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On Nonlinear Control and Synchronization Design for Autonomous Chaotic Systems

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Abstract: In this paper, a chaos control and synchronization technique is presented. The proposed technique is applied to achieve both control and synchronization for some autonomous chaotic dynamical systems. Numerical simulations are used to show the effectiveness of the proposed technique.

Keywords: *Chaos; autonomous systems; control; synchronization.*

Mathematics Subject Classification (2000): 34C15, 34C28, 49J15, 93B52.

1 Introduction

Control and synchronization of chaotic dynamical systems have received a great deal of interest among scientists from various fields [5, 13]. These two ideas were first proposed in 1990 [22, 24]. The idea of controlling chaos consists on stabilizing one of the unstable periodic orbits within the strange attractor of the chaotic dynamics, and the task was fulfilled by perturbing an accessible parameter around its nominal value. The idea of synchronizing chaotic systems refers to a process wherein two or many chaotic systems starting from different initial conditions adjust a given property of their motion to a common behaviour. Since then, many possible applications of chaos control and synchronization methods have been discussed by computer simulation and realized in laboratory condition [3, 8, 12, 14, 17, 19, 20, 21, 26, 28].

The Ott–Grebogi–Yorke method, known as OGY method, is a feedback control method, which uses the chaos in system to stabilize an unstable periodic orbit. The main idea of the method is to adjust the parameter perturbations for relatively small time in order to stabilize the desired unstable periodic orbit (UPO) and obtain an attracting time-periodic motion. This control technique is practical from an experimental standpoint because it requires no analytical model of the system. It just requires determining the fixed point and the stable and unstable directions. However, the success of

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the original OGY theory is limited by the fact that, it applies only to systems where the manifolds are constructed directly by using the Jacobian eigenvalues and eigenvectors. In most of dynamical systems, the dynamics is not confined to a lower-dimensional attractor. Chaos control in higher dimensional systems is technically difficult because it may be impossible to construct the stable and unstable directions.

The aim of this letter is to apply both control and synchronization to some chaotic dynamical systems. This is done by extending the OGY chaos control method. As a potential application of the proposed control strategy, we used it to study the synchronization of some high order chaotic systems.

The principles of control and synchronization of autonomous chaotic systems are given in Section 2. In Section 3 we apply control and synchronization to Lorenz dynamical system and numerical simulations are used to show this process. In Section 4 synchronization and control are applied to Chen chaotic system. Section 5 is devoted to the control and synchronization of Chua system. We conclude in Section 6.

2 Control and Synchronization Principles

Consider the two nonlinear systems

$$\dot{X}_1 = f(X_1, p), \tag{1}$$

$$\dot{X}_2 = g(X_1, X_2), \tag{2}$$

where $f : R^N \times R \to R^N$, $g : R^N \times R^N \to R^N$ are continuous, $X_1, X_2 \in R^N$ are the state variables and $p \in R$ is a parameter control.

The system given by equation (1) will be called the drive system and the system given by equation (2) will be called the response system.

2.1 Chaos control principle

The chaos control algorithm that we introduce in the following uses, in a large sense, the Poincaré section properties. Since chaos is the superposition of a number of periodic motions, it is represented in the Poincaré section by a number of fixed points, called the system chaotic attractor. The chaos control algorithm developed here relies on the knowledge of the chaotic attractor and its response to small perturbations of the system. It is based on the analysis of the Poincaré section to determine how the system approaches the desired orbit or fixed point. The analysis is carried out in three steps:

1. Among the unstable periodic orbits (UPO) of the attractor, choose the one that represents the desired performances.

2. Determine the influence of control parameter on the chosen UPO. For this, we vary the control parameter around the value for which we want to control the system and each time to generate the associated Poincaré section.

3. Determine the variation that should be applied to the control parameter in order to force the system to rejoin the desired UPO or fixed point.

After information about this Poincaré section has been gathered, the system is kept to remain on the desired orbit by perturbing the appropriate parameter. Similar to the original OGY control method, we wish to make only small controlling perturbations to the system. We do not envision creating new orbits with different properties from the already existing orbits.

The basic idea of our control algorithm is as follows. Given a periodic orbit represented by a fixed points at the Poincaré section, we wait for the system trajectory to come close to the control region (which will be defined later) of the desired UPO to bring the system trajectory near the control region. When the system state is in the control region, we will try to use a small parametric perturbation to control the unstable directions of the chaotic state variables x. In other words, we attempt to bring the deviation $\delta x = x - x_f$ to lie on the linearized stable direction. where x_f represents the unstable fixed point obtained by the Poincaré section.

The control law (3) below is directly derived from the Poincaré section and will be applied to the drive system as follows:

$$\delta p = \frac{\partial p}{\partial x_f}(x - x_f), \tag{3}$$

where $\frac{\partial p}{\partial x_f}$ determines the influence of small parametric variation on fixed points variation.

This perturbation control law acts instantaneously on the system. However, in real cases, the future system state of a chaotic system depends on the current parametric variation as well as the previous parametric variations, so the system must take sometime to react to the correction. It seems more sensitive, from a practical point of view, to introduce some delay between the computation of the control law and the effective modification of the control parameter. This is realized by adding to the computed law a term depending on the previous value of the control parameter weighted with a parameter γ, which is determined by trial and error.

Thus, equation (3) becomes:

$$\delta p_{new} = \frac{\partial p}{\partial x_f}(x - x_f) + \gamma \delta p_{old}. \tag{4}$$

However, in terms of the quality of control performance, once the control is activated, the controlled system must be maintained at its new trajectory along its evolution. This stability criterion is assured by a good choice of γ for each chaotic system to be controlled.

We expect that, under forward applications of the control law (4), points in the local neighbourhood of the fixed point will eventually fall into the local neighbourhood and then be controlled.

2.2 Chaos synchronization principle

Let $X_1(t, X_1(0))$ and $X_2(t, X_2(0))$ be solutions to the drive system (1) and to the response system (2) respectively.

In this framework, complete synchronization is defined as the identity between the trajectories of the response system X_2 and of one replica X_2' of it $\dot{X}_2' = g(X_1, X_2')$ for the same chaotic driving system X_1.

If the solutions $X_1(t, X_1(0))$ and $X_2(t, X_2(0))$ satisfy

$$\lim_{t\to\infty} \|X_1(t, X_1(0)) - X_2(t, X_2(0))\| = 0. \tag{5}$$

Then, the drive system and the response system are said *synchronized*. In other words, the response system *forgets* its initial conditions, though evolving on a chaotic attractor.

In [24, 25], authors established that this kind of synchronization can be achieved and provide that all the Lyapunov exponents of the response system under the action of the driver (the conditional Lyapunov exponents) are negative. This implies that the response system is asymptotically stable.

3 Control and Synchronization of Lorenz System

The Lorenz system is a differential system with a chaotic behaviour for some values of parameters, described by:

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = (r - z)x - y, \\ \dot{z} = xy - bz. \end{cases} \tag{6}$$

The parameters setting for the Lorenz system to display chaos are: $\sigma = 10, b = 8/3$ and $r = 28$.

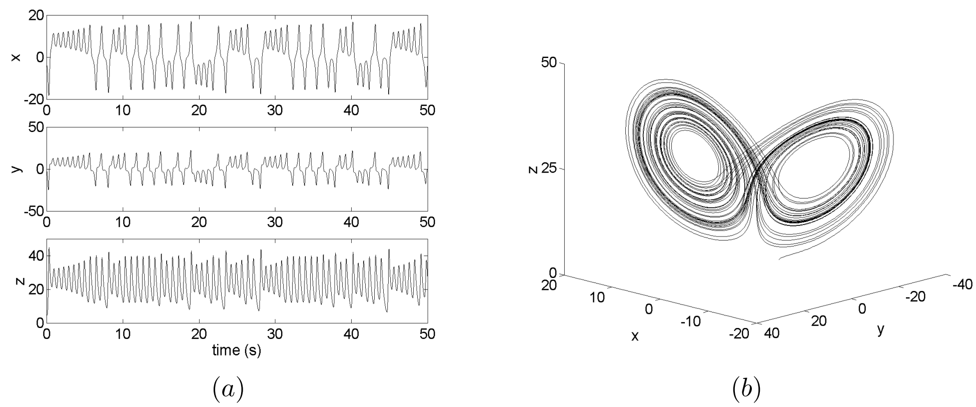


Figure 3.1: Lorenz chaotic attractor. (a) Time response. (b) Phase plane.

The drive system is given by:

$$\begin{cases} \dot{x}_1 = \sigma(y_1 - x_1), \\ \dot{y}_1 = ((r + \delta r) - z_1)x_1 - y_1, \\ \dot{z}_1 = x_1 y_1 - b z_1. \end{cases} \tag{7}$$

Here r is used as the control parameter with δr is the perturbing parameter control.

To apply the chaos control algorithm to the drive system, we have to determine the Poincaré section. This section is described by one dimensional map and corresponds to the set of points where attractor is at its maximum. That is $z = \max(z_1)$.

Figure 3.2 shows the plots given the maxima of $z(n+1)$ against those of $z(n)$. The fixed points are then obtained at the intersection of these plots with the straight line $z(n+1) = z(n)$.

The value of the third state variable of the fixed point is determined as $z_f = 39.82$.

To determine parametric influence of the small parametric variation on fixed-points variation, we generate a Poincaré section at $r = 28.2$ as shown in Figure 3.3.

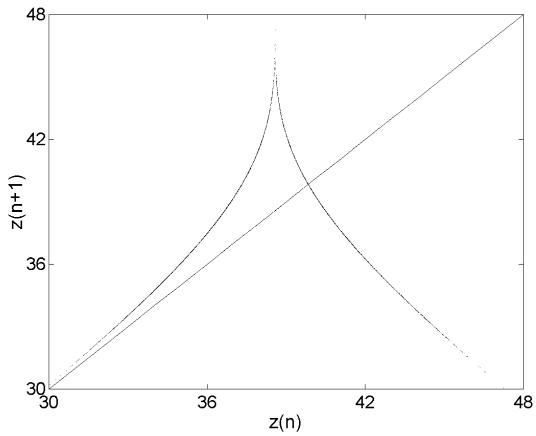


Figure 3.2: Successive maxima map of state variable z_1.

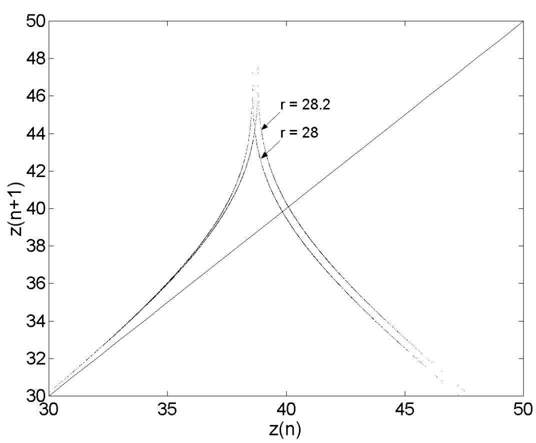


Figure 3.3: Superposition of two successive maxima map of state variable z_1.

In this case, we find $z'_f = 40.09$.

The drive system is under chaos control law (3) of the form:

$$\begin{aligned}\delta r_{new} &= \frac{\partial r}{\partial z_f}(z_1 - z_f) + \gamma\delta r_{old} \\ &= \frac{28.2 - 28}{40.09 - 39.82}(z_1 - z_f) + \gamma\delta r_{old}.\end{aligned} \tag{8}$$

Our control method needs to determine the stabilizer parameter γ in the feedback control. This parameter must be small, so it is chosen from the interval [0.01, 0.5].

The drive system is under control of the form:

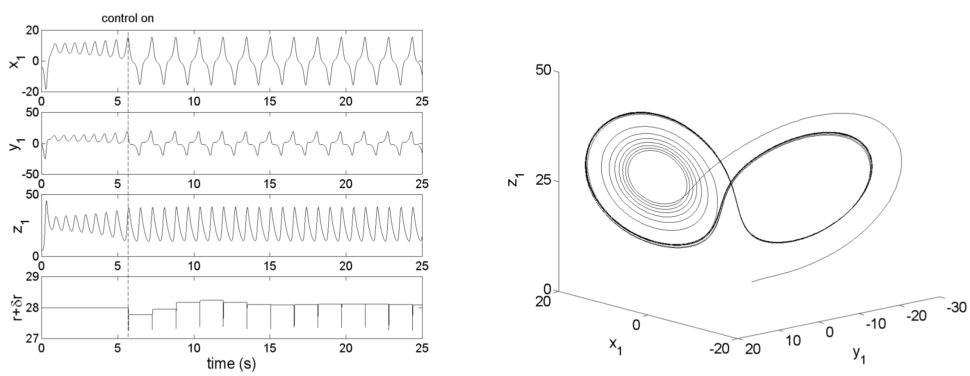
$$\delta r_{new} = 0.74(z_1 - 39.82) + 0.2\delta r_{old}. \tag{9}$$

This control law is activated only when the state variables x and z are located in the neighbourhood of the appropriates fixed points x_f and z_f respectively. The condition is defined by:

$$(x_1 - x_f)^2 + (z_1 - z_f)^2 < 1 \tag{10}$$

with $x_f = 14.89$.

The result of the control of the drive system is depicted in Figure 3.4.



(*a*) (*b*)

Figure 3.4: Control of the Lorenz chaotic driver. (a) Time response. (b) Phase plane.

To stabilize the chaos on its real unstable periodic orbit, one can see that control generate a pulse train, each pulse is activated automatically so that, at a sufficient amplitude, determined by the Poincaré section at each travelling from the fixed point, eventually the system orbit converges to the desired unstable periodic orbit. We also tested our chaos control strategy with different initial conditions and it was found to be robust.

Once controlled drive system is obtained, we construct a response system which exhibits a generalized kind of synchronization motion with the driver based on the Pecora and Caroll concept, by making a simple nonlinear transformation among the response variable x_2.

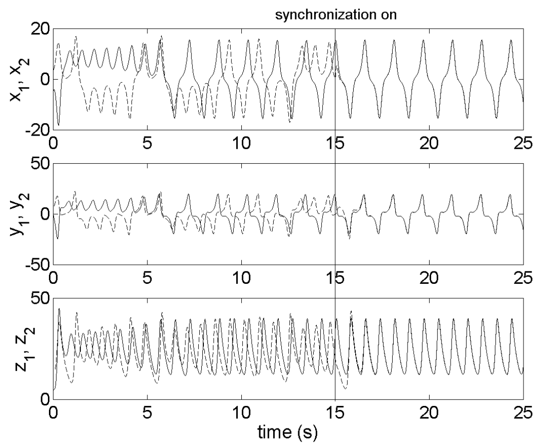


Figure 3.5: Synchronization of the Lorenz drive-response systems.

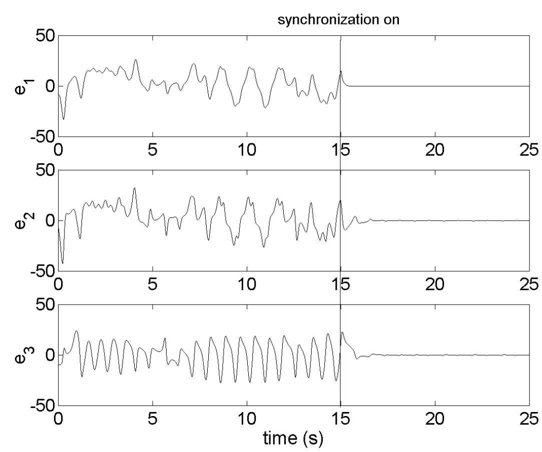


Figure 3.6: Time response of the error variables.

Thus, the response system will be given by:

$$\begin{cases} \dot{x}_2 = \sigma(y_1 - x_2), \\ \dot{y}_2 = (r - z_2)x_2 - y_2, \\ \dot{z}_2 = x_2 y_2 - b z_2. \end{cases} \tag{11}$$

Notice that (11) consists of a copy of (7) without control ($\delta r = 0$) and in the synchronization Pecora and Carroll concept, y_1 is the drive signal.

Introducing the error variables $e_1 = x_1 - x_2$, $e_2 = y_1 - y_2$ and $e_3 = z_1 - z_2$, we obtain the error dynamics

$$\begin{cases} \dot{e}_1 = -\sigma e_1, \\ \dot{e}_2 = r e_1 - e_2 - z_1 x_1 + z_2 x_2 + \delta r x_1, \\ \dot{e}_3 = -b e_3 + x_1 y_1 - x_2 y_2. \end{cases} \tag{12}$$

From (12), we should choose the synchronization subsystem such that an equilibrium state can be achieved. To find the equilibrium state, we set the three equations equals to zero, that is:

$$\begin{aligned} & \begin{cases} \dot{e}_1 = -\sigma e_1 = 0, \\ \dot{e}_2 = -e_2 + r e_1 - z_1 x_1 + z_2 x_2 + \delta r x_1 = 0, \\ \dot{e}_3 = -b e_3 + x_1 y_1 - x_2 y_2 = 0, \end{cases} \\ \Rightarrow & \begin{cases} -\sigma e_1 = 0, \\ -e_2 + r e_1 - z_1 x_1 + z_2 x_2 + x_1 z_2 - x_1 z_2 + \delta r x_1 = 0, \\ -b e_3 + x_1 y_1 + x_1 y_2 - x_1 y_2 - x_2 y_2 = 0, \end{cases} \\ \Rightarrow & \begin{cases} -\sigma e_1 = 0, \\ -e_2 + r e_1 - x_1 e_3 - z_2 e_1 + \delta r x_1 = 0, \\ -b e_3 - x_1 e_2 - y_2 e_1 = 0, \end{cases} \\ \Rightarrow & \begin{cases} -\sigma e_1 = 0, \\ -e_2 + (r - z_2) e_1 + (\delta r - e_3) x_1 = 0, \\ -b e_3 - x_1 e_2 - y_2 e_1 = 0. \end{cases} \end{aligned} \tag{13}$$

Because the parameters σ, b, r and the state variables x_1, y_2, z_2 are different from zero and $\delta r \to 0,$ it follows that the error states (e_1, e_2, e_3) asymptotically converges to $(0, 0, 0)$. In other words, the response system (11) asymptotically synchronizes with the drive system (7) no matter how they are initialized.

The initial values of the drive system are $(x_1(0), y_1(0), z_1(0)) = (-5, 0, 5)$ and the initial values of the response system are $(x_2(0), y_2(0), z_2(0)) = (3, 6, 15)$.

Synchronization law is applied for $t > 15$ and the drive-response systems are in a perfect synchronized state. The results of the simulation are shown in Figures 3.5 and 3.6.

4 Control and Synchronization of Chen System

Recently, Chen found another chaotic attractor, also in a simple three-dimensional autonomous system, which nevertheless is not topologically equivalent to the Lorenz's [6]:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a - z)x + cy, \\ \dot{z} = xy - bz. \end{cases} \tag{14}$$

System (14) has chaotic behaviour at the parameters values $c = 35, a = 28$ and $b = 3$. This system has the same complexity as the Lorenz equation – they are both three-dimensional autonomous with only two quadratic terms. The chaotic behaviour of the system is shown in Figure 4.1.

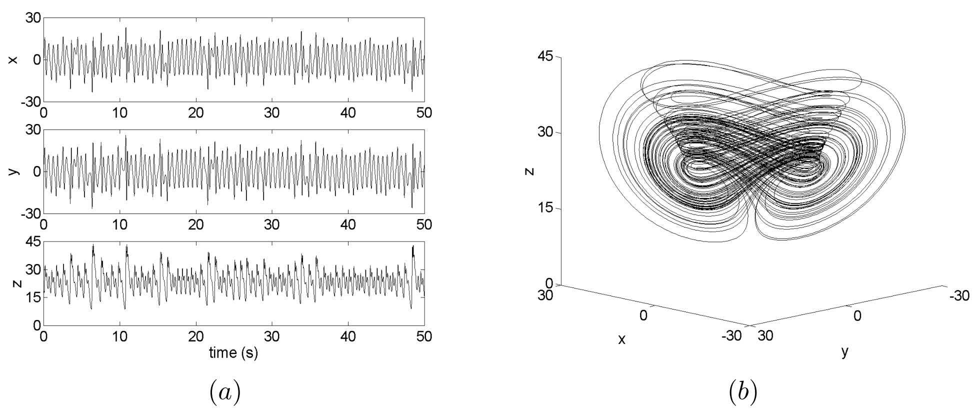


Figure 4.1: Chen chaotic attractor. (a) Time response. (b) Phase plane.

For Chen chaotic attractor, the drive system is defined as follows:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1), \\ \dot{y}_1 = ((c + \delta c) - a - z_1)x_1 + cy_1, \\ \dot{z}_1 = x_1 y_1 - bz_1. \end{cases} \tag{15}$$

Here c is used as the control parameter.

Figure 4.2 shows the Poincaré section realized on the third state variable for different values of parameter c.

At $c = 28$, the value of the third state variable of the fixed point is determined as $z_f = 27.29$ and at $c = 28.2$, $z'_f = 27.75$.

The control law is defined by

$$\begin{aligned} \delta c_{new} &= \frac{\partial c}{\partial z_f}(z_1 - z_f) + \gamma \delta c_{old} \\ &= \frac{28.2 - 28}{27.75 - 27.29}(z_1 - z_f) + \gamma \delta c_{old}. \end{aligned} \tag{16}$$

Then we obtain

$$\delta c_{new} = 0.43(z_1 - 27.29) + 0.1\delta c_{old}. \tag{17}$$

This control law is activated only when:

$$(x_1 - x_f)^2 + (z_1 - z_f)^2 < 1 \tag{18}$$

with $x_f = 14.89$.

The result of the control is shown in Figure 4.3. Figure 4.3(b) depicts the orbit of the controlled Chen's chaotic system in the phase space. From Figure 3(a), one can see

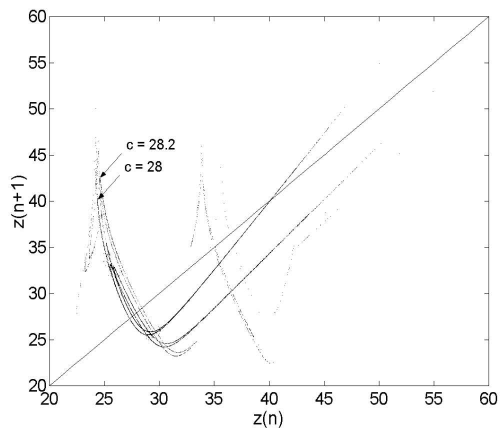
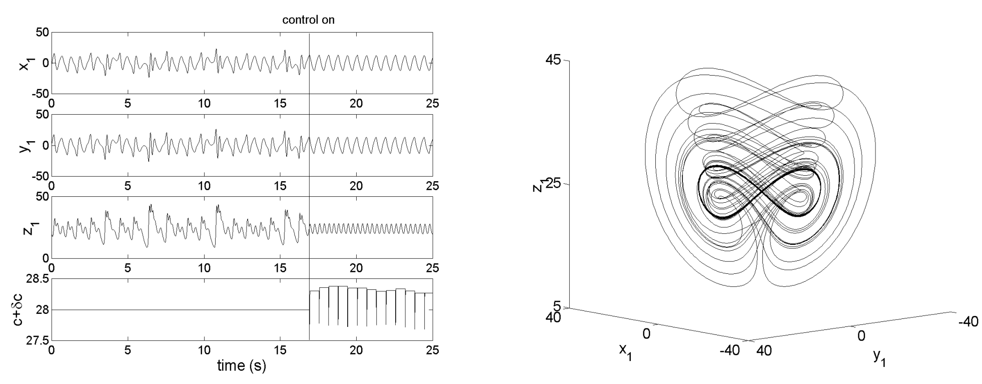


Figure 4.2: Return map of state variable z_1.



(a) (b)

Figure 4.3: Control of the Chen chaotic driver. (a) Time response. (b) Phase plane.

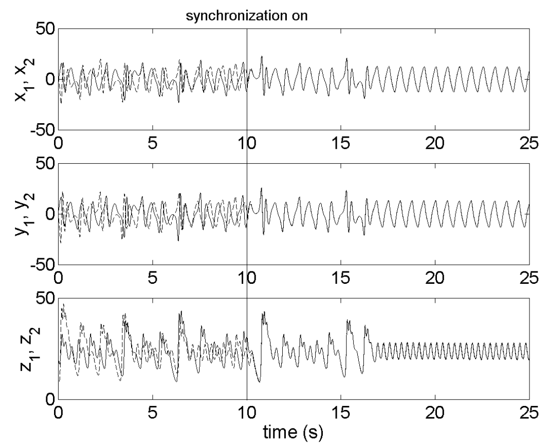


Figure 4.4: Synchronization of the Chen drive-response systems.

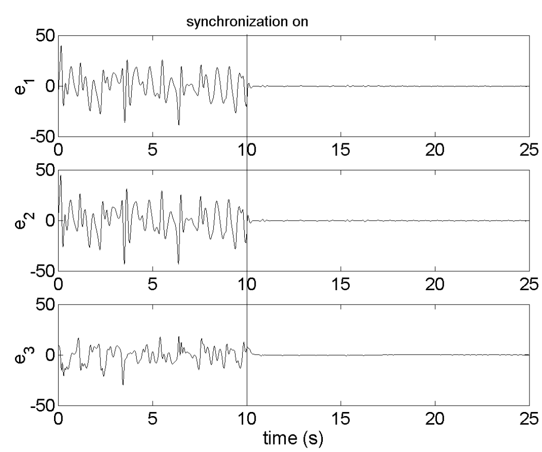


Figure 4.5: Time response of the error variables.

that, to stabilize chaos, the method works by applying instantaneous periodic kicks to the system variables and eventually the system orbits converge to the desired UPO.

We construct a Pecora-Caroll drive–response configuration with a drive signal y_1 introduced in the y_2 dynamics of the response system given by:

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2), \\ \dot{y}_2 = (c - a - z_2)x_2 + cy_1, \\ \dot{z}_2 = x_2 y_2 - bz_2. \end{cases} \tag{19}$$

The dynamic of the error variables will be given by:

$$\begin{cases} \dot{e}_1 = -a(e_1 - e_2), \\ \dot{e}_2 = (c - a)e_1 - z_1 x_1 + z_2 x_2 + \delta c x_1, \\ \dot{e}_3 = -be_3 + x_1 y_1 - x_2 y_2. \end{cases} \tag{20}$$

Demanding that all of the equations of system (20) are zero, we get the following:

$$\begin{cases} \dot{e}_1 = -a(e_1 - e_2) = 0, \\ \dot{e}_2 = (c - a)e_1 - z_1 x_1 + z_2 x_2 + \delta c x_1 = 0, \\ \dot{e}_3 = -be_3 + x_1 y_1 - x_2 y_2 = 0, \end{cases}$$
$$\Rightarrow \begin{cases} e_1 = e_2, \\ (c - a - z_2)e_1 + (\delta c - e_3)x_1 = 0, \\ -be_3 - x_1 e_2 - y_2 e_1 = 0, \end{cases}$$
$$\Rightarrow \begin{cases} e_1 = 0, \\ e_2 = 0, \\ e_3 = 0. \end{cases} \tag{21}$$

The initial values of the drive system are $(x_1(0), y_1(0), z_1(0)) = (-3, 2, 20)$ and the initial values of the response system are $(x_2(0), y_2(0), z_2(0)) = (5, -2, 10)$. In this case, synchronization was applied before applying the control law and simulation results are given in Figures 4.4 and 4.5.

5 Control and Synchronization of Chua System

The Chua circuit is a nonlinear circuit with chaotic behaviour for some values of parameters. The normalized equations representing the circuit are:

$$\begin{cases} \dot{x} = \alpha(y - x - h(x)), \\ \dot{y} = x - y + z, \\ \dot{z} = -\beta y, \end{cases} \tag{22}$$

where

$$h(x) = m_1 x + \frac{m_0 - m_1}{2}\left(|x + 1| - |x - 1|\right) \tag{23}$$

represents the nonlinear element of the circuit.

When $\alpha = 10, \beta = 14.87, m_0 = -1.27, m_1 = -0.68$, Chua attractor is chaotic and has a plot as shown in Figure 5.1.

The drive system is given by:

$$\begin{cases} \dot{x}_1 = (\alpha + \delta\alpha)(y_1 - x_1 - h(x_1)), \\ \dot{y}_1 = x_1 - y_1 + z_1, \\ \dot{z}_1 = -\beta y_1. \end{cases} \tag{24}$$

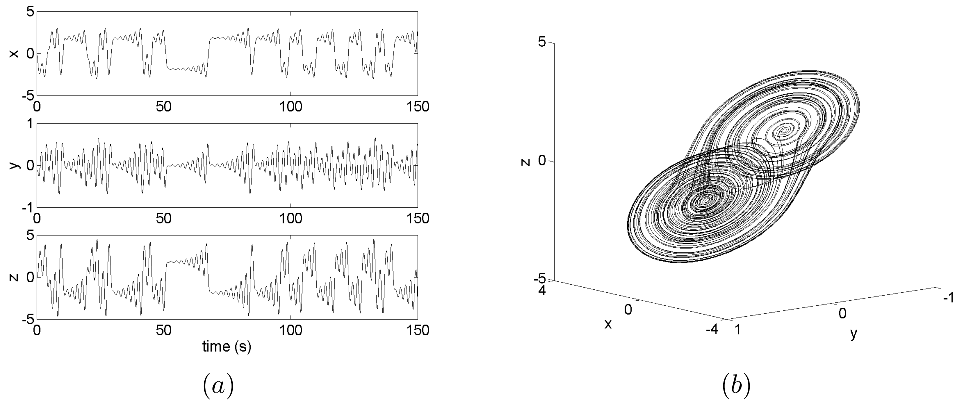


Figure 5.1: Chua's circuit. (a) Time response. (b) Phase plane.

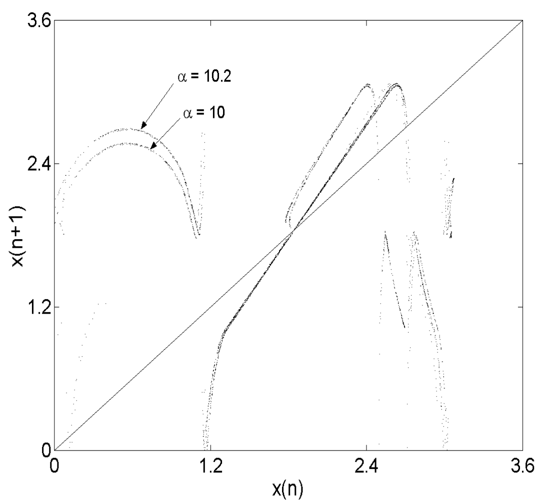


Figure 5.2: Return map of state variable x_1.

In Chua's circuit, we chose the Poincaré section by plotting the current maxima $x(n+1)$ against the previous maxima $x(n)$.

The first state of the fixed point is determined by $x_f = 2.70$. At $\alpha = 10.2$, in this case, $x'_f = 3.31$.

The deduced control law is

$$\begin{aligned} \delta\alpha_{new} &= \frac{\partial\alpha}{\partial x_f}(x_1 - x_f) + \gamma\delta\alpha_{old} \\ &= \frac{10.2 - 10}{3.31 - 2.70}(x_n - x_f) + \gamma\delta\alpha_{old}. \end{aligned} \tag{25}$$

Then we obtain

$$\delta\alpha_{new} = 0.32(x_n - x_f) + 0.1\delta\alpha_{old}. \tag{26}$$

The activation region of the control is defined by:

$$(x_n - x_f)^2 + (y_n - y_f)^2 < 1, \tag{27}$$

where $y_f = 0.27$. In the Chua system, the response system is chosen as follows:

$$\begin{cases} \dot{x}_2 = \alpha(y_2 - x_2 - h(x_2)), \\ \dot{y}_2 = x_1 - y_2 + z_2, \\ \dot{z}_2 = -\beta y_2. \end{cases} \tag{28}$$

Consequently, the error variables will be defined by:

$$\begin{cases} \dot{e}_1 = \alpha(e_1 - e_2 - h(x_1) + h(x_2)) + \delta\alpha(y_1 - x_1 - h(x_1)), \\ \dot{e}_2 = -e_2 + e_3, \\ \dot{e}_3 = -\beta e_{2.} \end{cases} \tag{29}$$

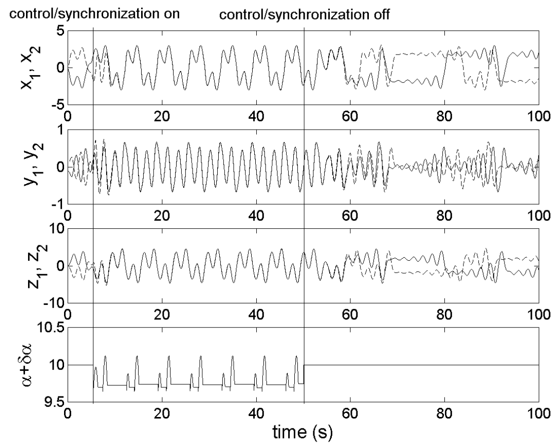
To find the equilibrium state of (29), we rewrite as follows:

$$\begin{aligned} &\begin{cases} \alpha(e_1 - e_2 - h(x_1) + h(x_2)) + \delta\alpha(y_1 - x_1 - h(x_1)) = 0, \\ -e_2 + e_3 = 0, \\ -\beta e_2 = 0, \end{cases} \\ \Rightarrow &\begin{cases} e_1 = 0, \\ e_2 = 0, \\ e_3 = 0. \end{cases} \end{aligned} \tag{30}$$

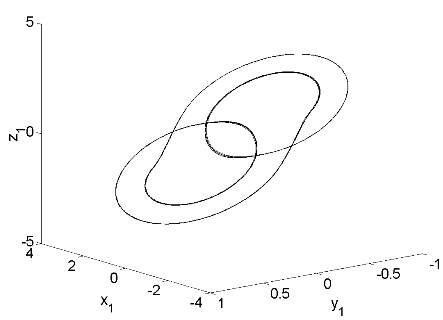
Starting from the initial values $(x_1(0), y_1(0), z_1(0)) = (-0.1, -0.1, -0.1)$ of the drive system and from $(x_2(0), y_2(0), z_2(0)) = (0.1, 0.1, 0.1)$ of the response system, controlled drive system (24), synchronization of the response system (28) with the controlled drive system and time response of the error variables (29) are shown together in Figure 5.3(a), (b) and (c) respectively.

6 Conclusion

This letter demonstrates that control and synchronization can be achieved in autonomous chaotic systems by different ways. The response system is synchronized with the drive system even if synchronization is activated before, after or simultaneously with the control law.



(a)



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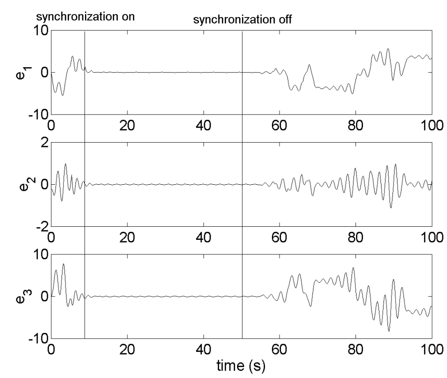

(c)

Figure 5.3: Control and synchronization of the Chua system. (a) Time response of the drive and response systems. (b) Phase plane of the controlled trajectory. (c) Time response of the error variables.

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