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Adaptive Control of Nonlinear in Parameters Chaotic Systems

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Abstract: This paper presents the adaptive control of chaotic systems, which are nonlinear in parameters (NLP). A method based on Lagrangian of an objective functional is used to identify the parameters of the system. Also this method is improved to result in better rate of convergence of the estimated parameters. Estimation results are used to calculate the Lyapunov exponents adaptively. Finally, the Lyapunov exponents placement method is used to assign the desired Lyapunov exponents of the closed loop system. Simulation results are provided to show the effectiveness of the results.

Keywords: *Chaotic system; nonlinear in parameter system identification; Lyapunov exponents placement; gradient method.*

Mathematics Subject Classification (2000): 93C10, 93C40, 34C28.

1 Introduction

Chaotic systems have been widely studied by many scientists and engineers from different viewpoints. The recent applications of chaotic systems have raised new questions regarding chaos control [1, 2, 3]. From a practical control system design point of view, an important issue in the analysis and control of chaotic systems can be the uncertainty associated with the system parameters. Equally important is the time varying nature of many system parameters. In [4, 5] an adaptive strategy is proposed for the on-line identification and control of chaotic systems. However, the method is restricted to chaotic systems that are linear in parameters. In this paper, the adaptive control of nonlinear in parameter chaotic systems is considered.

Parameter estimation methods for nonlinear chaotic systems, such as NARX (nonlinear autoregressive with exogenous inputs) [6], NARMAX (nonlinear autoregressive

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moving average with exogenous inputs) [7, 8] are employed for linear in parameter (LP) systems. For NLP systems, nonlinear programming methods can be used [9, 10]. In this paper, a proper local objective functional is defined and minimized based on the Lagrangian of an objective functional using the speed gradient (SG) method. The method results in fast convergence and estimated parameters are unbiased [11].

To achieve faster convergence rate of the estimated parameters, the SG method is improved. In the new improved SG (ISG) method, descent gradient method [9, 10] is used to optimize step length matrix of SG method in each iteration, to minimize another local objective functional.

Lyapunov exponents are commonly used for chaos identification in nonlinear dynamical systems. Lyapunov exponents show the average rate of growing or shrinking of a small volume of initial conditions. These exponents provide a quantitative measure for the sensitivity of the nonlinear system to the change of initial conditions. Also Lyapunov exponents demonstrate the chaotic behavior of the system [12, 13].

There are several methods for numerical calculation of Lyapunov exponents [14, 15]. Throughout this paper, we use a Jacobian approach with QR factorization to extract the Lyapunov exponents from long term product of the local Jacobian matrices.

Lyapunov exponents placement has been used for chaotization and anti-chaotization of a system [16]. It also could be used for the adaptive control of unknown or time varying LP chaotic systems [4]. This paper proposes an adaptive methodology for NLP chaotic systems.

The paper is organized as follows. Section 1 provides the introduction. In Section 2, the SG method for identification of NLP systems is described. In Section 3, we use descent gradient method to formulate the improved SG method. In Section 4, Lyapunov exponents of the system are determined. In Section 5, Lyapunov exponents placement strategy is described and is used to calculate the control input for the system. Finally, simulation results for the chaotic Duffing and Lorenz systems are provided in Section 6.

2 Adaptive Parameter Estimation

Consider the system described by:

$$\begin{cases} \dot{x}(t) = f(x(t), p(t)), \\ x(0): \text{ is given}, \end{cases} \tag{1}$$

where $t > 0$ is time, $x \in R^n$ is state vector of the plant, $p \in R^k$ is the vector of unknown parameters and $f\colon\ R^n \times R^k \to R$ is a vector function of state variables and parameters. Let the system estimator be described as follows:

$$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t), \hat{p}(t)), \\ \hat{x}(0): \text{ is given}, \end{cases} \tag{2}$$

where $\hat{x} \in R^n$ is the estimated state vector, $\hat{x} \in R^k$ is the vector of estimated parameters and $\hat{f}\colon\ R^n \times R^k \to R$ is a vector function of state variables and parameters. We make the assumption that the structure of the system is clear

$$f(\hat{x}(t), \hat{p}(t)) = \hat{f}(\hat{x}(t), \hat{p}(t)) = \left[\hat{f}_1(\hat{x}(t), \hat{p}(t)), \, \ldots, \, \hat{f}_n(\hat{x}(t), \hat{p}(t))\right]^{\mathrm{T}}, \tag{3}$$

where $\hat{f}_i\colon\ R^n \times R^k \to R$, $i = 1, 2, \dots, n$. The objective can be formulated as

$$\lim_{t\to\infty} |\hat{p}(t) - p(t)| = 0. \tag{4}$$

Define a positive scalar functional $J_1(x, \hat{x})$. We choose $J_1(x, \hat{x})$ in a way that when $J_1(x, \hat{x})$ is minimized, (4) is achieved. The total rate of change is given by the substantial derivatives of the $J_1(x, \hat{x})$

$$\frac{dJ_1}{dt} = \frac{\partial J_1}{\partial t} + \hat{f}(\hat{x}(t), \hat{p}(t))\nabla J_1, \tag{5}$$

where $\nabla = \left(\frac{\partial}{\partial \hat{x}_1}, \frac{\partial}{\partial \hat{x}_2}, \dots, \frac{\partial}{\partial \hat{x}_n}\right)^{\mathrm{T}}$ is the gradient operator in the estimated state space. Now, let

$$\frac{\partial \hat{p}}{\partial t} = -G\nabla_{\hat{p}}\frac{dJ_1}{dt}\,, \tag{6}$$

where $G > 0$ is the step length matrix. By defining,

$$J_1(x, \hat{x}) = \frac{w_1}{2}(\hat{x}_1 - x_1)^2 + \dots + \frac{w_n}{2}(\hat{x}_n - x_n)^2 = \sum_{i=1}^{n} \frac{w_i}{2}(e_i)^2, \tag{7}$$

where $e_i = \hat{x}_i - x_i$ are the state errors, we have

$$\frac{dJ_1}{dt} = \sum_{i=1}^{n} w_i e_i \left(\frac{de_i}{dt} + \hat{f}(\hat{x}(t), \hat{p}(t))\right). \tag{8}$$

As

$$\frac{de_i}{dt} = \frac{dx_i}{dt} - \frac{d\hat{x}_i}{dt} = \hat{f}(\hat{x}(t), \hat{p}(t)) - f(x(t), p(t)) \tag{9}$$

equation (8) reduces to

$$\frac{dJ_1}{dt} = \sum_{i=1}^{n} w_i e_i \left(2\hat{f}(\hat{x}(t), \hat{p}(t)) - f(x(t), p(t))\right). \tag{10}$$

Using equations (6) and (10) and because of $\nabla_{\hat{p}}\{f_i(x(t), p(t))\}$, we get

$$\frac{\partial \hat{p}}{dt} = -G\nabla_{\hat{p}}\frac{dJ_1}{dt} = -2G\nabla_{\hat{p}}\left\{\sum_{i=1}^{n} w_i e_i \hat{f}_i(\hat{x}(t), \hat{p}(t))\right\}. \tag{11}$$

This provides the updating law for estimated parameters in the identified system.

3 Improvement of the SG Method

In this part we adaptively select step length matrix, G, in a way to minimize a new local objective functional J_2. For this purpose we use descent gradient method

$$\frac{dg_{ij}}{dt} = -\alpha\frac{dJ_2}{dg_{ij}}\,, \tag{12}$$

where $G = |g_{ij}|,\ i,j = 1,2,\dots,k$ and $\alpha > 0$. Consider the following objective functional candidate

$$J_2(x,\hat{x}) = \frac{v_1}{2}(\hat{x}_1 - x_1)^2 + \cdots + \frac{v_n}{2}(\hat{x}_n - x_n)^2 = \sum_{i=1}^{n} \frac{v_i}{2}(e_i)^2, \tag{13}$$

where $e_i = \hat{x}_i - x_i$ are the state errors, we will have

$$\frac{dJ_2}{dg_{ij}} = \frac{\partial J_2}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial \hat{p}} \frac{\partial \hat{p}}{dg_{ij}}\,. \tag{14}$$

By using (13) we have

$$\frac{\partial J_2}{\partial \hat{x}} = [v_1 e_1,\ v_2 e_2,\ \dots,\ v_n e_n]_{n\times 1}. \tag{15}$$

For calculation of $\dfrac{\partial \hat{x}}{\partial \hat{p}}$ we use (2)

$$\frac{\partial \dot{\hat{x}}}{\partial \hat{p}} = \frac{\partial}{\partial \hat{p}} \frac{d\hat{x}}{dt} = \frac{d}{dt} \frac{\partial \hat{x}}{\partial \hat{p}} = \frac{\partial \hat{f}}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial \hat{p}} + \frac{\partial \hat{f}}{\partial \hat{p}}\,, \tag{16}$$

where $\hat{x} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]$ and it is assumed that

$$\frac{\partial}{\partial \hat{p}} \frac{d\hat{x}}{dt} = \frac{d}{dt} \frac{\partial \hat{x}}{\partial \hat{p}} \tag{17}$$

and the sufficient conditions for the validity of this equality are given in [23], page 279.

We have that $\hat{x}(t) = f(t,\hat{p})$ is continuous. $\dfrac{\partial \hat{x}}{\partial t}$, $\dfrac{\partial \hat{x}}{\partial \hat{p}}$ and $\dfrac{\partial}{\partial \hat{p}} \dfrac{\partial \hat{x}}{\partial t}$ exist and they are piece-wise continuous. Hence, equation (17) is valid. By using (3), (16) we get

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial \hat{p}}\right)_{n\times k} &= \frac{d}{dt}\begin{bmatrix} \dfrac{\partial \hat{x}_1}{\partial \hat{p}_1} & \cdots & \dfrac{\partial \hat{x}_1}{\partial \hat{p}_k} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \hat{x}_n}{\partial \hat{p}_1} & \cdots & \dfrac{\partial \hat{x}_n}{\partial \hat{p}_k} \end{bmatrix} \\
&= \begin{bmatrix} \displaystyle\sum_{i=1}^{n}\left(\frac{\partial \hat{f}_1}{\partial \hat{x}_i}\frac{\partial \hat{x}_i}{\partial \hat{p}_1}\right) + \frac{\partial \hat{f}_1}{\partial \hat{p}_1} & \cdots & \displaystyle\sum_{i=1}^{n}\left(\frac{\partial \hat{f}_1}{\partial \hat{x}_i}\frac{\partial \hat{x}_i}{\partial \hat{p}_k}\right) + \frac{\partial \hat{f}_1}{\partial \hat{p}_k} \\ \vdots & \ddots & \vdots \\ \displaystyle\sum_{i=1}^{n}\left(\frac{\partial \hat{f}_n}{\partial \hat{x}_i}\frac{\partial \hat{x}_i}{\partial \hat{p}_1}\right) + \frac{\partial \hat{f}_n}{\partial \hat{p}_1} & \cdots & \displaystyle\sum_{i=1}^{n}\left(\frac{\partial \hat{f}_n}{\partial \hat{x}_i}\frac{\partial \hat{x}_i}{\partial \hat{p}_k}\right) + \frac{\partial \hat{f}_n}{\partial \hat{p}_k} \end{bmatrix},
\end{aligned} \tag{18}$$

where $\hat{x} \in R^n$ is the estimated state vector, $\hat{p}(t) \in R^k$ is the vector of estimated parameters.

For calculation of $\dfrac{\partial \hat{p}}{\partial g_{ij}}$ we use (6)

$$\frac{\partial \hat{p}}{dt} = -G\nabla_{\hat{p}} \frac{dJ_1}{dt} = -G f_{\hat{p}}(\hat{x}(t), \hat{p}(t), g_{ij}, t), \tag{19}$$

where $f_{\hat{p}}(\hat{x}(t), \hat{p}(t), g_{ij}, t) = \left[f^1_{\hat{p}}(\hat{x}(t), \hat{p}(t), g_{ij}, t), \ldots, f^k_{\hat{p}}(\hat{x}(t), \hat{p}(t), g_{ij}, t)\right]^T$ and we have

$$\frac{\partial \dot{\hat{p}}}{\partial g_{ij}} = \frac{\partial}{\partial g_{ij}} \frac{d\hat{p}}{dt} = \frac{d}{dt} \frac{\partial \hat{p}}{\partial g_{ij}} = \frac{\partial f_{\hat{p}}}{\partial \hat{p}} \frac{\partial \hat{p}}{\partial g_{ij}} + \frac{\partial f_{\hat{p}}}{\partial g_{ij}}, \quad \hat{p} = [\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_k]. \tag{20}$$

Using (20) gives

$$\frac{d}{dt}\left(\frac{\partial \hat{p}}{\partial \hat{g}_{ij}}\right)_{k\times k^2} = \frac{d}{dt}\begin{bmatrix} \frac{\partial \hat{p}_1}{\partial \hat{g}_{11}} & \frac{\partial \hat{p}_1}{\partial \hat{g}_{12}} & \cdots & \frac{\partial \hat{p}_1}{\partial \hat{g}_{kk}} \\ \frac{\partial \hat{p}_2}{\partial \hat{g}_{11}} & \ddots & & \vdots \\ \vdots & & & \\ \frac{\partial \hat{p}_k}{\partial \hat{g}_{11}} & \cdots & & \frac{\partial \hat{p}_k}{\partial \hat{g}_{kk}} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{k}\left(\frac{\partial \hat{f}^1_{\hat{p}}}{\partial \hat{p}_i}\frac{\partial \hat{p}_i}{\partial \hat{g}_{11}}\right) + \frac{\partial \hat{f}^1_{\hat{p}}}{\partial \hat{g}_{11}} & \sum_{i=1}^{k}\left(\frac{\partial \hat{f}^1_{\hat{p}}}{\partial \hat{p}_i}\frac{\partial \hat{p}_i}{\partial \hat{g}_{12}}\right) + \frac{\partial \hat{f}^1_{\hat{p}}}{\partial \hat{g}_{12}} & \cdots & \sum_{i=1}^{k}\left(\frac{\partial \hat{f}^1_{\hat{p}}}{\partial \hat{p}_i}\frac{\partial \hat{p}_i}{\partial \hat{g}_{kk}}\right) + \frac{\partial \hat{f}^1_{\hat{p}}}{\partial \hat{g}_{kk}} \\ \sum_{i=1}^{k}\left(\frac{\partial \hat{f}^2_{\hat{p}}}{\partial \hat{p}_i}\frac{\partial \hat{p}_i}{\partial \hat{g}_{11}}\right) + \frac{\partial \hat{f}^2_{\hat{p}}}{\partial \hat{g}_{11}} & \ddots & & \vdots \\ \vdots & & & \\ \sum_{i=1}^{k}\left(\frac{\partial \hat{f}^k_{\hat{p}}}{\partial \hat{p}_i}\frac{\partial \hat{p}_i}{\partial \hat{g}_{11}}\right) + \frac{\partial \hat{f}^k_{\hat{p}}}{\partial \hat{g}_{11}} & \cdots & & \sum_{i=1}^{k}\left(\frac{\partial \hat{f}^k_{\hat{p}}}{\partial \hat{p}_i}\frac{\partial \hat{p}_i}{\partial \hat{g}_{kk}}\right) + \frac{\partial \hat{f}^k_{\hat{p}}}{\partial \hat{g}_{kk}} \end{bmatrix} \tag{21}$$

By using (15), (18), (21) in (14), using (12), and by using appreciate numerical methods we can adaptively calculate G in each iteration. With the calculated G and (6) we can estimate system parameters. For assigning the initial value of step length matrix, $G(0)$, we could use genetic algorithms. In (12) $\alpha > 0$ is arbitrary. But we can use a suitable 1-dimensional nonlinear programming method [17, 20], in each iteration, to calculate α.

4 Adaptive Calculation of Lyapunov Exponents

A Jacobian approach is used to calculate the Lyapunov exponents. Let the discrete time system be described by

$$x(k) = f(x(k-1)), \quad k = 0, 1, \ldots, \tag{22}$$

where $x(k) \in R^n$ is the state vector and $f(.)$ is a continuously differentiable smooth function. Linearization of the system gives

$$x_k = J(k-1)x(k-1), \quad J(k-1) = \left(\frac{\partial f}{\partial x}\right)\Bigg|_{k-1} \in R^{n\times n}. \tag{23}$$

Lyapunov exponents are defined as follows [14].

Definition 4.1 Let $Y^k = J_{k-1}J_{k-2}\ldots J_0$, then the following symmetric positive definite matrix

$$\wedge = \lim_{t\to\infty} \left((Y^k)^{\mathrm{T}} \cdot Y^k\right)^{\left(\frac{1}{2k}\right)} \tag{24}$$

exists and the logarithms of its eigenvalues are called Lyapunov exponents.

To compute the Lyapunov exponents a QR factorization is used to decompose the Jocobian matrix as $J = QR$, where Q is an orthonormal matrix and R is upper triangular with positive diagonal elements. Then, using (23) Lyapunov exponents become

$$\lambda_i = \lim_{t\to\infty} \frac{1}{k} \ln(R_k^i \dots R_0^i) = \lim_{t\to\infty} \frac{1}{k} \sum_{i=1}^{k} \ln(R_k^i), \tag{25}$$

where R_k^i is the i-th diagonal element of R in k-th step. We can rewrite equation (24) into following recursive form

$$\lambda_i = \frac{t_{n-1}}{t_n} \lambda_{k-1}^i + \frac{1}{t_n} Ln(R_k^i). \tag{26}$$

5 Controller Design Methodology

In this section, an adaptive controller based on Lyapunov exponents placement is proposed. Calculating the Lyapunov exponents of the open loop system (λ_{ol}), if there exists at least one positive Lyapunov exponent, then the system is chaotic. For suppressing chaotic behavior of system, we choose some suitable negative Lyapunov exponents for closed loop system (λ_{cl}). Then the control input (u_k) is applied to the open loop system, equation (22)

$$\begin{cases} x(k) = f(x(k-1)) + u_k, \\ x(0) \text{ is given,} \end{cases} \tag{27}$$

where u_k is calculated from an adaptive state feedback law [16]

$$u_k = B_k x_k. \tag{28}$$

Let λ_{ol} and λ_{cl} be the Jacobians of the open loop and closed loop systems, respectively. Then

$$J_{cl} = J_{ol} + B_k. \tag{29}$$

To assign the Lyapunov exponents of the closed loop system in the desired locations, feedback matrix B_k, is calculated from following equation

$$B_k = -J_{cl}(k) + \begin{bmatrix} e^{\lambda_{cl}^1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & e^{\lambda_{cl}^n} \end{bmatrix}. \tag{30}$$

The Lyapunov exponent of the closed loop system will be the desired ones and since they are negative the system will suppress chaos.

If the control action is large and we want a smaller control input for suppressing chaos, we can apply control action when we are in a neighborhood of the desired (equilibrium) point.

6 Simulation Results

In this section, simulation results are used to show the main points of the paper. The first example is the NLP chaotic Duffing system. The second example is the Lorenz chaotic system which is LP and the proposed method, with fixed step length matrix, is compared with the previous Lyapunov exponent identification strategy based on recursive least squares estimation. The third example compares proposed method with fixed step length matrix and adaptive step length matrix to show the effectiveness of the improved method.

Example 6.1 State equations of the forced Duffing's oscillator are

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -p_1 x_1 - p_2 x_1^3 - p x_2 + q\cos(wt), \end{cases} \tag{31}$$

where p_1, p_2, p, q and w are the parameters of the NLP system. And x_1, x_2 are the states of the system. This equation arises in models of the forced vibration of buckled beams and in electrical circuits [17, 18, 19, 20]. For $p = 0.168$, $p_1 = -0.5$, $p_2 = 0.5$, $q = 0.21$, $w = 1$ system is chaotic. Figure 6.1 is the Lyapunov exponents of this uncontrolled system. One of the exponents is positive then the system is chaotic.

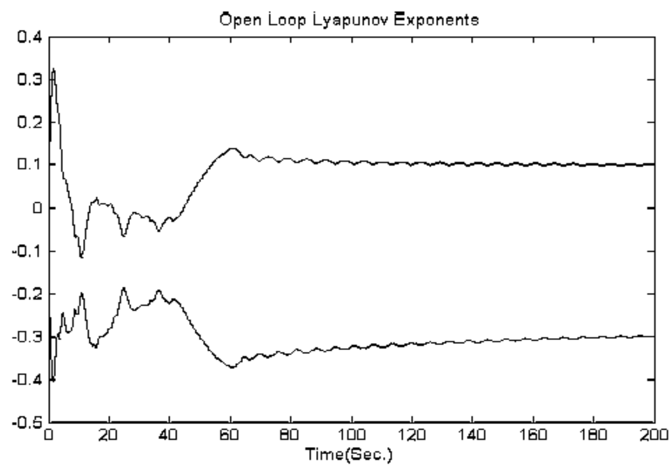

Figure 6.1: Lyapunov exponents of the open loop Duffing system.

Selected sampling period is 0.004 second and the desired closed loop Lyapunov exponents are $\lambda_{cl}^1 = -0.5$, $\lambda_{cl}^2 = -0.6$. Figure 6.2 shows the estimated parameters that converge to the real values. Figure 6.3 shows the states of the system where in the 30-th second, the control signal is applied to the system. Figure 6.3 shows the chaotic behavior of the open loop system. Chaotic System has aperiodic noise-like behavior.

Figure 6.4 shows the closed loop Lyapunov exponents that converge to the desired values. Prior to 30-th second Lyapunov exponents are for the open loop system and after that they converge to the desired values $(-0.5, -0.6)$. It is important to know that the control input implemented when system is chaotic. Figure 6.5 shows the control input. Figure 6.6 shows the states of the closed loop system that becomes a stable limit cycle.

Example 6.2 In this example, the proposed method is applied to the Lorenz system and the results are compared with the RLS method. State space equations of the system

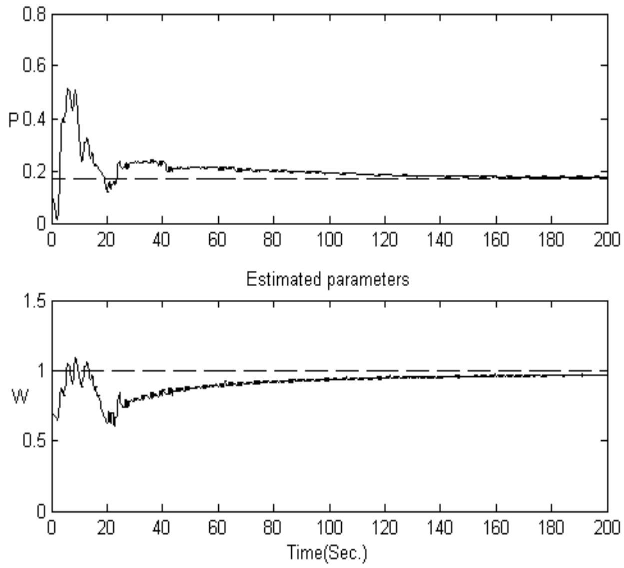

Figure 6.2: Estimated parameters.

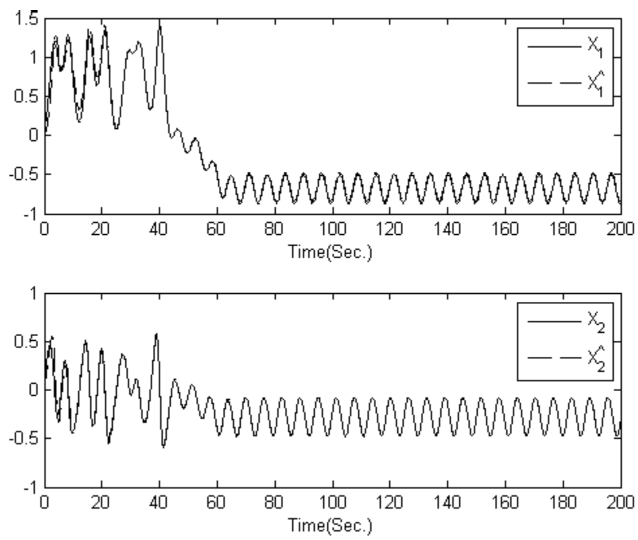

Figure 6.3: States of the Duffing system.

are

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = rx - y - zx, \\ \dot{z} = yx - bz, \end{cases} \tag{32}$$

where parameters are $r = 27$, $\sigma = 10$ and $b = 8/3$. This equation arises in models of

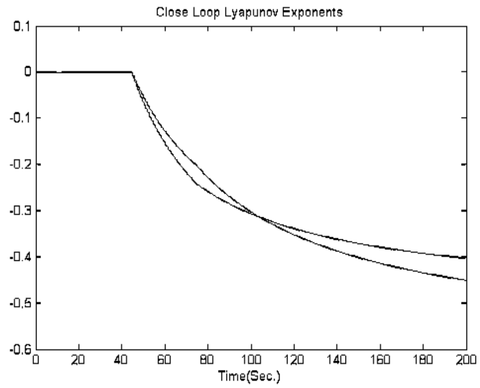

Figure 6.4: Closed loop system Lyapunov exponents.

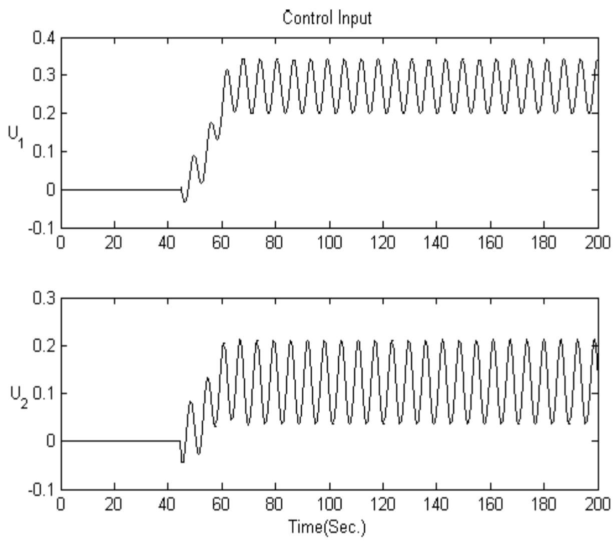

Figure 6.5: Control input.

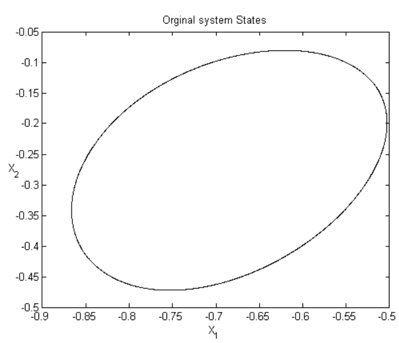

Figure 6.6: States of closed loop Duffing system.

the turbulent motion in convection systems [21, 22, 12]. The equilibrium points of this system are $(\pm\sqrt{b(r-1)}, \pm\sqrt{b(r-1)}, r-1)$ and $(0,0,0)$. Figure 6.7 shows estimated parameters with RLS method. And Figure 6.8 shows the estimated parameters with the proposed method.

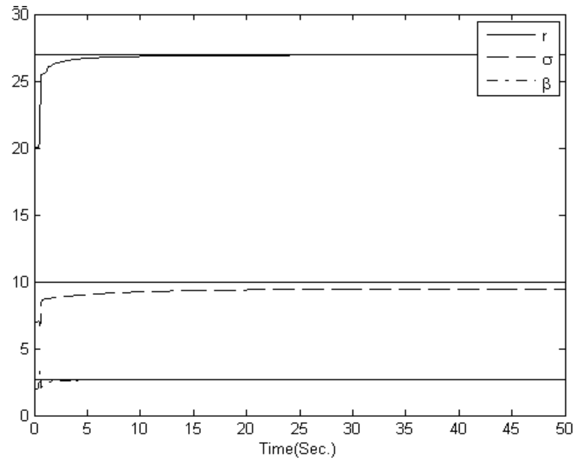

Figure 6.7: Estimated parameters of the system by RLS method.

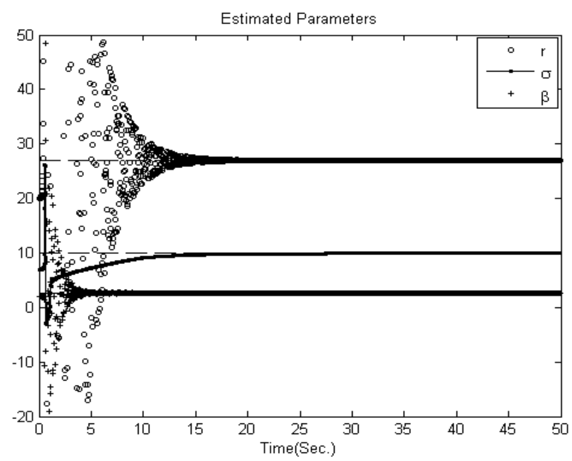

Figure 6.8: Estimated parameters of system by the proposed method.

It is obvious that the estimation performance of the proposed method is superior to the RLS based approach in faster convergence and unbiased estimates.

Example 6.3 In this example we want to show the effectiveness of the improved SG method. We have used the following $G(0)$, which is resulted from genetic algorithms. Figure 6.9 is resulted from the simulation of equation (6).

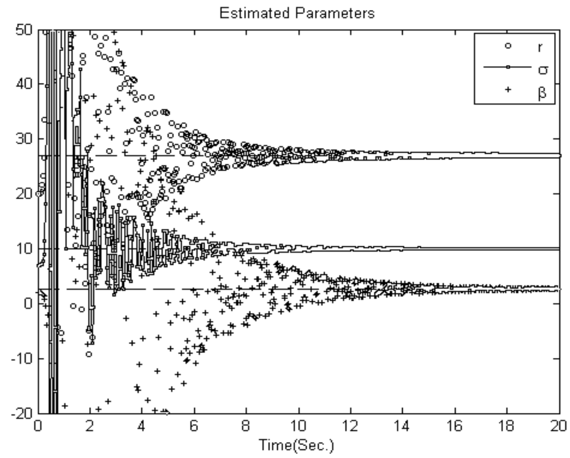

Figure 6.9: Estimated parameters of system by the proposed method.

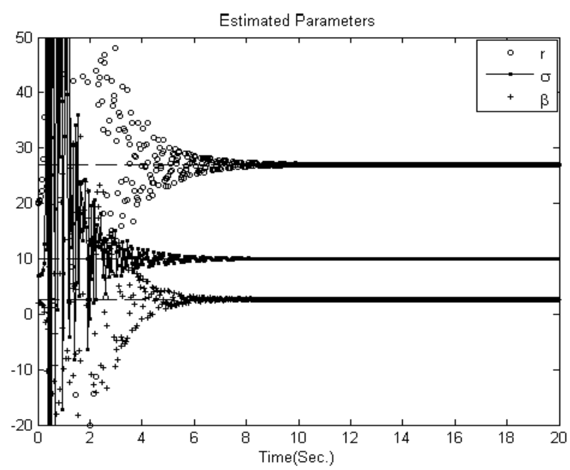

Figure 6.10: Estimated parameters of system by the Improved SG method.

Figure 6.10 is resulted from simulation of (6) and (12). Comparison of the figures intuitively clears that the Improved SG Method has faster convergence rate.

$$\begin{bmatrix} 48.5 & 0 & 5.5711 \\ 0 & 434.25 & 4815 \\ 1.2119 & 0.7902 & 48.1986 \end{bmatrix}.$$

7 Conclusions

This paper provides an estimation method for on-line identification of the Lyapunov exponents of nonlinear in parameters chaotic systems. This method is based on the minimization of two objective functionals. For faster convergence rate of the parameters, the new improved SG (ISG) method is developed. Also, the parameter estimation and Lyapunov exponent placement methods are combined for adaptive control of NLP chaotic systems. Simulation results are provided for adaptive control of Duffing's Oscillator. Also, a comparison with the RLS method in LP systems is given to show the superior performance of the proposed method. Finally, we showed the effectiveness of the improved SG Method in identification of the parameters of the Lorenz system.

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