Constrained Linear Quadratic Regulator:  
Continuous-Time Case

M.F. Hassan\textsuperscript{1} and E.K. Boukas\textsuperscript{2*}

\textsuperscript{1} Department of Electrical Engineering, University of Kuwait,  
P. O. Box 5969, Safat, 13060, Kuwait  
\textsuperscript{2} Mechanical Engineering Department, École Polytechnique de Montréal,  
P.O. Box 6079, Station “Centre-ville”, Montréal, Québec, Canada H3C 3A7.

Received: December 29, 2005; Revised: June 30, 2007

Abstract: This paper deals with the linear quadratic regulator with constraints on the state and the input vectors. Such an optimization problem has a wide applications in industry like chemical and manufacturing industries. Our goal in this paper consists of developing an efficient numerical algorithm to solve such problem. Our technique relays on an iterative approach that uses the solution of the standard linear quadratic regulator as an initial guess for the optimal solution and then iteratively, the solution is improved by designing a controller that compensates for the violation of the constraints at each iteration. A numerical example is given to show the effectiveness of this algorithm.

Keywords: Linear systems; linear quadratic regulator; constrained input; constrained state.

Mathematics Subject Classification (2000): 49N10, 49N35.

1 Introduction

The linear quadratic regulator (LQR) is one of the most studied control problem in the literature. It will require many pages to cite all the works that were reported in the literature on the subject. In fact there are many variants. If we restrict ourselves to the case of LQR with constrained states and inputs, this variant consists of designing a state feedback controller that drives the state from a nonzero initial condition to zero by respecting simultaneously the constraints on the state and the control vectors.

This control problem has many applications in industry. In fact to motivate our study, let us consider a deterministic manufacturing system that produces \( n \)-items that

\* Corresponding author: el-kebir.boukas@polymtl.ca

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