



Lyapunov Based on Cascaded Non-linear Control of Induction Machine

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Abstract: In this paper is developed Lyapunov based non-linear control to ensure the flux-speed tracking regime of voltage fed induction machine. The control law is determined in two steps, in the first the virtual control, based on Lyapunov function, is obtained in view to impose the flux-speed tracking. After this, is deduced the real control imposing the virtual control law. The simulation results of flux-speed tracking of induction machine show the validity of the proposed method in presence of strong parametric perturbations. Finally, an extension of the proposed method to most voltage alternating current (AC) machines is discussed. This allows to get a unified view for the control of electric AC machines.

Keywords: *Lyapunov method; virtual control; flux speed-tracking; induction machine; AC machines.*

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1 Introduction

One of the fundamental deficiencies of non linear control theory is the lack of a systematic design procedure for controllers synthesis. The earlier work of Lyapunov produced some of most powerful tools for control design that are still used up to date. In this work, the design problem is formulated in terms of finding a suitable state function (so called Lyapunov function) having some properties that guarantee boundedness of trajectories and convergence to an equilibrium point. Although this result is one of the most significant ones in control theory, there is no general theory for constructing such a Lyapunov

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function for a given non linear systems. Meanwhile, the backstepping methodology developed in [1] allows to construct recursively the non linear feedback law and its associated Lyapunov functions for a certain class of non linear systems. Beside this, a great effort was devoted to developing other methods for the control design of a certain class of non linear systems. Among these methods are found the following ones : a the diffeomorphic transformation, non linear feedback linearization, the sliding mode approach, and the dynamic linearization.

The analysis and control problem of cascaded non linear systems have been intensively studied during the last decades (see[2]-[7] and reference there in). In [2], based on the explicit construction of a Lyapunov function for a partially linear cascaded system, a stabilizing controller is designed for a special class of non linear cascaded systems. Sontag in [3] gave some sufficient conditions for asymptotic stabilization of two cascaded non linear systems. A passivity interpretation of this latter result is given in [4]. In [5], a wide class of time varying non linear systems is considered. The authors in [6] gave sufficient conditions under which an interconnected non linear system with parametric uncertainty is stabilizable. Singular H_∞ suboptimal control of a class of two blocks interconnected non linear systems is investigated [7].

Otherwise, the development of electrical machine drive grows more and more in order to follow the increasing need for various fields such as industry, electric cars, actuators, etc. By means of electrical machine drive, we can get high level of productivity in industry and product quality enhancement. Among, the most used electrical AC machine one can mention induction machine, permanent magnet synchronous machine, and synchronous machine. However, the induction machine is the machine of choice in many industrial applications due to its reliability, ruggedness and relatively low cost.

The control of electric machines has become an active domain of research over the last few years. Different control methods such as field oriented control, exact linearization, passivity approach and sliding mode control have been reported in literature. The field orientation control, which gives high dynamic response, ensures torque/flux decoupling of AC machines assuming exact knowledge of rotating field [8, 9, 10]. This assumption is difficult to realise in practice and the high performance of such strategy is often deteriorated due to significant plant uncertainties. These later include, in general, magnetic saturation or motor winding temperature change or motor internal parameters variance.

The control of AC machines can be decoupled and linear by means of non linear feedback linearization [11, 12]. However this method have some disadvantages:

- i) the necessary and sufficient condition for linearization can't be held all the time,
- ii) singular point exist,
- iii) requires relatively complicated differential geometry to derive the control law.

Contrary, the passivity based control does not decoupling the system, but it has an outstanding advantage-simplicity, because it does not cancel all the nonlinearities. As the result it does not have any preliminary requirement or singular point. The passivity theory based on the control of AC machines is developed in [13] and experimental results for induction machines are given in [14, 15].

Due to its simplicity and attractive robustness properties, the sliding mode theory is widely applied in electrical drives. In [16], the fundamental principles of sliding mode control and its application to electrical machines are formulated. Example of real time sliding mode application involving induction motors is reported in [17, 18]. The cascaded structure is exploited in [19] to obtain a nonlinear predictive control of induction machine.

The authors in [20, 21], use the backstepping to derive the control for the torque and the field amplitude of induction motors in rotating (d, q) reference frame. As pointed out by the authors the proposed control law is not robust in face to parameter variations and necessitate the adaptive strategy for parameters involved in control law. Moreover, no information is given about the generalization of this method for other AC machines.

In this paper is developed Lyapunov based non-linear control to ensure the flux-speed tracking regime of voltage fed induction machine (see also [24]). The control law is determined in two steps, in the first the virtual control, based on Lyapunov function, is obtained in view to get the flux-speed tracking. After this, is deduced the real control imposing the virtual control law. From the fact that most voltage fed AC machines belongs to the same class of cascaded non linear systems, we show how to generalize the proposed method to these AC machines. This generalization allows to get a unified view for the control of most voltage fed AC machine.

This paper is organized as follows. The formulated problem is given in Section 2 where the induction motor model is seen in the cascaded system form and also for other electric machines. Section 3 is devoted to the development of the real control law in order to involve the flux-speed tracking objectives for the induction machines and some remarks are pointed out in the end of this section. The stability analysis of the induction motor under the proposed control law is discussed in Section 4. The application and simulation results appears in Section 5.

2 Formulation problem

In order to control induction machine, we give in first its model. In the stator reference frame, the state space model of voltage fed induction machine is obtained from Park's model. The state vector is composed of the stator current components (i_α, i_β) , the rotor flux components $(\phi_\alpha, \phi_\beta)$ and the rotor rotating pulsation ω_r , whereas a vector control is composed of the stator voltage components (v_α, v_β) and the external disturbance is represented by the load torque Γ_r . By introducing our notation, the state vector and the control vector are respectively represented by :

$$\begin{aligned} \begin{pmatrix} \xi & \eta \end{pmatrix}^t &= \begin{pmatrix} \xi_1 & \xi_2 & \eta_1 & \eta_2 & \eta_3 \end{pmatrix}^t = \begin{pmatrix} i_\alpha & i_\beta & \phi_\alpha & \phi_\beta & \omega_r \end{pmatrix}^t, \\ u^t &= \begin{pmatrix} u_1 & u_2 \end{pmatrix}^t = \begin{pmatrix} v_\alpha & v_\beta \end{pmatrix}^t. \end{aligned}$$

Using these notations, the dynamic of voltage fed induction machine takes the form:

$$\begin{cases} \dot{\xi}_1 = f_1 + d_1 u_1, & f_1 = -a_1 \xi_1 + b_1 \eta_1 + c_1 \eta_2 \eta_3, \\ \dot{\xi}_2 = f_2 + d_1 u_2, & f_2 = -a_1 \xi_2 + b_1 \eta_2 - c_1 \eta_1 \eta_3, \\ \dot{\eta}_1 = F_1, & F_1 = a_3 \xi_1 - b_3 \eta_1 - \eta_2 \eta_3, \\ \dot{\eta}_2 = F_2, & F_2 = a_3 \xi_2 - b_3 \eta_2 + \eta_1 \eta_3, \\ \dot{\eta}_3 = F_3, & F_3 = -a_5 \eta_3 - c_5 \Gamma_r + b_5 (\eta_1 \xi_2 - \eta_2 \xi_1). \end{cases} \quad (1)$$

It is now well understood that flux reference can be used as an additional degree of freedom to improve motor efficiency (minimizing losses) or to maximize the delivered torque (minimum time). So, in this work, we are interested by the outputs represented by the rotor magnitude flux $\phi = \varphi_\alpha^2 + \varphi_\beta^2$ and the rotor rotating pulsation ω_r with $y = (y_1 \ y_2)^t = (\phi \ \omega_r)^t$ it leads to :

$$\begin{cases} y_1 = h_1(\eta), & h_1(\eta) = \eta_1^2 + \eta_2^2, \\ y_2 = h_2(\eta), & h_2(\eta) = \eta_3, \end{cases} \quad (2)$$

where the positive coefficients (a_1, \dots, c_5) are given by

$$a_1 = \frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r}, \quad b_1 = \frac{(1-\sigma)}{\sigma M T_r}, \quad c_1 = \frac{(1-\sigma)}{\sigma M}, \quad d_1 = \frac{1}{\sigma L_s}, \quad a_3 = \frac{M}{T_r}, \quad b_3 = \frac{1}{T_r},$$

$$a_5 = \frac{k_f}{J}, \quad b_5 = \frac{p^2 M}{J L_r}, \quad c_5 = \frac{p}{J}, \quad \sigma = 1 - \frac{M}{L_s L_r} < 1$$

and related to the following machine parameters :

- T_s, T_r : the stator and rotor electric time constant;
- σ : the leakage coefficient;
- L_s, L_r : the cyclic stator inductance, the cyclic rotor inductance;
- M : the cyclic mutual inductance between stator and rotor;
- k_f : the friction coefficient and Γ_r is a load torque;
- J : the inertia and p is the pairs of poles.

The induction motor dynamic (1) with associated outputs (2) is square non linear system where the input u and the output y are that $u \in R^2$ and $y \in R^2$. The functions $f(\cdot) = [f_1, f_2]^T$ and $F(\cdot) = [F_1 F_2 F_3]^T$ are continuous, moreover $h(\cdot) = [h_1 h_2]^T$ are continuous radially unbounded functions. Due to physical considerations, it is known that machine parameters are always positive and they may be constant or they change in continuous manner so, coefficients $(a_1 \dots c_5)$ are positive bounded. The state vector ξ and the output $y_2 = \eta_3$ which represent respectively the stator current components and the rotor speed are in practice easily measured. On the other hand, the output y_1 which is the magnitude flux is derived from flux components (η_1, η_2) . These later are generally observed and there exist enormous literature about this [9, 17].

We attach to the system (1), the outputs dynamic given by

$$\begin{cases} \dot{y}_1 = H_1(\xi, \eta) = \pi_1(\eta) + \psi_1(\xi, \eta), \\ \dot{y}_2 = H_2(\xi, \eta) = \pi_2(\eta) + \psi_2(\xi, \eta), \end{cases} \quad (3)$$

with

$$\begin{cases} \pi_1(\eta) = -2b_3(\eta_1^2 + \eta_2^2), \\ \pi_2(\eta) = -a_5\eta_3 - c_5\Gamma_r, \end{cases} \quad (4)$$

and

$$\begin{cases} \psi_1(\xi, \eta) = 2a_3(\eta_1\xi_1 + \eta_2\xi_2), \\ \psi_2(\xi, \eta) = b_5(\eta_1\xi_2 - \eta_2\xi_1). \end{cases} \quad (5)$$

Let us define the tracking errors e_1 and e_2 by

$$\begin{cases} e_1 = y_1 - y_{1d}, \\ e_2 = y_2 - y_{2d}, \end{cases} \quad (6)$$

their dynamics are:

$$\begin{cases} \dot{e}_1 = \pi_1(\eta) + \psi_1(\xi, \eta) - \dot{y}_{1d}, \\ \dot{e}_2 = \pi_2(\eta) + \psi_2(\xi, \eta) - \dot{y}_{2d}, \end{cases} \quad (7)$$

where y_{1d} and y_{2d} are desired trajectories.

The problem we are concerned with, consists of developing the control law u that allows the output $y_i (i = 1, 2)$ to track the desired trajectories $y_{id} (i = 1, 2)$. From the fact that the desired output trajectory may be defined by a signal external to the control system so that y_{id} and its time derivatives $(\ddot{y}_{id}, \dot{y}_{id}, \text{ for } i = 1, 2)$ may be measured or provided by a reference signal. Therefore, we assume that the reference signal y_{id} and its derivatives $(\ddot{y}_{id}, \dot{y}_{id}, \text{ for } i = 1, 2)$ are bounded and measurable. Our procedure to tackle

the control problem is similar in spirit to the backstepping methodology developed in [1]. In fact, the control problem is constructed in two steps :

i) Step1: For the tracking errors (e_1, e_2) , we determine, based on the Lyapunov method, the desired values ψ_{1d} and ψ_{2d} , for respectively the functions $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$, which ensure the asymptotic convergence of the tracking errors e_1 and e_2 to zero. Therefore, functions $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$ are seen as a virtual control signals for the output dynamic (7).

ii) Step 2: Based on the plant dynamic (1), we search for the real control signal u that constrain the functions $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$, to take respectively the desired values ψ_{1d} and ψ_{2d} and to ensure asymptotic converge of the tracking errors e_1 and e_2 .

Remark 2.1 In stator reference frame, the dynamic of two phase symmetric induction machine and voltage fed is similarly modelled by system (1), when the outputs are chosen as rotor flux and rotor speed. For the case of permanent magnet synchronous machine, the developments are given in Appendix. In general, the dynamic models of most voltage fed machines can be put under a same general class of non linear system. This class is a cascade non linear of the underlying form :

$$\begin{cases} \dot{\xi} = f(\xi, \eta) + g(\xi, \eta).u, \\ \dot{\eta} = F(\xi, \eta), \\ y = h(\eta), \end{cases}$$

with the outputs dynamic given by:

$$\dot{y} = \pi(\eta) + \psi(\xi, \eta) = H(\xi, \eta).$$

The control input u and the measured output vector y are that $u \in R^m$ and $y \in R^m$. The state vectors $\xi \in R^p$, $\eta \in R^q$ with $p \geq 1$ and $q \geq 1$ are available by measure or by observation. Functions $f(\cdot)$, $g(\cdot)$, $F(\cdot)$, $h(\cdot)$ are known continuous and $h(\cdot)$ is continuous radially unbounded functions.

3 Control Law Synthesis

Consider two continuous function $\Lambda(x)$ and $S(x)$ satisfying $\Lambda(x) > 0, \forall x \neq 0$ and $xS(x) > 0, \forall x \neq 0$, the following result can be established.

Proposition 3.1 *If the system (1) is in closed loop with the following real control law*

$$u = A^{-1}(\xi, \eta) (B(\xi, \eta) - e - k.S(z)) \quad \text{with } k_1, k_2 > 0, \tag{8a}$$

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = A^{-1}(\eta, \xi) \left[\begin{pmatrix} B_1(\eta, \xi) \\ B_2(\eta, \xi) \end{pmatrix} - \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} - \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} S(z_1) \\ S(z_2) \end{pmatrix} \right], \tag{8b}$$

$$A(\eta, \xi) = \begin{pmatrix} 2a_3d_1\eta_1 & 2a_3d_1\eta_2 \\ -b_5d_1\eta_2 & b_5d_1\eta_1 \end{pmatrix}, \quad z_i = \psi_i(\xi, \eta) - \psi_{id} \quad \text{with } i = 1, 2, \tag{8c}$$

$$\psi_{id} = -q_i e_i \Lambda(e_i) - \pi_i(\eta) + \dot{\psi}_{id} \quad \text{with } i = 1, 2, \tag{8d}$$

$$B_1(\eta, \xi) = -2a_3(\eta_1 f_1 + \eta_2 f_2 + \xi_1 F_1 + \xi_2 F_2) + \dot{\psi}_{1d}, \tag{8e}$$

$$B_2(\eta, \xi) = -b_5(\xi_2 F_1 + \eta_1 f_2 - \eta_2 f_1 - \xi_1 F_2) + \dot{\psi}_{2d}, \tag{8f}$$

then the outputs error $(e_i, i = 1, 2)$ are bounded and converge at least asymptotically to the origin.

Proof The proof is based on the two steps discussed in Section 2.

Step 1. Based on the dynamic (7), it is possible to search the desired values ψ_{1d} and ψ_{2d} that must take the functions $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$ in order to force the asymptotic converge of the errors (e_1, e_2) . $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$ constitute the virtual control laws for the tracking errors e_1 and e_2 so, they does not form the real control for the induction motor. To this end, let us consider the following Lyapunov function related to the system (7) :

$$V_i(e_i) = \frac{1}{2}(e_i)^2 \quad \text{for } i = 1, 2 \quad (9)$$

its time derivative is then

$$\dot{V}_i(e_i) = e_i \dot{e}_i \quad \text{for } i = 1, 2 \quad (10)$$

if the virtual control law $\psi_i(\xi, \eta)$ with $i = 1, 2$ are equal to the desired value ψ_{id} the dynamic tracking error, given by expression (7), can be rewrite under the form :

$$\dot{e}_i = \pi_i(\eta) + \psi_{id} - \dot{y}_{1d} \quad \text{with } i = 1, 2. \quad (11)$$

By replacing ψ_{id} by its expression (8d), the tracking error dynamic (11) is reduced to:

$$\dot{e}_i = -q_i e_i \Lambda(e_i) \quad (12)$$

$$= \pi_i(\eta) + \psi_{id} - \dot{y}_{1d}. \quad (13)$$

With relation (12), the time derivative of Lyapunov function (10) becomes:

$$\dot{V}_i = -q_i (e_i)^2 \Lambda(e_i) \quad \text{with } i = 1, 2. \quad (14)$$

To have $\dot{V}_i < 0 \forall e_i \neq 0$ it is sufficient that $q_i > 0$ and $\Lambda(e_i) > 0, \forall e_i \neq 0$. Hence, e_i tend to zero at least asymptotically.

Step 2. Now, we must determine the real control input u , which, in same time, constrain the functions $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$ to follow respectively the desired values ψ_{1d} and ψ_{2d} and the tracking errors (e_1, e_2) converge asymptotically to zero.

Indeed, adding and subtracting the desired values ψ_{1d} and ψ_{2d} in the equation (7), this latter becomes :

$$\begin{cases} \dot{e}_1 = \pi_1(\eta) + \psi_1(\xi, \eta) - \psi_{1d} + \psi_{1d} - \dot{y}_{1d}, \\ \dot{e}_2 = \pi_2(\eta) + \psi_2(\xi, \eta) - \psi_{2d} + \psi_{2d} - \dot{y}_{2d}, \end{cases} \quad (15)$$

and we define the error variables as:

$$\begin{cases} z_1 = \psi_1(\xi, \eta) - \psi_{1d}, \\ z_2 = \psi_2(\xi, \eta) - \psi_{2d}. \end{cases} \quad (16)$$

By introducing these two variables z_1 and z_2 in the system (15) it leads to :

$$\begin{cases} \dot{e}_1 = \pi_1(\eta) + \psi_{1d} - \dot{y}_{1d} + z_1, \\ \dot{e}_2 = \pi_2(\eta) + \psi_{2d} - \dot{y}_{2d} + z_2, \end{cases} \quad (17)$$

and replacing ψ_{1d} and ψ_{2d} by their expression (8d), the precedent relation becomes

$$\begin{cases} \dot{e}_1 = -q_1 e_1 \Lambda(e_1) + z_1, \\ \dot{e}_2 = -q_2 e_2 \Lambda(e_2) + z_2. \end{cases} \quad (18)$$

Besides, the time derivative of the variables z_1 and z_2 are obtained from relation (16) :

$$\begin{cases} \dot{z}_1 = 2a_3 (\dot{\eta}_1 \xi_1 + \eta_1 \dot{\xi}_1 + \dot{\eta}_2 \xi_2 + \eta_2 \dot{\xi}_2) - \dot{\psi}_{1d}, \\ \dot{z}_2 = b_5 (\dot{\eta}_1 \xi_2 + \eta_1 \dot{\xi}_2 - \dot{\eta}_2 \xi_1 - \eta_2 \dot{\xi}_1) - \dot{\psi}_{2d}. \end{cases} \quad (19)$$

By replacing the dynamics $(\dot{\eta}_1, \dot{\eta}_2, \dot{\xi}_1, \dot{\xi}_2)$ by their respective expressions from (1), it leads to :

$$\begin{cases} \dot{z}_1 = 2a_3(F_1 \xi_1 + \eta_1 f_1 + F_2 \xi_2 + \eta_2 f_2) - \dot{\psi}_{1d} + 2a_3 d_1 \eta_1 u_1 + \eta_2 d_1 u_2, \\ \dot{z}_2 = b_5(F_1 \xi_2 + \eta_1 f_2 - F_2 \xi_1 - \eta_2 f_1) - \dot{\psi}_{2d} - b_5 d_1 \eta_2 u_1 + b_5 d_1 \eta_1 u_2, \end{cases} \quad (20)$$

or in compact form :

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = - \begin{pmatrix} B_1(\xi, \eta) \\ B_2(\xi, \eta) \end{pmatrix} + A(\xi, \eta) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \quad (21)$$

Let be the Lyapunov functions candidate V_{1a} and V_{2a} related to the systems (18) and (21) which are defined by :

$$\begin{cases} V_{1a} = \frac{1}{2} e_1^2 + \frac{1}{2} z_1^2, \\ V_{2a} = \frac{1}{2} e_2^2 + \frac{1}{2} z_2^2, \end{cases} \quad (22)$$

exploiting relation (18), the time derivative of expression (22) is then:

$$\begin{pmatrix} \dot{V}_{a,1}(e_1, z_1) \\ \dot{V}_{a,2}(e_2, z_2) \end{pmatrix} = - \begin{pmatrix} q_1 e_1^2 \Lambda(e_1) \\ q_2 e_2^2 \Lambda(e_2) \end{pmatrix} + \begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix} \left[\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} + \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} \right] \quad (23)$$

introducing (21) in (23) induces

$$\begin{pmatrix} \dot{V}_{a,1}(e_1, z_1) \\ \dot{V}_{a,2}(e_2, z_2) \end{pmatrix} = - \begin{pmatrix} q_1 e_1^2 \Lambda(e_1) \\ q_2 e_2^2 \Lambda(e_2) \end{pmatrix} + \begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix} \left[\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} - \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} + A(\xi, \eta) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \right]. \quad (24)$$

By using the control law (8a) in (24) it leads to :

$$\begin{pmatrix} \dot{V}_{a,1}(e_1, z_1) \\ \dot{V}_{a,2}(e_2, z_2) \end{pmatrix} = - \begin{pmatrix} q_1 e_1^2 \Lambda(e_1) \\ q_2 e_2^2 \Lambda(e_2) \end{pmatrix} - \begin{pmatrix} k_1 z_1 S(z_1) \\ k_2 z_2 S(z_2) \end{pmatrix}. \quad (25)$$

The relation (25) allows to conclude that the variables (e_1, z_1, e_2, z_2) are bounded and they converge at least asymptotically to zero. So, the functions $\psi_1(\xi, \eta)$ and $\psi_2(\xi, \eta)$ follow respectively the desired value ψ_{1d} and ψ_{2d} and the outputs y_1 and y_2 track respectively their reference y_{1d} and y_{2d} . ■

Remark 3.1 $\Lambda(e_i)$ must be continuous satisfying $\Lambda_1(e_i) > 0, \forall e_i$ is realized by the function $\Lambda(e_i) = e_i^n$ for n even natural number or $\Lambda(e_i) = \cosh(e_i)$. The function $S(z_i)$ is continuous satisfying $z_i S(z_i) > 0, \forall z_i \neq 0$ can be implemented by any continuous function like a sign function by example smooth function defined by $S(z_i) = \frac{z_i}{|z_i| + \epsilon_i}$ with $i > 0$ and $i = 1, 2$ or by $S(z_i) = \tanh(z_i)$.

Remark 3.2 The determination of the input vector u is possible only if the matrix $A(\eta, \xi)$ has an inverse. Its determinant given by $d_1 a_3 b_5 (\eta_1^2 + \eta_2^2)$ is always positive if the rotor flux magnitude $\eta_1^2 + \eta_2^2$ is different from zero. This latter condition is verified since the machine is connected to the supply.

Remark 3.3 In the case where, the time derivative relating to the outputs y_1 and y_2 and the states η are used in place of their expressions from (1), for the determination of the control law, the vector $B(\eta, \xi)$ takes the form :

$$\begin{aligned} B_1(\eta, \xi) &= -2a_3(\eta_1 f_1 + \eta_2 f_2 + \xi_1 \dot{\eta}_1 + \xi_2 \dot{\eta}_2) + \dot{\psi}_{1d}, \\ B_2(\eta, \xi) &= -b_5(\xi_2 \dot{\eta}_1 + \eta_1 f_2 - \eta_2 f_1 - \xi_1 \dot{\eta}_2) + \dot{\psi}_{2d}, \end{aligned}$$

with

$$\dot{\psi}_{1d} = -q_1(\dot{y}_1 - \dot{y}_{1d}) + 2b_3\dot{y}_1 + \ddot{y}_{1d}, \quad \dot{\psi}_{2d} = -q_2(\dot{y}_2 - \dot{y}_{2d}) + a_5\dot{y}_2 + c_5\dot{\Gamma}_r + \ddot{y}_{2d},$$

if function $\Lambda(x)$ is taken in its simplest form: $\Lambda(x) = 1$. Meanwhile in practice, the time derivative of signals included in control law are generally not used due to the unavoidable noise affecting the measured signals and may produce important spikes on the time derivative signals.

Remark 3.4 The global Lyapunov function and the global augmented Lyapunov function for the original system are :

$$V(e) = \sum_{i=1}^2 V_i(e_i) \quad \text{and} \quad V_a(e, z) = \sum_{i=1}^2 V_{ai}(e_i, z_i) = V(e) + \frac{1}{2} \sum_{i=1}^2 z_i^2.$$

Remark 3.5 In the general case of the cascade non linear system introduced in Remark 2.1, the control input can be derived in the following form :

$$u = A^{-1}(\eta, \xi) \cdot (B(\eta, \xi) - k \cdot S(z)),$$

where

$$\begin{aligned} k \cdot S(z) &= \begin{pmatrix} k_1 \cdot S(z_1) \\ \vdots \\ k_m \cdot S(z_m) \end{pmatrix}, \quad z_i = \psi_i(\xi, \eta) - \psi_{id} \quad \text{with} \quad i = 1, \dots, m, \\ B(\xi, \eta) &= \begin{pmatrix} \dot{\psi}_{1d} \\ \vdots \\ \dot{\psi}_{md} \end{pmatrix} - \begin{pmatrix} \frac{\delta \psi_1}{\delta \eta} F(\xi, \eta) \\ \vdots \\ \frac{\delta \psi_m}{\delta \eta} F(\xi, \eta) \end{pmatrix} - \begin{pmatrix} \frac{\delta \psi_1}{\delta \xi} f(\xi, \eta) \\ \vdots \\ \frac{\delta \psi_m}{\delta \xi} f(\xi, \eta) \end{pmatrix} - \begin{pmatrix} \frac{\partial V_1(e_1)}{\partial e_1} \\ \vdots \\ \frac{\partial V_m(e_m)}{\partial e_m} \end{pmatrix}. \end{aligned}$$

4 Stability analysis

The convergence of e_i to zero does not implies that the state vector (ξ, η) remains bounded. As imposed by the control law the output $y_i(t)$ with $i = (1, 2)$ follows asymptotically its bounded reference $y_{id}(t)$ and from the fact that $h_i(\eta)$ is continuous function radially unbounded (see the expression form (2)) it induce to that the variable $\eta_i(t)$ takes a bounded values.

On the one hand, the functions ψ_{1d} and ψ_{2d} given by expression (8d) are bounded since the functions $\pi_1(\eta)$ and $\pi_2(\eta)$ are continuous radially unbounded, the desired trajectories $(y_{id}, \dot{y}_{id}, \ddot{y}_{id})$ and states are bounded. And in addition, the control input makes that the variables z_1 and z_2 bounded and they converge asymptotically to zero. So, according

to the expression (16) it induces that the continuous functions $\psi_1(\eta, \xi)$ and $\psi_2(\eta, \xi)$ take bounded values.

From (5), the states ξ are forced to be the solution of the following system :

$$\begin{pmatrix} 2a_3\eta_1 & 2a_3\eta_2 \\ -b_5\eta_2 & b_5\eta_1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \quad (26)$$

Since ψ_1 and ψ_2 are bounded values and the determinant of (26), given by $2a_3b_5(\eta_{1d}^2 + \eta_{2d}^2)$, is bounded positive scalar (see remark 3.2), then the states ξ are always bounded.

According to the control input expression (8a) and from the fact that the matrix $A(\eta, \xi)$ is non singular, the functions $f_i(\eta)$, $F_i(\eta, \xi)$ and ψ_{id} are bounded moreover the desired trajectories (y_{id} , \dot{y}_{id} , \ddot{y}_{id}) and the state variable (η, ξ) are bounded it follows that the control input is bounded.

5 Application and simulations

For the application, we must in first choice the function $\Lambda(x)$ and its simplest form is:

$$\Lambda(x) = 1 \quad (27)$$

and applying relation (8d), therefore the desired values ψ_{1d} and ψ_{2d} are then given by:

$$\begin{cases} \psi_{1d} = -q_1 (y_1 - y_{1d}) + 2b_3(\eta_1^2 + \eta_2^2) + \dot{y}_{1d}, \\ \psi_{2d} = -q_2 (y_2 - y_{2d}) + a_5\eta_3 + c_5\Gamma_r + \dot{y}_{2d}, \end{cases} \quad (28)$$

where y_{1d} and y_{2d} are respectively the desired flux and the desired speed.

Differentiating expression (28) gives:

$$\begin{cases} \dot{\psi}_{1d}(t) = -q_1 (H_1 - \dot{y}_{1d}) + 2b_3H_1 + \ddot{y}_{1d}, \\ \dot{\psi}_{2d}(t) = -q_2 (H_2 - \dot{y}_{2d}) + a_5F_3 + c_5\dot{\Gamma}_r + \ddot{y}_{2d}. \end{cases} \quad (29)$$

Therefore, the terms $B_1(\xi, \eta)$ and $B_2(\xi, \eta)$ given in (8e) and (8f) take the final expression :

$$B_1(\eta, \xi) = -2a_3(\eta_1 f_1 + \eta_2 f_2 + \xi_1 F_1 + \xi_2 F_2) - q_1(H_1 - \dot{y}_{1d}) + 2b_3H_1 + \ddot{y}_{1d}, \quad (30)$$

$$B_2(\eta, \xi) = -b_5(\xi_2 F_1 + \eta_1 f_2 - \eta_2 f_1 - \xi_1 F_2) - q_2(H_2 - \dot{y}_{2d}) + a_5F_3 + c_5\dot{\Gamma}_r + \ddot{y}_{2d}. \quad (31)$$

The simulations are performed for three phase induction machine characterised by :

$$Pn = 3.7Kw, 220/380, 8.54/14.8A,$$

$$M = 0.048H, L_s = 0.17H, L_r = 0.015H, \sigma = 0.0964,$$

$$T_s = 0.151s, T_r = 0.136s, J = 0.135mN/radS^{-2}, K_f = 0.0018mN/radS^{-1}.$$

The function $S(z_i)$ for $i = (1, 2)$ is implemented by $S(z_i) = \frac{z_i}{|z_i| + \epsilon_i}$ where the threshold values ϵ_1 and ϵ_2 are fixed to unity. The desired flux and speed tracking are involved with the regulator coefficients tuned to :

$$k_1 = 8000; k_2 = 2000; q_1 = 1000; q_2 = 2000.$$

Figures 5.1 and 5.2 give the machine responses in tracking regime (for both $\omega_{ref} > 0$ and $\omega_{ref} < 0$). It appears clearly that the flux and speed track their references with a good accuracy. More over, the initial stator peak current are attenuated by reducing the control inputs only in the beginning of the transient stage (for time $t \leq 0.175s$). This reduction affects the tracking during this interval of time. In order to maintain

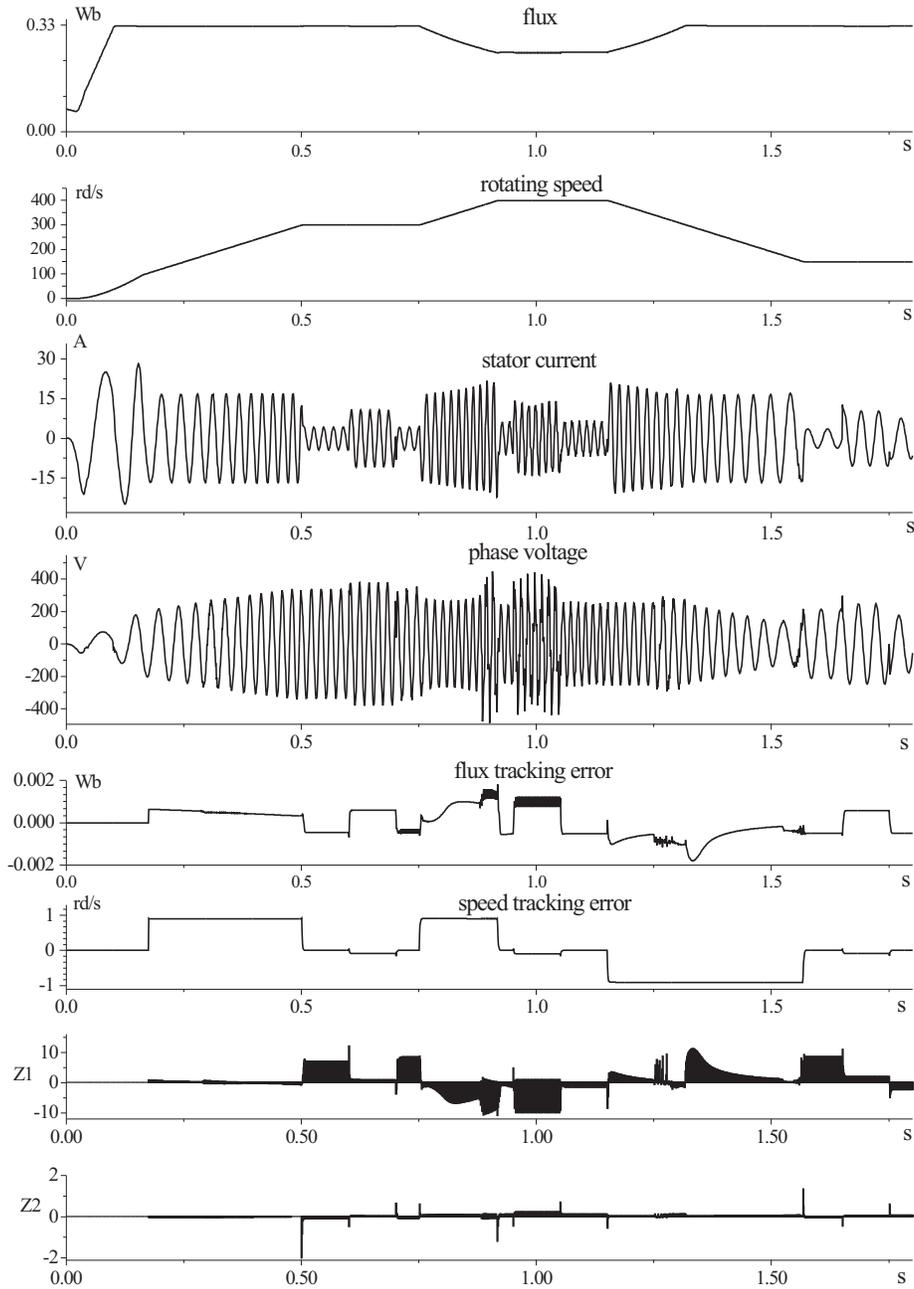


Figure 5.1: Induction machine responses in tracking regime for positive reference speed with the disturbances applied during only 0.1s respectively at time $t=0.6$, 0.95 s and $t=1.75$ s.

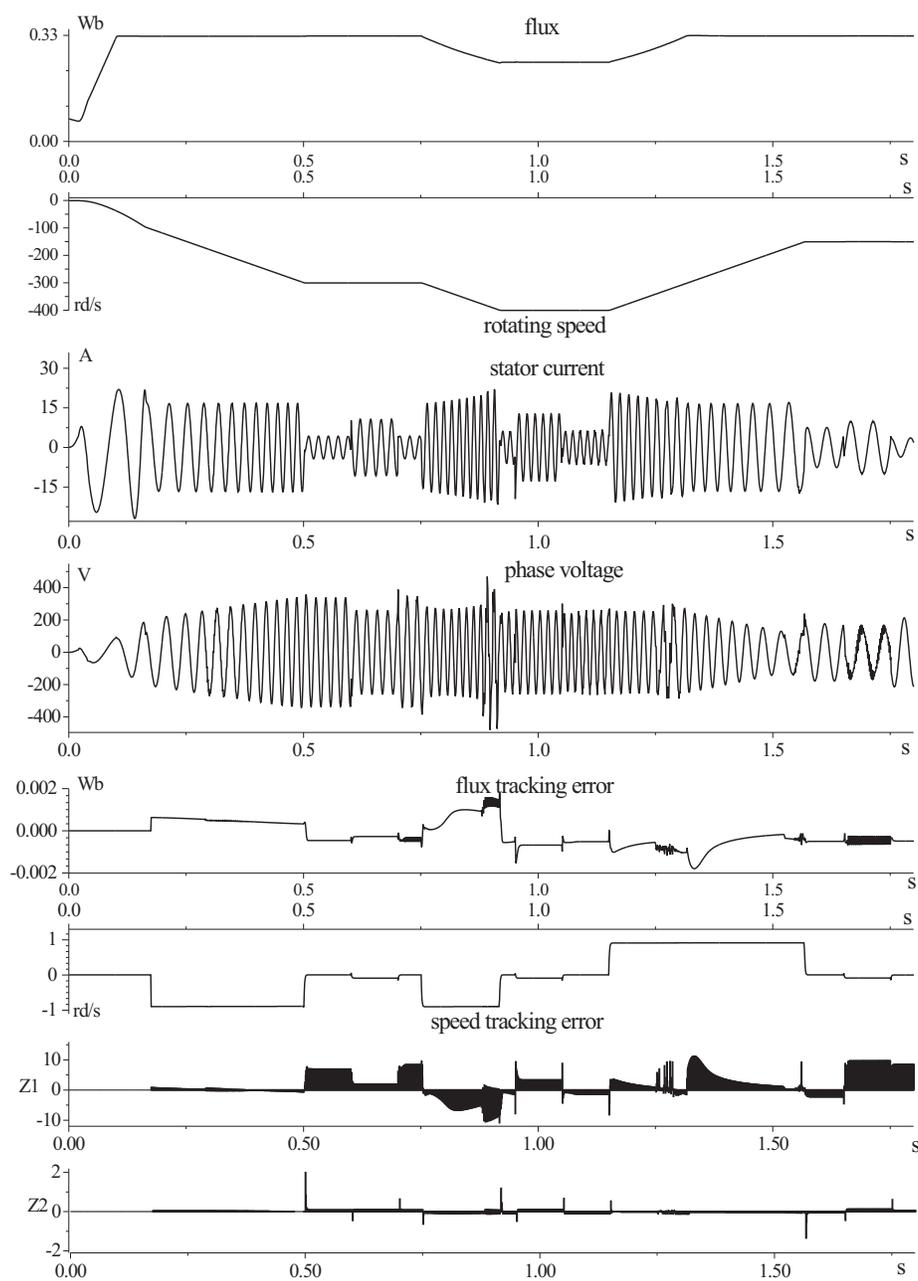


Figure 5.2: Induction machine responses in tracking regime for negative reference speed with the disturbances applied during only 0.1s at time $t=0.6$, 0.95 s and $t=1.75$ s.

the voltage in admissible range when the speed reference ω_{ref} grows up to nominal value $\omega_n = 300rd/s$, the reference flux ϕ_{ref} is reduced down to the nominal flux ϕ_n ($\phi_n = 0.33Wb$) as $\phi_{ref} = \phi_n \omega_n / \omega_{ref}$.

Further, it is noted that the speed and flux tracking reveal a good robustness against disturbances represented by parametric variations and nominal load torque occurring at the same time. These disturbances are applied during $0.1s$ respectively at the time $t = 0.6, 0.95s$ and $t = 1.75s$. The robustness tests are performed for the parameter variations around nominal values as that the stator and rotor resistors increase respectively by an amount of 50% and 100%, the stator and rotor inductors decrease respectively by an amount of 25% and 50%. Meanwhile, these variations affect only the machine model coefficients and who that appearing in the control (u_1, u_2), desired values (ψ_{1d}, ψ_{2d}) and variables (z_1, z_2), are maintained constant. The maximal absolute values of tracking errors (see Table 5.1) reveals that this control law is highly robust in face parameters variation when the state vector is completely known. Despite this highly disturbances, the stator voltage remains in admissible range.

Maximal Tracking error	$ \phi_{ref} - \phi $	$ \omega_{ref} - \omega $	$ z_1 $	$ z_2 $
Positive reference	$2.10^{-3}Wb$	$0.92rd/s$	12.1	2
Negative reference	$2.10^{-3}Wb$	$0.92rd/s$	11.3	2

Table 5.1: Maximal tracking error and maximal (z_1, z_2) values.

6 Conclusion

This paper develops a control design procedure for flux-speed tracking of voltage fed induction motor. This design procedure is based on the Lyapunov theory and is similar in spirit to the backstepping methodology. So, in the first step, the virtual control law is derived as that flux and speed follow at last asymptotically their desired trajectory. Then, in second step, is deduced the real control, by imposing this virtual control law. Noticing that the proposed control law does not include the derivatives of states and outputs hence, it avoids the presence of spikes which often affect the derivative signals. The simulation results involving the flux-speed tracking are given a good results and highlight usability of the suggested approach. Moreover, the control law reveals a strong robustness in face to disturbances generated at the same moment by application of the nominal load torque and large parametric variations. The immediate interest of the proposed procedure comes from the fact that it can be easily extended to the most of voltage fed machines.

6.1 Appendix

In the field reference frame (i. e. the rotor), the state model of the permanent magnets synchronous machine (PMSM) and voltage fed is obtained from the Park equations [22, 23]. This model is derived using the state vector constituted by stator current components (i_{ds}, i_{qs}) and the rotor rotating pulsation ω_r , whereas a vector control is composed of the stator voltage components (v_{ds}, v_{qs}). It is known that the PMSM produce optimal electromagnetic torque when the stator current component i_{ds} takes a determined value i_{dref} . This latter must be zero ($i_{dref} = 0$) when the magnets are mounted on the rotor surface. So, the control objective is to constrain the component i_{ds} to take the value i_{dref} and to control the pulsation rotor rotation ω_r . So, the PMSM dynamic is separated into two interconnected systems : the first one concerns

the control for the output ids and the second, which form cascaded non linear system, is related to the control output ω_r . Using the precedent notation for the state vector, the control input vector and the output vector :

$$\begin{pmatrix} \eta_1 & \eta_2 & \xi_1 \end{pmatrix} = \begin{pmatrix} i_{ds} & \omega_r & i_{qs} \end{pmatrix}, \quad u^T = \begin{pmatrix} u_1 & u_2 \end{pmatrix}^T = \begin{pmatrix} u_{ds} & u_{qs} \end{pmatrix}^T,$$

$$y^T = \begin{pmatrix} y_1 & y_2 \end{pmatrix}^T = (i_{ds} \ \omega_r)^T$$

and the PMSM dynamic takes the form :

$$\begin{cases} \dot{\eta}_1 = -a_1\eta_1 + b_1\eta_2\xi_1 + c_1u_1, \\ y_1 = \eta_1, \end{cases} \quad (32)$$

$$\begin{cases} \dot{\xi}_1 = -a_2\xi_1 - b_2\eta_1\eta_2 - c_2\eta_2 + d_2u_2, \\ \dot{\eta}_2 = -c_3\eta_2 - d_3\Gamma_r + (a_3\eta_1 + b_3)\xi_1, \\ y_2 = \eta_2. \end{cases} \quad (33)$$

It is obviously that the second subsystem has the same form as the studied one. Another way to control the PMSM is to regulate only the speed. In this case, the state vector, the input vector and the output are then respectively represented by :

$$\begin{pmatrix} \xi_1 & \xi_2 & \eta_1 \end{pmatrix} = \begin{pmatrix} i_{ds} & i_{qs} & \omega_r \end{pmatrix}, \quad u^T = \begin{pmatrix} u_1 & u_2 \end{pmatrix}^T = \begin{pmatrix} v_{ds} & v_{qs} \end{pmatrix}^T, \quad y_1 = \omega_r$$

and the PMSM state model takes the form :

$$\begin{cases} \dot{\xi}_1 = -a_1\xi_1 + b_1\xi_2\eta_1 + c_1u_1, \\ \dot{\xi}_2 = -a_2\xi_2 - b_2\xi_1\eta_1 - c_2\eta_1 + d_2u_2, \\ \dot{\eta}_1 = (a_3\xi_1 + b_3)\xi_2 - c_3\eta_1 - d_3\Gamma_r, \\ y_1 = \eta_1. \end{cases} \quad (34)$$

It appears that the precedent dynamic is the same class as indicated in 1. Meanwhile, the action on the speed is carried out by the two inputs (u_1 et u_2) so, this degree of freedom can be exploited in order to introduce another constraint.

The coefficients (a_1, \dots, c_4) are related to the machine parameters by :

$$a_1 = \frac{R_s}{L_d}, \quad a_2 = \frac{L_q}{L_d}, \quad a_3 = \frac{1}{L_d}, \quad b_1 = \frac{R_s}{L_q}, \quad b_2 = \frac{L_d}{L_q}, \quad b_3 = \frac{\phi_f}{L_q}, \quad b_4 = \frac{1}{L_q},$$

$$c_1 = \frac{3 \cdot (p)^2}{2 \cdot J} (L_d - L_q), \quad c_2 = \frac{3 \cdot (p)^2 \cdot \phi_f}{2 \cdot J}, \quad c_3 = \frac{k_f}{J}, \quad c_4 = \frac{p}{J}$$

and the physical parameters represent :

R_s : stator phase resistor,

L_d/L_q : cyclic stator/roto inductance related to (d, q) axe,

f : flux produced by rotor magnets,

J : inertia and p is the pairs of poles,

k_f : friction coefficient and Γ_r is a load torque.

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