Uniform Convergence to Global Attractors for Discrete Disperse Dynamical Systems

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Abstract: In this paper we study uniform convergence of trajectories of discrete disperse dynamical systems generated by set-valued mappings to their global attractors. In particular, we show that this convergence holds even in the presence of computational errors.

Keywords: Compact metric space; set-valued mapping; trajectory; global attractor.

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1 Introduction

Dynamical systems theory has been a rapidly growing area of research which has various applications to physics, engineering, biology and economics. In this theory one of the goals is to study the asymptotic behavior of the trajectories of a dynamical system. A discrete-time dynamical system is described by a space of states and a transition operator which can be set-valued. Usually in the dynamical systems theory a transition operator is single-valued. In the present paper we study a class of dynamical systems introduced in [3] and studied in [4, 5] with a compact metric space of states and a set-valued transition operator. Such dynamical systems describe economical models [1, 2, 6].

Let $(X, ρ)$ be a compact metric space and let $a: X \rightarrow 2^X \setminus \{∅\}$ be a set-valued mapping whose graph

\[ \text{graph}(a) = \{(x, y) \in X \times X : y \in a(x)\} \]

is a closed subset of $X \times X$. For each nonempty subset $E \subset X$ set

\[ a(E) = \bigcup \{a(x) : x \in E\} \quad \text{and} \quad a^0(E) = E. \]

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