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Robust Stability for Nonlinear Uncertain Neural Networks with Delay

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Abstract: The robust stability of nonlinear uncertain neural networks with constant or time-varying delays is studied. An approach combining the Lyapunov-Krasovskii functional with the linear matrix inequality is taken to study the problems. Some criteria for robust stability of neural networks with time delays are derived.

Keywords: *Nonlinear uncertain neural network; delay; robust stability; linear matrix inequality; Lyapunov-Krasovskii functional*

Mathematics Subject Classification (2000): 93C10, 92B20

1 Introduction

In applications of neural networks with or without delays to some practical problems, such as optimization solvers [1], pattern recognition, image compression [2], and quadratic programming problems [3, 4], the stability properties of system play an important role. The stability analysis for the neural network has received considerable attention in recent years. It is well known that the stability of neural network is prerequisite for the applications either as pattern recognition or as optimization solvers. There have been extensive results presented on the stability analysis of neural network and its applications. Moreover, parameter fluctuation in neural network implementation on very large scale integration (VLSI) chips is also unavoidable. This fact implies that a good neural network should have certain robustness which paves the way for introducing the theory of interval matrices and interval dynamics to investigate the global stability of interval neural networks. There exist several related results on robust stability, we refer to [5–9]. In recent years, the dynamics of neural network systems have been deeply investigated

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and many important results on the global asymptotic stability and global exponential stability have been established(see for example [10–13, 19]).

In this paper, we shall investigate the problem of the robust stability analysis for nonlinear uncertain neural networks with constant or time-varying delays. Some sufficient conditions for the delay independent robust stability of the neural networks are developed. All of the results are presented in terms of linear matrix inequalities (LMIs).

The paper is organized as follows. In Section 2, the problem to be investigated is stated and some definitions and lemmas are listed. Based on the Lyapunov-Krasovskii stability theory and the LMI approach, some criteria are obtained in Section 3. Two cases of time delay, i.e. constant delays case and time-varying delays case are discussed. Then, an exponential stability criterion for the considered neural networks is provided. A numerical example is given in Section 4. Finally, some conclusions are drawn in Section 5.

2 System Description and Preliminaries

Consider a nonlinear uncertain neural network with time delay, which is described by a set of functional differential equations as follows

$$\begin{aligned} \dot{x}_i(t) = -a_i x_i(t) - a_{di} x_i(t-\tau) + \sum_{j=1}^{n} b_{ij} f_j[x_j(t)] \\ + \sum_{j=1}^{n} b_{dij} f_j[x_j(t-\tau)] + c_i g_i(x_i(t), x_i(t-\tau)), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

or, the considered neural networks can be represented in vector state space form as follows

$$\dot{x}(t) = -Ax(t) - A_1 x(t-\tau) + Bf[x(t)] + B_1 f[x(t-\tau)] + Cg(x(t), x(t-\tau)), \tag{2}$$

with initial values

$$x(t) = \phi_x(t), \quad t \in [-\tau, 0), \tag{3}$$

where $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$, $A_1 = \mathrm{diag}(a_{d1}, a_{d2}, \ldots, a_{dn})$, $B = (b_{ij})_{n\times n}$, $B_1 = (b_{dij})_{n\times n}$, $C = \mathrm{diag}(c_1, c_2, \ldots, c_n)$; $x(t) = (x_1(t), \ldots, x_n(t))^T$ is the state vector of the neural network; $x(t-\tau) = (x_1(t-\tau), \ldots, x_n(t-\tau))^T$ is the delayed state vector of the neural networks; $\tau > 0$ denotes the delay; $g(x(t), x(t-\tau))$ is the uncertain perturbation with the form of $g(x(t), x(t-\tau)) = [g_1(x_1(t), x_1(t-\tau)), \ldots, g_n(x_n(t), x_n(t-\tau))]^T$; the activation function is $f[x(t)] = \{f_1[x_1(t)], \ldots, f_n[x_n(t)]\}^T$.

Though out this paper, we assume that the activation function $f_i[x_i(t)]$ ($i = 1, 2, \ldots, n$) and the nonlinear uncertain perturbation function $g(x(t), x(t-\tau))$ satisfy the following conditions:

(H_1) If there exist positive constants k_i, $i = 1, \ldots, n$, such that

$$0 < \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \le k_i$$

for all $\xi_1, \xi_2 \in R$, $\xi_1 \ne \xi_2$, $i = 1, \ldots, n$.

(H_2) There exist positive constant matrices M and M_1, such that

$$\|g(x(t), x(t-\tau))\| \le \|Mx(t)\| + \|M_1 x(t-\tau)\|.$$

In order to obtain our results, we need the following definitions and lemmas.

Definition 2.1 For any continuous function $V : R \to R$, Dini's time-derivative of $V(t)$ is defined as

$$D^+V(t) = \lim_{h\to 0^+} \sup \frac{V(t+h) - V(t)}{h} \,. \tag{4}$$

It is easy to see that if $V(t)$ is locally Lipschitz, then $|D^+V(t)| < \infty$.

Lemma 2.1 [14] (Lyapunov-Krasovskii stability theorem) *Consider the following functional-differential equation of the retarded type:*

$$\dot{x}(t) = f(t, x_t), \quad t \ge t_0, \quad x_{t_0} = \phi(\theta), \quad \forall\, \theta \in [-\tau, 0], \tag{5}$$

where $x_t(\cdot)$, for given $t \ge t_0$, denotes the restriction of $x(\cdot)$ to the interval $[t-\tau, t]$ translated to $[-\tau, 0]$, namely $x_t(\theta) = x(t+\theta)$, $\forall\, \theta \in [-\tau, 0]$.

Assume that there exists a continuous functional $V(t, \phi)$ such that

(i) $V_1(\|\phi(0)\|) \le V(t, \phi) \le V_2(\|\phi(\theta)\|)$;

(ii) $\dot{V}(t, x_t) \le -V_3(\|x(t)\|)$,

where $V_1, V_2, V_3 :\ R_+ \to R_+$ are continuous nondecreasing functions, $V_1(s)$, $V_2(s)$ are positive for $s > 0$, and $V_1(0) - V_2(0) = 0$, $V_3(s) > 0$ for $s > 0$, and $\dot{V}(t, x_t)$ is Dini's time-derivative of $V(t, x_t)$ along the solution of equation (5). Then, the trivial solution of equation (5) is uniformly asymptotically stable.

Notice that condition (i) means that the function $V(t, \phi)$ is positive definite and has an infinitesimal upper limit.

Lemma 2.2 [15] *Given any real matrices A, B, C of appropriate dimensions and a scalar $\varepsilon > 0$ such that $0 < C = C^T$. Then, the following inequality holds:*

$$A^TB + B^TA \le \varepsilon A^TCA + \varepsilon^{-1}B^TC^{-1}B, \tag{6}$$

where the superscript T means the transpose of a matrix.

Lemma 2.3 [15] (Schur complement) *Linear matrix inequality:*

$$\begin{pmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{pmatrix} > 0, \tag{7}$$

with $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ is the same as

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$$

3 Main Results

In this section, stability criteria for uncertain neural networks with time delay are given.

Theorem 3.1 *Consider the delayed neural networks with nonlinear perturbation (1), if there exist positive matrices $X > 0$, $W > 0$, positive diagonal matrices $S > 0$, $S_1 > 0$, and constants $\xi_1 > 0$, $\xi_2 > 0$, satisfying the LMI*

$$\Omega = \begin{bmatrix} \Omega_{11} & XK^T & XA_1^T & XM_1^T & XM^T \\ KX & -S - S_1 & 0 & 0 & 0 \\ A_1X & 0 & -W & 0 & 0 \\ M_1X & 0 & 0 & -\xi_2 I & 0 \\ MX & 0 & 0 & 0 & -\xi_1 I \end{bmatrix} < 0, \tag{8}$$

where

$$\Omega_{11} = -XA^T - AX + W + B_1S_1B_1^T + C(\xi_1+\xi_2)C^T,$$

then the system (1) is globally asymptotically stable. Here, $K = \mathrm{diag}\{k_1,\ldots,k_n\}$.

Proof Consider the Lyapunov functional

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-\tau}^{t} f^T[x(s)]S_1^{-1}f[x(s)]\,ds \\ &\quad + \int_{t-\tau}^{t} x^T(s)A_1^TW^{-1}A_1x(s)\,ds + \xi_2^{-1}\int_{t-\tau}^{t} x^T(s)M_1^TM_1x(s)\,ds. \end{aligned} \tag{9}$$

It is easy to obtain

$$\begin{aligned} \lambda_m(P)\|x(t)\|^2 \le V(t) \le \Big\{\lambda_M(P) + \tau\Big[\lambda_M(K^TS_1^{-1}K) \\ + \lambda_M(A_1^TW^{-1}A_1) + \xi_2^{-1}\lambda_M(M_1^TM_1)\Big]\Big\}\|x(t)\|^2, \end{aligned}$$

where $\lambda_m(P)$ and $\lambda_M(P)$ denote the minimum and maximum eigenvalues of P, respectively.

Calculating the upper right derivative D^+V of (9) along the solution of (2), we have that

$$\begin{aligned} D^+V(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &\quad + f^T[x(t)]S_1^{-1}f[x(t)] - f^T[x(t-\tau)]S_1^{-1}f[x(t-\tau)] \\ &\quad + x^T(t)A_1^TW^{-1}A_1x(t) - x^T(t-\tau)A_1^TW^{-1}A_1x(t-\tau) \\ &\quad + \xi_2^{-1}x^T(t)M_1^TM_1x(t) - \xi_2^{-1}x^T(t-\tau)M_1^TM_1x(t-\tau) \\ &= \{-Ax(t) - A_1x(t-\tau) + Bf[x(t)] \\ &\quad + B_1f[x(t-\tau)] + Cg(x(t),x(t-\tau))\}^T\,Px(t) \\ &\quad + x^T(t)P\,\{-Ax(t) - A_1x(t-\tau) + Bf[x(t)] \\ &\quad + B_1f[x(t-\tau)] + Cg(x(t),x(t-\tau))\} \\ &\quad + f^T[x(t)]S_1^{-1}f[x(t)] - f^T[x(t-\tau)]S_1^{-1}f[x(t-\tau)] \\ &\quad + x^T(t)A_1^TW^{-1}A_1x(t) - x^T(t-\tau)A_1^TW^{-1}A_1x(t-\tau) \\ &\quad + \xi_2^{-1}x^T(t)M_1^TM_1x(t) - \xi_2^{-1}x^T(t-\tau)M_1^TM_1x(t-\tau). \end{aligned} \tag{10}$$

From Lemma 2.2 and (10), we have that

$$\begin{aligned} D^+V(t) &\le [-x^T(t)A^TPx(t) - x^T(t)PAx(t)] \\ &\quad + x^T(t-\tau)A_1^TW^{-1}A_1x(t-\tau) + x^T(t)PWPx(t) \\ &\quad + f^T[x(t)]S^{-1}f[x(t)] + x^T(t)PBSB^TPx(t) \\ &\quad + f^T[x(t-\tau)]S_1^{-1}f[x(t-\tau)] + x^T(t)PB_1S_1B_1^TPx(t) \\ &\quad + x^T(t)PC(\xi_1+\xi_2)C^TPx(t) \\ &\quad + \xi_1^{-1}x^T(t)M^TMx(t) + \xi_2^{-1}x^T(t-\tau)M_1^TM_1x(t-\tau) \\ &\quad + f^T[x(t)]S_1^{-1}f[x(t)] - f^T[x(t-\tau)]S_1^{-1}f[x(t-\tau)] \\ &\quad + x^T(t)A_1^TW^{-1}A_1x(t) - x^T(t-\tau)A_1^TW^{-1}A_1x(t-\tau) \end{aligned}$$

$$
\begin{aligned}
&+ \xi_2^{-1} x^T(t) M_1^T M_1 x(t) - \xi_2^{-1} x^T(t-\tau) M_1^T M_1 x(t-\tau) \\
\le\; &[-x^T(t) A^T P x(t) - x^T(t) P A x(t)] + x^T(t) P W P x(t) \\
&+ f^T[x(t)] S^{-1} f[x(t)] + x^T(t) P B S B^T P x(t) + x^T(t) P B_1 S_1 B_1^T P x(t) \\
&+ x^T(t) P C(\xi_1 + \xi_2) C^T P x(t) + \xi_1^{-1} x^T(t) M^T M x(t) \\
&+ f^T[x(t)] S_1^{-1} f[x(t)] + x^T(t) A_1^T W^{-1} A_1 x(t) + \xi_2^{-1} x^T(t) M_1^T M_1 x(t) \\
\le\; &x^T(t) \Xi x(t),
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi = \big[&-A^T P - PA + PWP + K^T S^{-1} K + PBSB^T P \\
&+ PB_1 S_1 B_1^T P + PC(\xi_1 + \xi_2) C^T P + \xi_1^{-1} M^T M \\
&+ K^T S_1^{-1} K + A_1^T W^{-1} A_1 + \xi_2^{-1} M_1^T M_1 \big].
\end{aligned} \tag{11}
$$

Pre- and post-multiply (11) with $X = P^{-1}$. By the Schur complement, $\Xi < 0$ if and only if inequality (8) holds.

This completes the proof. □

Remark 3.1 Noting that the conditions of Theorem 3.1 do not include any information of the delay, that is, the theorem provides a delay-independent robust stability criterion for time-delayed neural networks with nonlinear perturbations in terms of LMIs. The results can be extended to time-varying delay case.

Consider the time-varying delay neural networks as follows

$$
\dot{x}(t) = -Ax(t) - A_1 x(t-\tau(t)) + Bf[x(t)] + B_1 f[x(t-\tau(t))] + Cg(x(t), x(t-\tau(t))), \tag{12}
$$

where τ is a function, $\tau\colon [0, +\infty) \to [0, +\infty]$. Furthermore, we assume that τ is differentiable and $\dot{\tau}(t) \le \tau^* < 1$.

We have the following result.

Theorem 3.2 *Consider the delayed neural networks with nonlinear perturbation (1), if there exist positive matrices $X > 0$, $W > 0$, positive diagonal matrices $S > 0$, $S_1 > 0$ satisfying the LMI*

$$
\Omega = \begin{bmatrix}
\Omega_{11} & XK^T & XA_1^T & XM_1^T & XM^T \\
KX & -S - S_1 & 0 & 0 & 0 \\
A_1 X & 0 & -W & 0 & 0 \\
M_1 X & 0 & 0 & -I & 0 \\
MX & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \tag{13}
$$

where

$$
\Omega_{11} = -XA^T - AX + \frac{1}{1-\tau^*} W + \frac{1}{1-\tau^*} B_1 S_1 B_1^T + C\left(1 + \frac{1}{1-\tau^*}\right) C^T.
$$

Then the system (1) is globally asymptotically stable. Here, $K = \operatorname{diag}\{k_1, \dots, k_n\}$.

Proof Consider the Lyapunov functional

$$
\begin{aligned}
V(t) = x^T(t) P x(t) &+ \int_{t-\tau(t)}^{t} f^T[x(s)] S_1^{-1} f[x(s)]\, ds \\
&+ \int_{t-\tau(t)}^{t} x^T(s) A_1^T W^{-1} A_1 x(s)\, ds + \int_{t-\tau(t)}^{t} x^T(s) M_1^T M_1 x(s)\, ds.
\end{aligned} \tag{14}
$$

It is easy to obtain

$$\lambda_m(P)\|x(t)\|^2 \le V(t) \le \left\{\lambda_M(P) + \tau\left[\lambda_M(K^T S_1^{-1} K) + \lambda_M(A_1^T W^{-1} A_1) + \lambda_M(M_1^T M_1)\right]\right\}\|x(t)\|^2,$$

where $\lambda_m(P)$ and $\lambda_M(P)$ denote the minimum and maximum eigenvalues of P, respectively.

Calculating the upper right derivative D^+V of (14) along the solution of (12), we obtain that

$$\begin{aligned} D^+V(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &\quad + f^T[x(t)]S_1^{-1}f[x(t)] - (1-\tau^*)f^T[x(t-\tau(t))]S_1^{-1}f[x(t-\tau(t))] \\ &\quad + x^T(t)A_1^T W^{-1}A_1x(t) - (1-\tau^*)x^T(t-\tau(t))A_1^T W^{-1}A_1x(t-\tau(t)) \\ &\quad + x^T(t)M_1^T M_1x(t) - (1-\tau^*)x^T(t-\tau(t))M_1^T M_1x(t-\tau(t)) \\ &= \{-Ax(t) - A_1x(t-\tau(t)) + Bf[x(t)] \\ &\quad + B_1f[x(t-\tau(t))] + Cg(x(t), x(t-\tau(t)))\}^T(t)Px(t) \\ &\quad + x^T(t)P\{-Ax(t) - A_1x(t-\tau(t)) + Bf[x(t)] \\ &\quad + B_1f[x(t-\tau(t))] + Cg(x(t), x(t-\tau(t)))\} \\ &\quad + f^T[x(t)]S_1^{-1}f[x(t)] - (1-\tau^*)f^T[x(t-\tau(t))]S_1^{-1}f[x(t-\tau(t))] \\ &\quad + x^T(t)A_1^T W^{-1}A_1x(t) - (1-\tau^*)x^T(t-\tau(t))A_1^T W^{-1}A_1x(t-\tau(t)) \\ &\quad + x^T(t)M_1^T M_1x(t) - (1-\tau^*)x^T(t-\tau(t))M_1^T M_1x(t-\tau(t)). \end{aligned} \tag{15}$$

From Lemma 2.2 and (15), it follows that

$$\begin{aligned} D^+V(t) &\le [-x^T(t)A^TPx(t) - x^T(t)PAx(t)] \\ &\quad + (1-\tau^*)x^T(t-\tau(t))A_1^T W^{-1}A_1x(t-\tau(t)) \\ &\quad + f^T[x(t)]S^{-1}f[x(t)] + x^T(t)PBSB^TPx(t) \\ &\quad + (1-\tau^*)f^T[x(t-\tau(t))]S_1^{-1}f[x(t-\tau(t))] \\ &\quad + \frac{1}{1-\tau^*}x^T(t)PB_1S_1B_1^TPx(t) + \frac{1}{1-\tau^*}x^T(t)PWPx(t) \\ &\quad + x^T(t)PC\left(1 + \frac{1}{1-\tau^*}\right)C^TPx(t) \\ &\quad + x^T(t)M^TMx(t) + (1-\tau^*)x^T(t-\tau(t))M_1^T M_1x(t-\tau(t)) \\ &\quad + f^T[x(t)]S_1^{-1}f[x(t)] - (1-\tau^*)f^T[x(t-\tau(t))]S_1^{-1}f[x(t-\tau(t))] \\ &\quad + x^T(t)A_1^T W^{-1}A_1x(t) - (1-\tau^*)x^T(t-\tau(t))A_1^T W^{-1}A_1x(t-\tau(t)) \\ &\quad + x^T(t)M_1^T M_1x(t) - (1-\tau^*)x^T(t-\tau(t))M_1^T M_1x(t-\tau(t)) \\ &\le [-x^T(t)A^TPx(t) - x^T(t)PAx(t)] \\ &\quad + f^T[x(t)]S^{-1}f[x(t)] + x^T(t)PBSB^TPx(t) \\ &\quad + \frac{1}{1-\tau^*}x^T(t)PB_1S_1B_1^TPx(t) + \frac{1}{1-\tau^*}x^T(t)PWPx(t) \\ &\quad + x^T(t)PC\left(1 + \frac{1}{1-\tau^*}\right)C^TPx(t) + x^T(t)M^TMx(t) \end{aligned}$$

$$
\begin{aligned}
&+ f^T[x(t)]S_1^{-1}f[x(t)] + x^T(t)A_1^TW^{-1}A_1x(t) + x^T(t)M_1^TM_1x(t) \\
&\le x^T(t)\Xi x(t),
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi = \Big[&-A^TP - PA + \frac{1}{1-\tau^*}PWP + K^TS^{-1}K + PBSB^TP \\
&+ \frac{1}{1-\tau^*}PB_1S_1B_1^TP + PC\left(1 + \frac{1}{1-\tau^*}\right)C^TP + \xi_1^{-1}M^TM \\
&+ K^TS_1^{-1}K + A_1^TW^{-1}A_1 + \xi_2^{-1}M_1^TM_1 \Big].
\end{aligned}
$$

Pre- and post-multiply (16) with $X = P^{-1}$. By the Schur complement, $\Xi < 0$ if and only if inequality (13) holds.

This completes the proof. □

Remark 3.2 In Theorem 3.1 and Theorem 3.2, the delay-independent stability criteria are developed, however, no information on the state convergence degree of the neural networks is given. Here, we investigate the problem of exponential stability analysis for delayed neural networks.

Theorem 3.3 *Consider the delayed neural networks with nonlinear perturbation (1), if there exist positive matrices $X > 0$, $W > 0$, positive diagonal matrices $S > 0$, $S_1 > 0$, and constants $\xi_1 > 0, \xi_2 > 0, \alpha > 0$ satisfying the LMI*

$$
\Omega = \begin{bmatrix}
\Omega_{11} & e^{\alpha\tau}XK^T & e^{\alpha\tau}XA_1^T & e^{\alpha\tau}XM_1^T & e^{\alpha\tau}XM^T \\
e^{\alpha\tau}KX & -S-S_1 & 0 & 0 & 0 \\
e^{\alpha\tau}A_1X & 0 & -W & 0 & 0 \\
e^{\alpha\tau}M_1X & 0 & 0 & -\xi_2 I & 0 \\
e^{\alpha\tau}MX & 0 & 0 & 0 & -\xi_1 I
\end{bmatrix} < 0, \tag{17}
$$

where

$$
\Omega_{11} = -XA^T - AX + W + B_1S_1B_1^T + 2\alpha X + C(\xi_1 + \xi_2)C^T. \tag{18}
$$

Then the system (1) is exponential asymptotically stable. Here, $K = \mathrm{diag}\{k_1, \dots, k_n\}$.

Proof Let's introduce a transformation $x(t) = e^{-\alpha t}\eta(t)$, and define the Lyapunov functional as follows:

$$
\begin{aligned}
V(t) = \eta^T(t)P\eta(t) &+ \int_{t-\tau}^{t} f^T[\eta(s)]S_1^{-1}f[\eta(s)]\,ds \\
&+ \int_{t-\tau}^{t} \eta^T(s)A_1^TW^{-1}A_1\eta(s)\,ds + \xi_2^{-1}\int_{t-\tau}^{t} \eta^T(s)M_1^TM_1\eta(s)\,ds.
\end{aligned} \tag{19}
$$

Then follows the proof of Theorem 3.1, this theorem can be proved easily. □

Remark 3.3 As we can see, if $B \equiv 0$ and the uncertain perturbation $g(x(t), x(t-\tau)) \equiv 1$ in (2), then the neural network (1) or (2) represents the Hopfield's original neural network model and cellular neural networks [6, 7, 9, 10, 16, 17, 18].

4 An illustrative example

In this section, we present a numerical example to validate our results.

Example 4.1. We consider two-dimension nonlinear uncertain neural network (2) with time delay. The associated data are:

$$A = \begin{bmatrix} 1.8 & 0 \\ 0 & 1.8 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.01 & 0.02 \\ 0.03 & 1.08 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.32 & 0.45 \\ 0.30 & 0.50 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Suppose the activation function is described by $f_i(x) = \frac{1}{2}(|x+1| + |x-1|)$, $i = 1, 2$. Then we have $K = \mathrm{diag}(1, 1)$, and $f_i(x)$ satisfies (H_1). Now using the MATLAB LMI toolbox, we can obtain a feasible solution for LMI (8) as follows:

$$X = \begin{bmatrix} 56.5666 & 19.4526 \\ 19.4526 & 7.6137 \end{bmatrix} > 0, \quad W = \begin{bmatrix} 235.3002 & 178.2599 \\ 178.2599 & 146.4787 \end{bmatrix} > 0,$$
$$S = \begin{bmatrix} 983.9634 & 0 \\ 0 & 983.9634 \end{bmatrix} > 0, \quad S_1 = \begin{bmatrix} 802.8313 & 0 \\ 0 & 802.8313 \end{bmatrix} > 0, \quad \xi = 13.8307.$$

Then the conditions of Theorem 3.1 are satisfied. Therefore, the system (1) is globally asymptotically stable. Moreover, we can see from the behavior (see Figure 1) of the state variables, the solutions of system (1) converge upon the zero with the initial condition $\phi(s) = [0.1, -0.1]^T$.

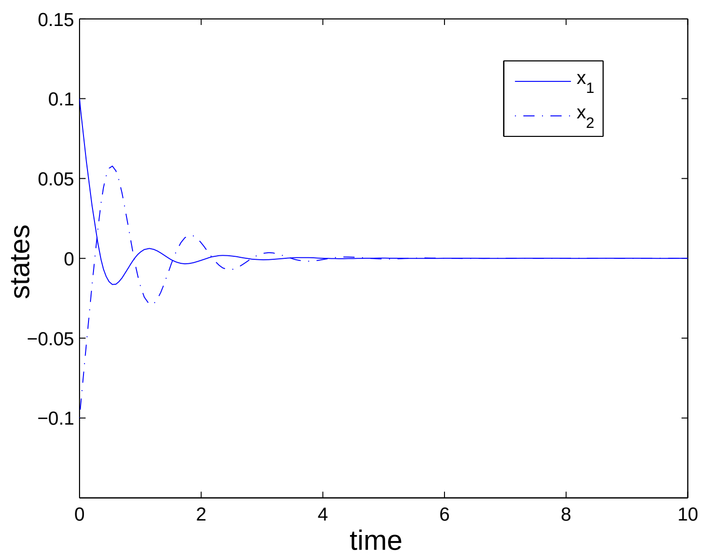

Figure 4.1: The time respond behavior of the system (1).

5 Conclusions

In this paper, the problem of robust stability analysis for uncertain neural networks with time delay is investigated. Based on Lyapunov stability theory, the robust stable criteria are given in terms of linear matrix inequalities. The proposed approach is more flexible in computation, and the results are more efficient then other existing results.

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