Nonlinear Dynamics of a Two-Degrees of Freedom Hamiltonian System: Bifurcations and Integration

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Abstract: In this paper we treat the motion induced by a starting pulse on a system of two-degrees of freedom \( s, \theta \). Decoupling the motion equations, we obtain the \( s \)-nonlinear ordinary differential equation

\[
\ddot{s} = c^2 \frac{s}{(d^2 + s^2)^2} - \lambda^2 s,
\]

where \((c, d, \lambda) > 0\), and the dots mean time derivatives. A bifurcation analysis has revealed the onset of periodic motions for \( \lambda \neq 0 \) (presence of elastic forces inside the system), whilst for \( \lambda = 0 \) nonperiodic motions will appear. Almost all the cases (five for \( \lambda \neq 0 \), three for \( \lambda = 0 \)) have been integrated by obtaining \( t = t(s) \) by means of the Jacobi elliptic functions.

The other (angle) coordinate \( \theta \) has been in any case brought to the quadratures by knowing \( s \).

Keywords: Nonlinear differential equations; Hamiltonian systems; bifurcations; elliptic functions.

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