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Nonlinear Dynamics of a Two-Degrees of Freedom Hamiltonian System: Bifurcations and Integration ⋆

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Abstract: In this paper we treat the motion induced by a starting pulse on a system of two-degrees of freedom s, θ. Decoupling the motion equations, we obtain the s-nonlinear ordinary differential equation

$$\ddot{s} = c^2 \frac{s}{\left(d^2 + s^2\right)^2} - \lambda^2 s,$$

where $(c, d, \lambda) > 0$, and the dots mean time derivatives. A bifurcation analysis has revealed the onset of periodic motions for $\lambda \neq 0$ (presence of elastic forces inside the system), whilst for $\lambda = 0$ nonperiodic motions will appear. Almost all the cases (five for $\lambda \neq 0$, three for $\lambda = 0$) have been integrated by obtaining $t = t(s)$ by means of the Jacobi elliptic functions.
The other (angle) coordinate θ has been in any case brought to the quadratures by knowing s.

Keywords: *Nonlinear differential equations; Hamiltonian systems; bifurcations; elliptic functions.*

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