Approximation of Solutions to a Class of Second Order History-valued Delay Differential Equations

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Abstract: In this paper we shall study the approximations of solutions to a class of second order history-valued delay differential equations in a separable Hilbert space. Using a pair of associated nonlinear integral equations and projection operators we consider a pair of approximate nonlinear integral equations. We first show the existence and uniqueness of solutions to this pair of approximate integral equations and then establish the convergence of the sequences of the approximate solutions to the solution and the pair of associated integral equations, respectively. Also, we consider the Faedo–Galerkin approximations of the solution and prove some convergence results. Finally, we give an example.

Keywords: Second order history-valued delay differential equations; analytic semigroup; Banach fixed point theorem; Faedo-Galerkin approximation.

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1 Introduction

We consider the following second order history-valued abstract delay differential equation in a separable Hilbert space \((H, \| \cdot \|, \langle \cdot, \cdot \rangle)\):

\[
\begin{align*}
  &u''(t) + Av(t) = f(t, u(t), v(t), u(t-\tau), v(t-\tau)), \quad t \in (0, T], \\
  &u(t) = h(t), \quad v(t) = g(t), \quad t \in [-\tau, 0],
\end{align*}
\]

\(1\)

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