Chaotic Dynamics in Hybrid Systems

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Abstract: In this paper we give an overview of some aspects of chaotic dynamics in hybrid systems, which comprise different types of behaviour. Hybrid systems may exhibit discontinuous dependence on initial conditions leading to new dynamical phenomena. We indicate how methods from topological dynamics and ergodic theory may be used to study hybrid systems, and review existing bifurcation theory for one-dimensional non-smooth maps, including the spontaneous formation of robust chaotic attractors. We present case studies of chaotic dynamics in a switched arrival system and in a system with periodic forcing.

Keywords: Chaotic dynamics; hybrid systems; symbolic dynamics; nonsmooth bifurcations.

Mathematics Subject Classification (2000): 34A37, 37B10, 37A40, 34A36, 37G35.

1 Introduction

A hybrid system is a dynamic system which comprises different types of behaviour. Classic examples of hybrid dynamical systems in the literature are impacting mechanical systems, for which the behaviour consists of continuous evolution interspersed by instantaneous jumps in the velocity, and dc-dc power converters, in which the behaviour depends on the state of a diode and a switch. Hybrid control systems occur when a continuous system is controlled using discrete sensors and actuators, such as thermostats and switched heating/cooling devices. Hybrid dynamics may also occur due to saturation effects on components of a system, and in idealized models of hysteresis. Finally, we mention that hybrid systems can be derived as singular limits of systems operating in multiple time-scales; indeed we may consider almost all hybrid systems to arise in this way.

From a mathematical point of view, hybrid systems typically exhibit non-smoothness or discontinuities in the dynamics, and these properties induce new dynamical phenomena.

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