The non-linearity is consequently interpreted, under these conditions, as a parametric disturbance. By recourse to the instrumental determinants ratio test for the structural estimation, and to the recursive least squares method for the parametric identification, the transfer function $H(q^{-1})$of the global model "M", worked out by the exploitation of an input-output measurements'file experimentally picked out on the reactor, can be written as follows:

$$H(q^{-1}) = \frac{-0.00010162q^{-1} + 0.0012255q^{-2}}{1 - 1.0425q^{-1} + 0.058094q^{-2}}. \tag{27}$$

7.2 Evaluation of the modeling results

To validate the obtained models, we have considered a new input-output measurements'file picked out for the real system. The effective output $y_{MM}(k)$ of the multimodel "MM" is calculated by fusion of the three base outputs pondered by their respective validities. Figure 7.4 represents the evolutions of the real, the global model "M" and the multimodel "MM" outputs. This figure shows that the "MM" approach, using the elaborated base, offers a very satisfactory precision relatively to the case in which classical modeling, based on one global model "M", is considered. Indeed, the relative error between the real output and the model "M" and the multimodel "MM" outputs confirms this last conclusion (Figure 7.5). The evolutions of the different validities relative to the different models of the base are given on Figure 7.6. It gives information about the complementarities of the different models in the operation area of the reactor which can be divided into three zones of heating, reaction and cooling.

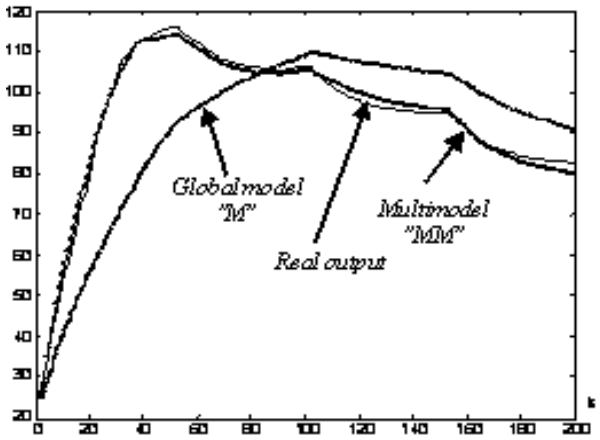

Figure 7.4: Experimental validation of the models (classical and multimodel approaches).

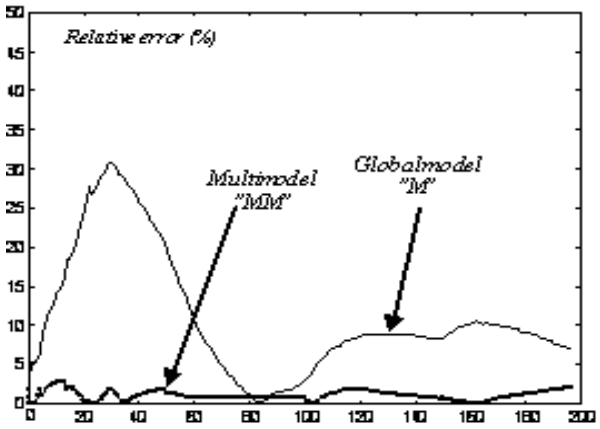

Figure 7.5: Evolutions of relative errors.

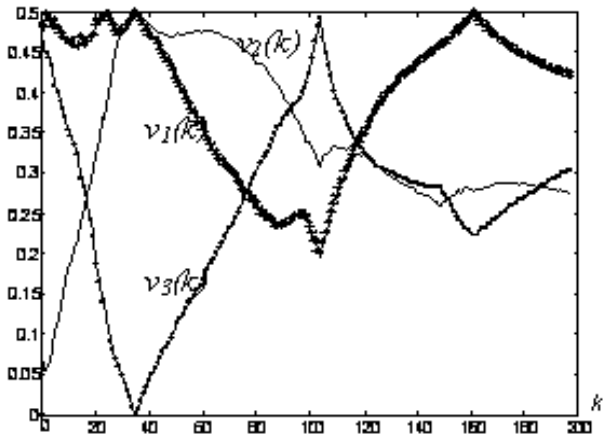

Figure 7.6: Evolutions of different models validities of the elaborate base.

8 Conclusion

In this paper, we have presented firstly a new systematic determination approach of models base for multimodel approach. This approach does not require a priori knowledge about the studied system and can generate automatically the number, the structures and the parameters of base models. Indeed, it can be applied on three steps. The primary step consists in determining the suitable number of base models. The second one consists in an off-line classification of identification data. The structural and parametric estimations of the base models from the obtained vectors in the classification step, form the third step. Secondly, a new technique of validities computation is developed. This last technique consists in minimizing a quadratic criterion exploiting the clusters centers obtained in the stage of determination of the models base. The application of these contributions is carried out, first, on a simulation example, then on a real process corresponding to a semi-batch chemical reactor. These applications showed the efficiency and the very good performances of the two proposed methods, with regard to the classical modeling method based on unique model and to the residues approach.

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