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$\mathcal{H}_\infty$ Filtering for Uncertain Bilinear Stochastic Systems †

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Abstract: This paper is concerned with the problem of $\mathcal{H}_\infty$ filtering for continuous-time uncertain stochastic systems. The model under consideration contains both state-dependent stochastic noises and deterministic parameter uncertainties residing in a polytope. According to the online availability of the information on the uncertain parameters, we propose two approaches, namely robust stochastic $\mathcal{H}_\infty$ filtering and parameter-dependent stochastic $\mathcal{H}_\infty$ filtering. Both approaches solve the filtering problems based on a modified (improved) bounded real lemma for continuous-time stochastic systems, which decouples the product terms between the Lyapunov matrix and systems matrices and enables us to exploit parameter-dependent stability idea in the filter designs. Sufficient conditions for the existence of admissible robust stochastic $\mathcal{H}_\infty$ filters and parameter-dependent stochastic $\mathcal{H}_\infty$ filters are obtained in terms of linear matrix inequalities, upon which the filter designs are cast into convex optimization problems. Since the filter designs make full use of the parameter-dependent stability idea, the obtained results are less conservative than the existing one in the quadratic framework. A numerical example is provided to illustrate the effectiveness and advantage of the filter design methods proposed in this paper.

Keywords: *Linear matrix inequality; $\mathcal{H}_\infty$ filtering; parameter uncertainty; robust filtering; stochastic systems.*

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1 Introduction

During the past decades, stochastic modeling has come to play an important role in many branches of science such as biology, economics and engineering applications. Therefore, much attention has been drawn to systems with stochastic perturbations from researchers working in related areas. By stochastic systems, we generally refer to systems whose parameter uncertainties are modeled as white noise processes. These parameter uncertainties are usually due to some stochastic environment, and thus it is a natural way to represent them in the model by stochastic parameters fluctuating around some deterministic nominal values. This kind of systems has been called systems with random parametric excitation [2], stochastic bilinear systems [18] and linear stochastic systems with multiplicative noise [15, 31]. Analysis and synthesis of stochastic systems have been investigated extensively and many fundamental results for deterministic systems have been extended to stochastic cases. To mention a few, the analysis of asymptotic behavior can be found in [19, 21, 24]; the optimal control problems were reported in [15, 31]; and recently with the development of H_∞ control theory, the robust control and filtering results have also been extended to stochastic systems through Riccati-like approaches as well as by means of linear matrix inequality (LMI) [3, 4, 9, 16, 29, 33].

On the other hand, for the purpose of analysis and synthesis, estimating the state variables of a dynamic model is important in helping to improve our knowledge about the system concerned [1]. Hence, state estimation has long been an important and interesting problem in the control and signal processing area. Among the existing approaches for estimating the state variables of a linear system described by a state-space equation, arguably, the most popular and useful one is the celebrated Kalman filter [6, 7, 17] which has been applied to a wide range of problems (biology, economics, aerospace, and even population analysis etc. [23, 26]). Usually, it is supposed that a precisely known system model is available and that the dynamic and measurement equations are additively affected by white noise processes satisfying standard assumptions. In many practical situations, however, the availability of the *a priori* information about the external noise is unrealistic. In this case, the filtering problem is more involved and many researchers have made great efforts in proposing useful algorithms in different contexts (see, for instance, [11, 13, 27, 34, 35] and the references therein). Among these available filtering results, the $\mathcal{H}_\infty$ filtering approach provides both a guaranteed noise attenuation level and robustness against unmodeled dynamics. In the presence of both unknown statistics of the external noises and uncertain parameters in the system model, a common approach is to design robust $\mathcal{H}_\infty$ filters. The problem of robust $\mathcal{H}_\infty$ filtering consists on designing a linear stationary asymptotically stable filter that assures a prescribed $\mathcal{H}_\infty$ performance for the filtering error system, irrespective of modeling uncertainties. In general, two popular approaches used to solve the aforementioned filtering problem are Riccati equation approach [30] and linear matrix inequality (LMI) approach [22, 32, 33], and two kinds of parameter uncertainty have been widely used in the literature: norm-bounded uncertainty and polytopic uncertainty. In solving the robust $\mathcal{H}_\infty$ filtering problem, most of the reported results are based on quadratic Lyapunov functions, which have been largely used for robust analysis and synthesis in the past decades. Although being able to ensure stability for systems with arbitrarily fast time-varying parameters, methods based on quadratic stability can produce conservative results since the same parameter-independent Lyapunov function must be used for the entire uncertainty domain. One recognized way to overcome this conservativeness is to consider a parameter-dependent Lyapunov function. An example of a less conservative stability condition based on parameter-dependent Lyapunov functions can be found in [8].

Recently, the problem of robust $\mathcal{H}_\infty$ filtering for uncertain stochastic systems has been investigated in [14] by using LMI technique. It is worth mentioning that the filter designs are based on the quadratic stability notion, which requires a common Lyapunov function for the entire uncertainty domain, and thus much overdesign has been introduced in the derivation process. In this paper, we revisit the problem solved in [14], and present two approaches to solve the $\mathcal{H}_\infty$ filtering problem for continuous-time stochastic systems with parameter uncertainties residing in a polytope. One approach is concerned with the robust stochastic $\mathcal{H}_\infty$ filter design, where stationary constant filters are designed to ensure the filtering error system to be asymptotically stable and has a guaranteed $\mathcal{H}_\infty$ performance for the entire uncertainty domain. The other approach designs parameter-dependent filters whose system matrices are dependent on the available information of the uncertain parameters. Both approaches solve the filtering problems based on a modified (improved) bounded real lemma for continuous-time stochastic systems, which decouples the product terms between the Lyapunov matrix and systems matrices and enables us to exploit parameter-dependent stability idea in the filter designs. Sufficient conditions for the existence of admissible robust stochastic $\mathcal{H}_\infty$ filters and parameter-dependent stochastic $\mathcal{H}_\infty$ filters are obtained in terms of LMIs, upon which the filter designs are cast into convex optimization problems. Since the filter designs make full use of the parameter-dependent stability idea, the obtained results are less conservative than the existing one in the quadratic framework. A numerical example is provided to illustrate the effectiveness and advantage of the filter design methods proposed in this paper.

The remainder of this paper is organized as follows. The problem of $\mathcal{H}_\infty$ filtering for uncertain continuous-time stochastic systems is formulated in Section 2. Sections 3 and 4 present results for parameter-dependent and robust stochastic $\mathcal{H}_\infty$ filtering problems respectively. An illustrative example is provided to show the effectiveness and advantages of the proposed filter designs in Section 5. Finally, some concluding remarks are given in Section 6.

Notations: The notations used throughout the paper are fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m \times n$ and the notation $P > 0$ means that P is real symmetric and positive definite. $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; the notation $|\cdot|$ refers to the Euclidean vector norm and $\|\cdot\|_2$ stands for the usual $L_2[0,\infty)$ norm. In symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry and $\mathrm{diag}\{\dots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In addition, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. the filtration contains all $\mathcal{P}$-null sets and is right continuous) and $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respect to the probability measure $\mathcal{P}$.

2 Problem Description

Consider a mean-square stable system $\mathcal{S}$ with state-dependent noise:

$$\mathcal{S}: \quad \begin{aligned} dx(t) &= [A(\lambda)x(t) + B(\lambda)w(t)]\,dt + E(\lambda)x(t)d\beta(t),\\ dy(t) &= [C(\lambda)x(t) + D(\lambda)w(t)]\,dt + F(\lambda)x(t)d\zeta(t),\\ z(t) &= L(\lambda)x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $y(t) \in \mathbb{R}^m$ is the measured output; $z(t) \in \mathbb{R}^p$ is the signal to be estimated; $w(t) \in \mathbb{R}^q$ is the disturbance input which belongs to $L_2[0,\infty)$. The variables $\beta(t)$ and $\zeta(t)$ are zero-mean real scalar Wiener processes that satisfy

$$\begin{aligned} \mathbb{E}\{d\beta(t)\} = 0, \quad \mathbb{E}\left\{d\beta(t)^2\right\} = dt, \\ \mathbb{E}\{d\zeta(t)\} = 0, \quad \mathbb{E}\left\{d\zeta(t)^2\right\} = dt, \\ \mathbb{E}\{d\beta(t)d\zeta(t)\} = \alpha dt, \quad |\alpha| < 1, \end{aligned}$$

$A(\lambda)$, $B(\lambda)$, $E(\lambda)$, $C(\lambda)$, $D(\lambda)$, $F(\lambda)$ and $L(\lambda)$ are appropriately dimensioned matrices. It is assumed that

$$\Omega(\lambda) \triangleq (A(\lambda), B(\lambda), E(\lambda), C(\lambda), D(\lambda), F(\lambda), L(\lambda)) \in \mathcal{R},$$

where $\mathcal{R}$ is a given convex bounded polyhedral domain described by s vertices:

$$\mathcal{R} \triangleq \left\{\Omega(\lambda): \ \Omega(\lambda) = \sum_{i=1}^{s} \lambda_i \Omega_i; \ \sum_{i=1}^{s} \lambda_i = 1, \ \lambda_i \geq 0\right\}$$

and $\Omega_i \triangleq (A_i, B_i, E_i, C_i, D_i, F_i, L_i)$ denotes the vertex of the polytope.

Since the signal $z(t)$ cannot be measured directly, our purpose in this paper is to estimate $z(t)$ via the available measurement $y(t)$, such that the estimation error is small in the $\mathcal{H}_\infty$ sense with respect to the energy bounded noise $w(t)$.

According to practical situations, we make two different assumptions on the uncertain parameter λ.

Assumption 1 *The uncertain parameter λ is unknown, and cannot be measured online.*

Assumption 2 *The uncertain parameter λ does not depend explicitly on the time variable but can be measured online. The uncertain parameter λ can vary slowly due to changes in temperature, wind, pressure, humidity, atmosphere, or operating points [20].*

For Assumption 1, since the uncertain parameter λ cannot be measured online, a natural way to deal with the filtering problem is to consider a robust filter of the following form (whose filter matrices are not dependent on the parameter λ):

$$\begin{aligned} \mathcal{F}_R: \quad dx_F(t) &= A_F x_F(t)dt + B_F dy(t), \quad x_F(0) = 0, \\ z_F(t) &= C_F x_F(t). \end{aligned} \tag{2}$$

In some situations, however, the uncertain parameter λ does not depend explicitly on the time variable but can be measured online. In such cases (Assumption 2), it may be desirable to utilize the available information on parameter λ to reduce the conservatism of the robust filter designs. That is, to design a parameter-dependent filter of the following form (whose filter matrices are explicitly dependent on the parameter λ):

$$\begin{aligned} \mathcal{F}_P: \quad dx_F(t) &= A_F(\lambda) x_F(t)dt + B_F(\lambda) dy(t), \quad x_F(0) = 0, \\ z_F(t) &= C_F(\lambda) x_F(t). \end{aligned} \tag{3}$$

Throughout the paper, the estimation error is denoted by $e(t) \triangleq z(t) - z_F(t)$. We define, for a given scalar $\gamma > 0$, the following performance index:

$$\mathcal{J} \triangleq \|e\|_E^2 - \gamma^2 \|w\|_2^2,$$

where

$$\|e\|_E^2 \triangleq \mathbb{E}\left\{\int_0^\infty |e(t)|^2 \, dt\right\}.$$

In the following sections, we will present LMI-based approaches to solve the above two stochastic filtering problems. We first present results on the parameter-dependent stochastic $\mathcal{H}_\infty$ filtering problem, and then solve the robust stochastic $\mathcal{H}_\infty$ filtering problem.

3 Parameter-Dependent Stochastic $\mathcal{H}_\infty$ Filtering

In the parameter-dependent stochastic $\mathcal{H}_\infty$ filtering problem, by augmenting the model of $\mathcal{S}$ to include the states of the filter $\mathcal{F}_P$, we obtain the filtering error system $\mathcal{E}_P$:

$$\mathcal{E}_P: \quad \begin{aligned} d\xi(t) &= \left[\bar{A}(\lambda)\xi(t) + \bar{B}(\lambda)w(t)\right] dt + \bar{E}(\lambda)\xi(t)\, d\beta(t) + \bar{F}(\lambda)\xi(t)\, d\zeta(t), \\ e(t) &= \bar{C}(\lambda)\xi(t), \end{aligned} \tag{4}$$

where $\xi(t) = \left[x^{\mathrm{T}}(t),\ x_F^{\mathrm{T}}(t)\right]^{\mathrm{T}}$ and

$$\begin{gathered} \bar{A}(\lambda) = \begin{bmatrix} A(\lambda) & 0 \\ B_F(\lambda)C(\lambda) & A_F(\lambda) \end{bmatrix}, \quad \bar{B}(\lambda) = \begin{bmatrix} B(\lambda) \\ B_F(\lambda)D(\lambda) \end{bmatrix}, \\ \bar{E}(\lambda) = \begin{bmatrix} E(\lambda) & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{F}(\lambda) = \begin{bmatrix} 0 & 0 \\ B_F(\lambda)F(\lambda) & 0 \end{bmatrix}, \\ \bar{C}(\lambda) = \left[L(\lambda), \quad -C_F(\lambda)\right]. \end{gathered} \tag{5}$$

Then, the parameter-dependent stochastic $\mathcal{H}_\infty$ filtering problem to be addressed in this section can be expressed as follows.

Problem PDSHinfF (Parameter-dependent Stochastic $\mathcal{H}_\infty$ Filtering): Given system $\mathcal{S}$ in (1), determine the parameter-dependent matrices $(A_F(\lambda), B_F(\lambda), C_F(\lambda))$ of the filter $\mathcal{F}_P$ in (3), such that the filtering error system $\mathcal{E}_P$ in (4) is mean-square asymptotically stable and $\mathcal{J} < 0$ for all nonzero $w(t) \in L_2[0,\infty)$. Filters satisfying the above conditions are called parameter-dependent stochastic $\mathcal{H}_\infty$ filters.

3.1 Preliminaries

To solve Problem PDSHinfF, we need the following lemma (see, for instance, Lemma 1 in [14]).

Lemma 3.1 *Suppose system $\mathcal{S}$ in (1) and filter $\mathcal{F}_P$ in (3) are given, the filtering error system $\mathcal{E}_P$ in (4) is mean-square asymptotically stable with $\mathcal{J} < 0$ for all nonzero $w(t) \in L_2[0,\infty)$ under zero initial conditions if and only if there exists a matrix function $Q(\lambda) > 0$ satisfying*

$$\begin{aligned} &\bar{A}^{\mathrm{T}}(\lambda)Q(\lambda) + Q(\lambda)\bar{A}(\lambda) + \bar{C}^{\mathrm{T}}(\lambda)\bar{C}(\lambda) + \gamma^{-2}Q(\lambda)\bar{B}(\lambda)\bar{B}^{\mathrm{T}}(\lambda)Q(\lambda) + \bar{E}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{E}(\lambda) \\ &\quad + \bar{F}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{F}(\lambda) + \alpha\bar{E}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{F}(\lambda) + \alpha\bar{F}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{E}(\lambda) < 0 \end{aligned} \tag{6}$$

The above lemma characterizes the $\mathcal{H}_\infty$ performance for continuous-time stochastic systems by using matrix inequality. Denoting $\bar{\alpha} \triangleq \sqrt{1-\alpha^2}$, by Schur complement [5], condition (6) in Lemma 3.1 can be transformed into

$$\begin{bmatrix} -Q(\lambda) & 0 & \bar{\alpha}Q(\lambda)\bar{E}(\lambda) & 0 & 0 \\ * & -Q(\lambda) & Q(\lambda)\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right) & 0 & 0 \\ * & * & \bar{A}^{\mathrm{T}}(\lambda)Q(\lambda)+Q(\lambda)\bar{A}(\lambda) & Q(\lambda)\bar{B}(\lambda) & \bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{7}$$

(7) is an LMI formulation of the $\mathcal{H}_\infty$ performance presented in Lemma 3.1 for continuous-time stochastic systems. A robust stochastic $\mathcal{H}_\infty$ filtering result has been presented in [14] based on the performance condition (7). Due to the existence of product terms between the Lyapunov matrix $Q(\lambda)$ and system matrices, the robust filtering result in [14] is obtained by imposing $Q(\lambda) \equiv Q$, which leads to a filtering result within the quadratic framework. In the following, we will present an improved version of (7) by decoupling the product terms between the Lyapunov matrix $Q(\lambda)$ and system matrices, which will be used in our filter designs.

Proposition 3.1 *Suppose system $\mathcal{S}$ in (1) and filter $\mathcal{F}_P$ in (3) are given, the filtering error system $\mathcal{E}_P$ in (4) is mean-square asymptotically stable with $\mathcal{J} < 0$ for all nonzero $w(t) \in L_2[0,\infty)$ under zero initial conditions if and only if for a sufficiently small scalar $\epsilon > 0$, there exist matrix functions $Q(\lambda) > 0$ and $W(\lambda)$ satisfying*

$$\begin{bmatrix} \Upsilon & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}W^{\mathrm{T}}(\lambda)\bar{E}(\lambda) & 0 & 0 \\ * & \Upsilon & 0 & \sqrt{\epsilon}W^{\mathrm{T}}(\lambda)\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right) & 0 & 0 \\ * & * & \Upsilon & W^{\mathrm{T}}(\lambda)\left(I+\epsilon\bar{A}(\lambda)\right) & \sqrt{\epsilon}W^{\mathrm{T}}(\lambda)\bar{B}(\lambda) & 0 \\ * & * & * & -Q(\lambda) & 0 & \sqrt{\epsilon}\bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{8}$$

where

$$\Upsilon \triangleq Q(\lambda) - W^{\mathrm{T}}(\lambda) - W(\lambda).$$

Proof We first show that (8) is equivalent to

$$\begin{bmatrix} -Q(\lambda) & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}Q(\lambda)\bar{E}(\lambda) & 0 & 0 \\ * & -Q(\lambda) & 0 & \sqrt{\epsilon}Q(\lambda)\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right) & 0 & 0 \\ * & * & -Q(\lambda) & Q(\lambda)\left(I+\epsilon\bar{A}(\lambda)\right) & \sqrt{\epsilon}Q(\lambda)\bar{B}(\lambda) & 0 \\ * & * & * & -Q(\lambda) & 0 & \sqrt{\epsilon}\bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0. \tag{9}$$

The equivalence between (8) and (9) can be proved as follows. On one hand, if there exists a matrix function $Q(\lambda) > 0$ satisfying (9), (8) is readily established by choosing $W^{\mathrm{T}}(\lambda) = W(\lambda) = Q(\lambda)$. On the other hand, if there exist matrix functions $Q(\lambda) > 0$ and $W(\lambda)$ satisfying (8), we can easily see that $W(\lambda)$ is nonsingular. In addition, we have $(Q(\lambda) - W(\lambda))^{\mathrm{T}} Q^{-1}(\lambda) (Q(\lambda) - W(\lambda)) \geq 0$, which implies that $\Gamma \triangleq -W^{\mathrm{T}}(\lambda)Q^{-1}(\lambda)W(\lambda) \leq Q(\lambda) - W^{\mathrm{T}}(\lambda) - W(\lambda)$. Therefore we can conclude from

(8) that

$$\begin{bmatrix} \Gamma & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}W^{\mathrm{T}}(\lambda)\bar{E}(\lambda) & 0 & 0 \\ * & \Gamma & 0 & \sqrt{\epsilon}W^{\mathrm{T}}(\lambda)\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right) & 0 & 0 \\ * & * & \Gamma & W^{\mathrm{T}}(\lambda)\left(I+\epsilon\bar{A}(\lambda)\right) & \sqrt{\epsilon}W^{\mathrm{T}}(\lambda)\bar{B}(\lambda) & 0 \\ * & * & * & -Q(\lambda) & 0 & \sqrt{\epsilon}\bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0. \tag{10}$$

Performing a congruence transformation to (10) by $\operatorname{diag}\{W^{-1}(\lambda)Q(\lambda), W^{-1}(\lambda)Q(\lambda), W^{-1}(\lambda)Q(\lambda), I, I, I\}$ yields (9).

Now, performing a congruence transformation to (9) by $\operatorname{diag}\left\{I, I, I, \epsilon^{-1/2}I, I, I\right\}$, we obtain

$$\begin{bmatrix} -Q(\lambda) & 0 & 0 & \bar{\alpha}Q(\lambda)\bar{E}(\lambda) & 0 & 0 \\ * & -Q(\lambda) & 0 & Q(\lambda)\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right) & 0 & 0 \\ * & * & -Q(\lambda) & Q(\lambda)\left(\epsilon^{-1/2}I+\sqrt{\epsilon}\bar{A}(\lambda)\right) & \sqrt{\epsilon}Q(\lambda)\bar{B}(\lambda) & 0 \\ * & * & * & -\epsilon^{-1}Q(\lambda) & 0 & \bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{11}$$

by Schur complement, (11) is equivalent to

$$\begin{bmatrix} -Q(\lambda) & 0 & \bar{\alpha}Q(\lambda)\bar{E}(\lambda) & 0 & 0 \\ * & -Q(\lambda) & Q(\lambda)(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)) & 0 & 0 \\ * & * & \begin{matrix} Q(\lambda)\bar{A}(\lambda)+\bar{A}^{\mathrm{T}}(\lambda)Q(\lambda)+ \\ \epsilon\bar{A}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{A}(\lambda) \end{matrix} & \begin{matrix} Q(\lambda)\bar{B}(\lambda)+ \\ \epsilon\bar{A}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{B}(\lambda) \end{matrix} & \bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & -\gamma^2 I+\epsilon\bar{B}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{B}(\lambda) & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{12}$$

which is further equivalent to

$$\begin{bmatrix} \tilde{\Upsilon} & Q(\lambda)\bar{B}(\lambda) \\ * & -\gamma^2 I \end{bmatrix} + \epsilon \begin{bmatrix} \bar{A}^{\mathrm{T}}(\lambda) \\ \bar{B}^{\mathrm{T}}(\lambda) \end{bmatrix} Q(\lambda) \begin{bmatrix} \bar{A}(\lambda) & \bar{B}(\lambda) \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned} \tilde{\Upsilon} \triangleq\ & Q(\lambda)\bar{A}(\lambda)+\bar{A}^{\mathrm{T}}(\lambda)Q(\lambda)+\bar{C}^{\mathrm{T}}(\lambda)\bar{C}(\lambda)+\bar{\alpha}^2\bar{E}^{\mathrm{T}}(\lambda)Q(\lambda)\bar{E}(\lambda) \\ & +\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right)^{\mathrm{T}} Q(\lambda)\left(\alpha\bar{E}(\lambda)+\bar{F}(\lambda)\right). \end{aligned}$$

Since $Q(\lambda) > 0$ and ϵ is sufficiently small positive, (13) is in fact equivalent to (6), and the proof is completed. □

The advantage of Proposition 3.1 lies in the fact that by introducing the slack (in the sense that no structural restriction is imposed) matrix function $W(\lambda)$ and a sufficient small positive constant ϵ, (8) does not contain product terms between the Lyapunov matrix $Q(\lambda)$ and system matrices. This decoupling property has been proved to be an advantage for polytopic uncertain systems concerning reducing conservativeness [25]. In the following (sub)sections, we will develop parameter-dependent and robust stochastic $\mathcal{H}_\infty$ filters based on Proposition 3.1.

It is noted that if the filter matrices $(A_F(\lambda), B_F(\lambda), C_F(\lambda))$ are given, (8) is a linear matrix inequality over the matrix variables $Q(\lambda)$ and $W(\lambda)$ for fixed λ. However, since our purpose is to determine the filter matrices $(A_F(\lambda), B_F(\lambda), C_F(\lambda))$, condition (8) is actually a nonlinear matrix inequality. In addition, to test the feasibility of these conditions is an infinite-dimensional problem in terms of the uncertain parameter λ. Our main objective hereafter is to transform (8) into finite-dimensional LMI condition.

3.2 Main Results

Our result depends on the following proposition.

Proposition 3.2 *Given system $\mathcal{S}$ in (1). For a sufficiently small scalar $\epsilon > 0$, there exist matrix functions $Q(\lambda) > 0$ and $W(\lambda)$ satisfying (8) if and only if there exist matrices $\bar{Q}(\lambda) \triangleq \begin{bmatrix} \bar{Q}_1(\lambda) & \bar{Q}_2(\lambda) \\ * & \bar{Q}_3(\lambda) \end{bmatrix} > 0$, $R(\lambda)$, $S(\lambda)$, $T(\lambda)$, $\bar{A}_F(\lambda)$, $\bar{B}_F(\lambda)$, and $\bar{C}_F(\lambda)$ satisfying*

$$\Psi(\lambda) \triangleq \begin{bmatrix} \Pi_1 & \Pi_2 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}R^{\mathrm{T}}(\lambda)E(\lambda) & 0 & 0 & 0 \\ * & \Pi_3 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}S^{\mathrm{T}}(\lambda)E(\lambda) & 0 & 0 & 0 \\ * & * & \Pi_1 & \Pi_2 & 0 & 0 & \Pi_4 & 0 & 0 & 0 \\ * & * & * & \Pi_3 & 0 & 0 & \Pi_5 & 0 & 0 & 0 \\ * & * & * & * & \Pi_1 & \Pi_2 & \Pi_6 & T(\lambda)+\epsilon\bar{A}_F(\lambda) & \Pi_8 & 0 \\ * & * & * & * & * & \Pi_3 & \Pi_7 & T(\lambda)+\epsilon\bar{A}_F(\lambda) & \Pi_9 & 0 \\ * & * & * & * & * & * & -\bar{Q}_1(\lambda) & -\bar{Q}_2(\lambda) & 0 & \sqrt{\epsilon}L^{\mathrm{T}}(\lambda) \\ * & * & * & * & * & * & * & -\bar{Q}_3(\lambda) & 0 & -\sqrt{\epsilon}\bar{C}_F^{\mathrm{T}}(\lambda) \\ * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{14}$$

where

$$\Pi_1 = \bar{Q}_1(\lambda) - R^{\mathrm{T}}(\lambda) - R(\lambda), \quad \Pi_2 = \bar{Q}_2(\lambda) - T(\lambda) - S(\lambda),$$
$$\Pi_3 = \bar{Q}_3(\lambda) - T(\lambda) - T^{\mathrm{T}}(\lambda), \quad \Pi_4 = \sqrt{\epsilon}\alpha R^{\mathrm{T}}(\lambda)E(\lambda) + \bar{B}_F(\lambda)F(\lambda),$$
$$\Pi_5 = \sqrt{\epsilon}\alpha S^{\mathrm{T}}(\lambda)E(\lambda) + \bar{B}_F(\lambda)F(\lambda), \quad \Pi_6 = R^{\mathrm{T}}(\lambda) + \epsilon R^{\mathrm{T}}(\lambda)A(\lambda) + \epsilon\bar{B}_F(\lambda)C(\lambda),$$
$$\Pi_7 = S^{\mathrm{T}}(\lambda) + \epsilon S^{\mathrm{T}}(\lambda)A(\lambda) + \epsilon\bar{B}_F(\lambda)C(\lambda), \quad \Pi_8 = \sqrt{\epsilon}R^{\mathrm{T}}(\lambda)B(\lambda) + \sqrt{\epsilon}\bar{B}_F(\lambda)D(\lambda),$$
$$\Pi_9 = \sqrt{\epsilon}S^{\mathrm{T}}(\lambda)B(\lambda) + \sqrt{\epsilon}\bar{B}_F(\lambda)D(\lambda).$$

Moreover, under the above condition, the matrix functions for an admissible parameter-dependent stochastic $\mathcal{H}_\infty$ filter $\mathcal{F}_P$ in the form of (3) are given by

$$\begin{bmatrix} A_F(\lambda) & B_F(\lambda) \\ C_F(\lambda) & 0 \end{bmatrix} = \begin{bmatrix} T^{-1}(\lambda) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_F(\lambda) & \bar{B}_F(\lambda) \\ \bar{C}_F(\lambda) & 0 \end{bmatrix}. \tag{15}$$

Proof *Necessity.* Given a sufficiently small scalar $\epsilon > 0$, suppose there exist filter matrices $(A_F(\lambda),\ B_F(\lambda),\ C_F(\lambda))$ and matrices $Q(\lambda) > 0$ and $W(\lambda)$ satisfying (8). Let the matrix functions $Q(\lambda)$ and $W(\lambda)$ be partitioned as

$$Q(\lambda) = \begin{bmatrix} Q_1(\lambda) & Q_2(\lambda) \\ Q_2^{\mathrm{T}}(\lambda) & Q_3(\lambda) \end{bmatrix}, \quad W(\lambda) = \begin{bmatrix} W_1(\lambda) & W_2(\lambda) \\ W_4(\lambda) & W_3(\lambda) \end{bmatrix}. \tag{16}$$

By invoking a small perturbation if necessary, we can assume that $W_4(\lambda)$ and $W_3(\lambda)$ are nonsingular. Define the following invertible matrix functions

$$J(\lambda) = \begin{bmatrix} I & 0 \\ 0 & W_3^{-1}(\lambda)W_4(\lambda) \end{bmatrix}, \quad K(\lambda) = \mathrm{diag}\{J(\lambda),\ J(\lambda),\ J(\lambda),\ J(\lambda),\ I,\ I\} \tag{17}$$

and define

$$\bar{Q}(\lambda) = \begin{bmatrix} \bar{Q}_1(\lambda) & \bar{Q}_2(\lambda) \\ * & \bar{Q}_3(\lambda) \end{bmatrix} = J^{\mathrm{T}}(\lambda)Q(\lambda)J(\lambda). \tag{18}$$

Then, performing a congruence transformation to (8) by $K(\lambda)$ together with the consideration of (5) yields

$$\begin{bmatrix} \bar{Q}(\lambda) - \Psi_1 - \Psi_1^{\mathrm{T}} & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}\Psi_5 & 0 & 0 \\ * & \bar{Q}(\lambda) - \Psi_1 - \Psi_1^{\mathrm{T}} & 0 & \sqrt{\epsilon}(\alpha\Psi_5 + \Psi_6) & 0 & 0 \\ * & * & \bar{Q}(\lambda) - \Psi_1 - \Psi_1^{\mathrm{T}} & \Psi_1^{\mathrm{T}} + \epsilon\Psi_3 & \sqrt{\epsilon}\Psi_2 & 0 \\ * & * & * & -\bar{Q}(\lambda) & 0 & \sqrt{\epsilon}\Psi_4 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{19}$$

where

$$\Psi_1 = \begin{bmatrix} W_1(\lambda) & W_2(\lambda)W_3^{-1}(\lambda)W_4(\lambda) \\ W_4^{\mathrm{T}}(\lambda)W_3^{-T}(\lambda)W_4(\lambda) & W_4^{\mathrm{T}}(\lambda)W_3^{-T}(\lambda)W_4(\lambda) \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} W_1^{\mathrm{T}}(\lambda)B(\lambda) + W_4^{\mathrm{T}}(\lambda)B_F(\lambda)D(\lambda) \\ W_4^{\mathrm{T}}(\lambda)W_3^{-T}(\lambda)W_2^{\mathrm{T}}(\lambda)B(\lambda) + W_4^{\mathrm{T}}(\lambda)B_F(\lambda)D(\lambda) \end{bmatrix},$$

$$\Psi_3 = \begin{bmatrix} W_1^{\mathrm{T}}(\lambda)A(\lambda) + W_4^{\mathrm{T}}(\lambda)B_F(\lambda)C(\lambda) & W_4^{\mathrm{T}}(\lambda)A_F(\lambda)W_3^{-1}(\lambda)W_4(\lambda) \\ \begin{array}{c} W_4^{\mathrm{T}}(\lambda)W_3^{-T}(\lambda)W_2^{\mathrm{T}}(\lambda)A(\lambda) + \\ W_4^{\mathrm{T}}(\lambda)B_F(\lambda)C(\lambda) \end{array} & W_4^{\mathrm{T}}(\lambda)A_F(\lambda)W_3^{-1}(\lambda)W_4(\lambda) \end{bmatrix},$$

$$\Psi_4 = \begin{bmatrix} L^{\mathrm{T}}(\lambda) \\ -W_4^{\mathrm{T}}(\lambda)W_3^{-T}(\lambda)C_F^{\mathrm{T}}(\lambda) \end{bmatrix},$$

$$\Psi_5 = \begin{bmatrix} W_1^{\mathrm{T}}(\lambda)E(\lambda) & 0 \\ W_4^{\mathrm{T}}(\lambda)W_3^{-T}(\lambda)W_2^{\mathrm{T}}(\lambda)E(\lambda) & 0 \end{bmatrix},$$

$$\Psi_6 = \begin{bmatrix} W_4^{\mathrm{T}}(\lambda)B_F(\lambda)F(\lambda) & 0 \\ W_4^{\mathrm{T}}(\lambda)B_F(\lambda)F(\lambda) & 0 \end{bmatrix}.$$

By defining

$$R(\lambda) = W_1(\lambda), \tag{20}$$

$$S(\lambda) = W_2(\lambda)W_3^{-1}(\lambda)W_4(\lambda), \tag{21}$$

$$T(\lambda) = W_4^{\mathrm{T}}(\lambda)W_3^{-1}(\lambda)W_4(\lambda), \tag{22}$$

$$\begin{bmatrix} \bar{A}_F(\lambda) & \bar{B}_F(\lambda) \\ \bar{C}_F(\lambda) & 0 \end{bmatrix} = \begin{bmatrix} W_4^{\mathrm{T}}(\lambda) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_F(\lambda) & B_F(\lambda) \\ C_F(\lambda) & 0 \end{bmatrix} \begin{bmatrix} W_3^{-1}(\lambda)W_4(\lambda) & 0 \\ 0 & I \end{bmatrix}, \tag{23}$$

(19) is equivalent to (14), and the necessity is proved.

Sufficiency. Suppose for a sufficiently small scalar $\epsilon > 0$, there exist matrix functions $\bar{Q}(\lambda) > 0$, $R(\lambda)$, $S(\lambda)$, $T(\lambda)$, $\bar{A}_F(\lambda)$, $\bar{B}_F(\lambda)$, and $\bar{C}_F(\lambda)$ satisfying (14), we will prove that there must exist filter matrices ($A_F(\lambda)$, $B_F(\lambda)$, $C_F(\lambda)$) and matrices $Q(\lambda) > 0$ and $W(\lambda)$ satisfying (8).

First (14) implies $T(\lambda) + T^{\mathrm{T}}(\lambda) - \bar{Q}_3(\lambda) > 0$, then we know that $T(\lambda)$ is nonsingular due to $\bar{Q}_3(\lambda) > 0$. Thus one can always find square and nonsingular matrix functions $W_3(\lambda)$ and $W_4(\lambda)$ satisfying (22). Now introduce the matrix functions $J(\lambda)$, $K(\lambda)$ as defined in (17) and

$$W(\lambda) = \begin{bmatrix} R(\lambda) & S(\lambda)W_4^{-1}(\lambda)W_3(\lambda) \\ W_3(\lambda) & W_4(\lambda) \end{bmatrix},$$
$$Q(\lambda) = J^{-T}(\lambda)\bar{Q}(\lambda)J^{-1}(\lambda),$$
$$\begin{bmatrix} A_F(\lambda) & B_F(\lambda) \\ C_F(\lambda) & 0 \end{bmatrix} = \begin{bmatrix} W_4^{-T}(\lambda) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_F(\lambda) & \bar{B}_F(\lambda) \\ \bar{C}_F(\lambda) & 0 \end{bmatrix} \begin{bmatrix} W_4^{-1}(\lambda)W_3(\lambda) & 0 \\ 0 & I \end{bmatrix}. \tag{24}$$

Then, we have $Q(\lambda) > 0$. Now, by some algebraic matrix manipulations, it can be established that (14) is equivalent to

$$\begin{bmatrix} \widetilde{\Phi} & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}J^{\mathrm{T}}(\lambda)W^{\mathrm{T}}(\lambda)\bar{E}(\lambda)J(\lambda) & 0 & 0 \\ * & \widetilde{\Phi} & 0 & \begin{array}{c}\sqrt{\epsilon}J^{\mathrm{T}}(\lambda)W^{\mathrm{T}}(\lambda)\times \\ (\alpha\bar{E}(\lambda)+\bar{F}(\lambda))J(\lambda)\end{array} & 0 & 0 \\ * & * & \widetilde{\Phi} & \begin{array}{c}J^{\mathrm{T}}(\lambda)W^{\mathrm{T}}(\lambda)\times \\ (I+\epsilon\bar{A}(\lambda))J(\lambda)\end{array} & \sqrt{\epsilon}J^{\mathrm{T}}(\lambda)W^{\mathrm{T}}(\lambda)\bar{B}(\lambda) & 0 \\ * & * & * & -J^{\mathrm{T}}(\lambda)Q(\lambda)J(\lambda) & 0 & \sqrt{\epsilon}J^{\mathrm{T}}(\lambda)\bar{C}^{\mathrm{T}}(\lambda) \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{25}$$

where $\widetilde{\Phi} = J^{\mathrm{T}}(\lambda)\Upsilon J(\lambda)$. Now, performing a congruence transformation to (25) by $K^{-1}(\lambda)$ yields (8), and the sufficiency proof is completed.

Proof of Second Part. If the condition in Proposition 3.2 has a set of feasible solutions $\{\bar{Q}(\lambda),\ R(\lambda),\ S(\lambda),\ T(\lambda),\ \bar{A}_F(\lambda),\ \bar{B}_F(\lambda),\ \bar{C}_F(\lambda)\}$, from the above proof we know that the filter with a state-space realization $\left(\bar{A}(\lambda), \bar{B}(\lambda), \bar{C}(\lambda)\right)$ defined in (24) guarantees the filtering error system $\mathcal{E}_P$ in (4) to be mean-square asymptotically stable with $\mathcal{J} < 0$ for all nonzero $w(t) \in L_2[0,\infty)$. Now denote the operator from $y(t)$ to $z_F(t)$ by $\mathcal{T}_{z_F y}(\lambda) = (A_F(\lambda),\ B_F(\lambda), C_F(\lambda))$, then we have $\mathcal{T}_{z_F y}(\lambda)$ is equivalent to $\mathcal{G}_{z_F y}(\lambda)$ under a similarity transformation, where

$$\begin{aligned}&\mathcal{G}_{z_F y}(\lambda)\\ &= \left(W_4^{-1}(\lambda)W_3(\lambda)A_F(\lambda)W_3^{-1}(\lambda)W_4(\lambda),\ W_4^{-1}(\lambda)W_3(\lambda)B_F(\lambda), C_F(\lambda)W_3^{-1}(\lambda)W_4(\lambda)\right).\end{aligned}$$

By substituting the matrices with (24) and by considering the relationship (22), we have

$$\mathcal{G}_{z_F y}(\lambda) = \left(T^{-1}(\lambda)\bar{A}_F(\lambda),\ T^{-1}(\lambda)\bar{B}_F(\lambda), \bar{C}_F(\lambda)\right).$$

Therefore, an admissible filter can be given by (15), and the proof is completed. □

Proposition 3.2 is a preliminary result for solving the parameter-dependent $\mathcal{H}_\infty$ filtering problem. It casts the nonlinear matrix inequality in Lemma 3.1 into an LMI condition by using linearization procedures, upon which desired filters can be constructed by using the obtained matrix functions $\bar{Q}(\lambda)$, $R(\lambda)$, $S(\lambda)$, $T(\lambda)$, $\bar{A}_F(\lambda)$, $\bar{B}_F(\lambda)$, and $\bar{C}_F(\lambda)$. However, this LMI condition still cannot be implemented due to it infinite-dimensional nature in the parameter λ. Our purpose hereafter is to transform the infinite-dimensional condition in Proposition 3.2 into finite-dimensional condition that depends only on the vertex matrices of the polytope $\mathcal{R}$. Then, we have the main filtering result in the following theorem.

Theorem 3.1 (Parameter-Dependent Stochastic H_∞ Filtering) *Given system $\mathcal{S}$ in (1), an admissible parameter-dependent stochastic $\mathcal{H}_\infty$ filter in the form of $\mathcal{F}_P$*

in (3) exists if for a sufficiently small scalar $\epsilon > 0$*, there exist matrices* R_i*,* S_i*,* T_i*,* $\bar{A}_{Fi}$*,* $\bar{B}_{Fi}$*,* $\bar{C}_{Fi}$ *and* $\bar{Q}_i = \begin{bmatrix} \bar{Q}_{1i} & \bar{Q}_{2i} \\ * & \bar{Q}_{3i} \end{bmatrix} > 0$*, satisfying*

$$\Psi_{ii} < 0, \quad i = 1, \dots, s, \tag{26}$$

$$\Psi_{ij} + \Psi_{ji} \le 0, \quad 1 \le i < j \le s, \tag{27}$$

where

$$\Psi_{ij} = \begin{bmatrix} \Phi_1 & \Phi_2 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha} R_i^{\mathrm{T}} E_j & 0 & 0 & 0 \\ * & \Phi_3 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha} S_i^{\mathrm{T}} E_j & 0 & 0 & 0 \\ * & * & \Phi_1 & \Phi_2 & 0 & 0 & \begin{matrix}\sqrt{\epsilon}\alpha R_i^{\mathrm{T}} E_j + \\ \bar{B}_{Fi} F_j\end{matrix} & 0 & 0 & 0 \\ * & * & * & \Phi_3 & 0 & 0 & \begin{matrix}\sqrt{\epsilon}\alpha S_i^{\mathrm{T}} E_j + \\ \bar{B}_{Fi} F_j\end{matrix} & 0 & 0 & 0 \\ * & * & * & * & \Phi_1 & \Phi_2 & \Phi_4 & T_i + \epsilon \bar{A}_{Fi} & \begin{matrix}\sqrt{\epsilon} R_i^{\mathrm{T}} B_j + \\ \sqrt{\epsilon}\bar{B}_{Fi} D_j\end{matrix} & 0 \\ * & * & * & * & * & \Phi_3 & \Phi_5 & T_i + \epsilon \bar{A}_{Fi} & \begin{matrix}\sqrt{\epsilon} S_i^{\mathrm{T}} B_j + \\ \sqrt{\epsilon}\bar{B}_{Fi} D_j\end{matrix} & 0 \\ * & * & * & * & * & * & -\bar{Q}_{1i} & -\bar{Q}_{2i} & 0 & \sqrt{\epsilon} L_j^{\mathrm{T}} \\ * & * & * & * & * & * & * & -\bar{Q}_{3i} & 0 & -\sqrt{\epsilon}\bar{C}_{Fi}^{\mathrm{T}} \\ * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned} &\Phi_1 = \bar{Q}_{1i} - R_i^{\mathrm{T}} - R_i, \quad \Phi_2 = \bar{Q}_{2i} - T_i - S_i, \quad \Phi_3 = \bar{Q}_{3i} - T_i - T_i^{\mathrm{T}}, \\ &\Phi_4 = R_i^{\mathrm{T}} + \epsilon R_i^{\mathrm{T}} A_j + \epsilon \bar{B}_{Fi} C_j, \quad \Phi_5 = S_i^{\mathrm{T}} + \epsilon S_i^{\mathrm{T}} A_j + \epsilon \bar{B}_{Fi} C_j. \end{aligned} \tag{28}$$

Moreover, under the above conditions, the matrix functions for an admissible parameter-dependent stochastic $\mathcal{H}_\infty$ *filter* $\mathcal{F}_P$ *in the form of (3) are given by*

$$\begin{bmatrix} A_F(\lambda) & B_F(\lambda) \\ C_F(\lambda) & 0 \end{bmatrix} = \begin{bmatrix} \left(\sum\limits_{i=1}^{s} \lambda_i T_i\right)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \sum\limits_{i=1}^{s} \lambda_i \bar{A}_{Fi} & \sum\limits_{i=1}^{s} \lambda_i \bar{B}_{Fi} \\ \sum\limits_{i=1}^{s} \lambda_i \bar{C}_{Fi} & 0 \end{bmatrix}. \tag{29}$$

Proof From Propositions 3.1 and 3.2, an admissible parameter-dependent stochastic $\mathcal{H}_\infty$ filter $\mathcal{F}_P$ in the form of (3) exists if there exist matrix functions $\bar{Q}(\lambda) > 0$, $R(\lambda)$, $S(\lambda)$, $T(\lambda)$, $\bar{A}_F(\lambda)$, $\bar{B}_F(\lambda)$, and $\bar{C}_F(\lambda)$ satisfying (14). Now assume the above matrix functions to be of the following form

$$\begin{aligned} \bar{Q}(\lambda) &= \sum_{i=1}^{s} \lambda_i \bar{Q}_i = \sum_{i=1}^{s} \lambda_i \begin{bmatrix} \bar{Q}_{1i} & \bar{Q}_{2i} \\ * & \bar{Q}_{3i} \end{bmatrix}, \\ R(\lambda) &= \sum_{i=1}^{s} \lambda_i R_i, \quad S(\lambda) = \sum_{i=1}^{s} \lambda_i S_i, \quad T(\lambda) = \sum_{i=1}^{s} \lambda_i T_i, \\ \bar{A}_F(\lambda) &= \sum_{i=1}^{s} \lambda_i \bar{A}_{Fi}, \quad \bar{B}_F(\lambda) = \sum_{i=1}^{s} \lambda_i \bar{B}_{Fi}, \quad \bar{C}_F(\lambda) = \sum_{i=1}^{s} \lambda_i \bar{C}_{Fi}. \end{aligned} \tag{30}$$

With (30) it is not difficult to rewrite $\Psi(\lambda)$ in (14) as

$$\Psi(\lambda) = \sum_{j=1}^{s}\sum_{i=1}^{s}\lambda_i\lambda_j\Psi_{ij} = \sum_{i=1}^{s}\lambda_i^2\Psi_{ii} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s}\lambda_i\lambda_j\left(\Psi_{ij}+\Psi_{ji}\right), \tag{31}$$

where Ψ_{ij} is defined in (28). Then, (26) and (27) guarantee $\Psi(\lambda) < 0$, and the first part of the proof is completed.

By substituting the matrices defined in (30) into (15), we readily obtain (29) and the proof is completed. □

Remark 3.1 The idea behind Theorem 3.1 is to use convex combinations of vertex matrices in the form of (30) to substitute the matrix functions in Proposition 3.2. With the introduction of these matrices, the infinite-dimensional nonlinear matrix inequality condition in Proposition 3.2 is cast into finite-dimensional LMI condition, which depends only on the vertex matrices of the polytope $\mathcal{R}$, and therefore can be readily checked by using standard numerical software [10].

Remark 3.2 Note that the condition in Theorem 3.1 is an LMI not only over the matrix variables, but also over the scalar γ. This implies that the scalar γ can be included as an optimization variable to obtain a reduction of the attenuation level bound. Then the minimum (in terms of the feasibility of Theorem 3.1) attenuation level of $\mathcal{H}_\infty$ filters can be readily found by solving the following convex optimization problem:

$$\text{Minimize } \gamma \text{ subject to (26) and (27) for sufficiently small } \epsilon > 0.$$

4 Robust Stochastic $\mathcal{H}_\infty$ Filtering

In the robust stochastic $\mathcal{H}_\infty$ filtering problem, by augmenting the model of $\mathcal{S}$ to include the states of the filter $\mathcal{F}_R$, we obtain the filtering error system $\mathcal{E}_R$:

$$\begin{aligned} \mathcal{E}_R: \quad & d\xi(t) = \left[\bar{A}(\lambda)\xi(t) + \bar{B}(\lambda)w(t)\right]dt + \bar{E}(\lambda)\xi(t)\,d\beta(t) + \bar{F}(\lambda)\xi(t)\,d\zeta(t), \\ & e(t) = \bar{C}(\lambda)\xi(t), \end{aligned} \tag{32}$$

where

$$\begin{gathered} \bar{A}(\lambda) = \begin{bmatrix} A(\lambda) & 0 \\ B_F C(\lambda) & A_F \end{bmatrix}, \quad \bar{B}(\lambda) = \begin{bmatrix} B(\lambda) \\ B_F D(\lambda) \end{bmatrix}, \quad \bar{E}(\lambda) = \begin{bmatrix} E(\lambda) & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{F}(\lambda) = \begin{bmatrix} 0 & 0 \\ B_F F(\lambda) & 0 \end{bmatrix}, \quad \bar{C}(\lambda) = \begin{bmatrix} L(\lambda) & -C_F \end{bmatrix}. \end{gathered} \tag{33}$$

Then, the robust stochastic $\mathcal{H}_\infty$ filtering problem to be addressed in this section can be expressed as follows:

Problem RSHinfF (Robust Stochastic $\mathcal{H}_\infty$ Filtering): Given system $\mathcal{S}$ in (1), determine the matrices (A_F, B_F, C_F) of the filter $\mathcal{F}_R$ in (2), such that the filtering error system $\mathcal{E}_R$ in (32) is mean-square asymptotically stable and $\mathcal{J} < 0$ for all nonzero $w(t) \in L_2[0,\infty)$ under zero initial conditions . Filters satisfying the above conditions are called robust stochastic $\mathcal{H}_\infty$ filters.

In the following, we will solve the robust stochastic $\mathcal{H}_\infty$ filtering problem. First according to Proposition 3.1, when system $\mathcal{S}$ in (1) and filter $\mathcal{F}_R$ in (2) are given, the filtering error system $\mathcal{E}_R$ in (32) is mean-square asymptotically stable with $\mathcal{J} < 0$ for all nonzero $w(t) \in L_2[0,\infty)$ under zero initial conditions if and only if for a sufficiently small scalar $\epsilon > 0$, there exist matrix functions $Q(\lambda) > 0$ and $W(\lambda)$ satisfying (8). It is worth noting that if we solve the robust filter design problem by following the idea in previous references [12, 28], we need to set the general-structured matrix $W(\lambda) \equiv W$ for the entire uncertainty domain. To reduce the conservativeness while keeping the filter synthesis problem tractable simultaneously, here we assume $W(\lambda)$ takes the following structure:

$$W(\lambda) = \begin{bmatrix} W_1(\lambda) & W_2(\lambda) \\ W_4 & W_3 \end{bmatrix}.$$

Then, by following similar lines as in the proof of Proposition 3.2, we have the following proposition.

Proposition 4.1 *Given system $\mathcal{S}$ in (1), an admissible robust stochastic $\mathcal{H}_\infty$ filter in the form of $\mathcal{F}_R$ in (2) exists if for a sufficiently small scalar $\epsilon > 0$, there exist matrices $\bar{Q}(\lambda) \triangleq \begin{bmatrix} \bar{Q}_1(\lambda) & \bar{Q}_2(\lambda) \\ * & \bar{Q}_3(\lambda) \end{bmatrix} > 0$, $R(\lambda)$, $S(\lambda)$, T, $\bar{A}_F$, $\bar{B}_F$, and $\bar{C}_F$ satisfying*

$$\Delta(\lambda) \triangleq \begin{bmatrix} \bar{\Pi}_1 & \bar{\Pi}_2 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}R^{\mathrm{T}}(\lambda)E(\lambda) & 0 & 0 & 0 \\ * & \bar{\Pi}_3 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}S^{\mathrm{T}}(\lambda)E(\lambda) & 0 & 0 & 0 \\ * & * & \bar{\Pi}_1 & \bar{\Pi}_2 & 0 & 0 & \bar{\Pi}_4 & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_3 & 0 & 0 & \bar{\Pi}_5 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Pi}_1 & \bar{\Pi}_2 & \bar{\Pi}_6 & T(\lambda)+\epsilon\bar{A}_F & \bar{\Pi}_8 & 0 \\ * & * & * & * & * & \bar{\Pi}_3 & \bar{\Pi}_7 & T(\lambda)+\epsilon\bar{A}_F & \bar{\Pi}_9 & 0 \\ * & * & * & * & * & * & -\bar{Q}_1(\lambda) & -\bar{Q}_2(\lambda) & 0 & \sqrt{\epsilon}L^{\mathrm{T}}(\lambda) \\ * & * & * & * & * & * & * & -\bar{Q}_3(\lambda) & 0 & -\sqrt{\epsilon}\bar{C}_F^{\mathrm{T}} \\ * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{34}$$

where

$$\bar{\Pi}_1 = \bar{Q}_1(\lambda) - R^{\mathrm{T}}(\lambda) - R(\lambda), \quad \bar{\Pi}_2 = \bar{Q}_2(\lambda) - T - S(\lambda), \quad \bar{\Pi}_3 = \bar{Q}_3(\lambda) - T - T^{\mathrm{T}},$$
$$\bar{\Pi}_4 = \sqrt{\epsilon}\alpha R^{\mathrm{T}}(\lambda)E(\lambda) + \bar{B}_F F(\lambda), \quad \bar{\Pi}_5 = \sqrt{\epsilon}\alpha S^{\mathrm{T}}(\lambda)E(\lambda) + \bar{B}_F F(\lambda),$$
$$\bar{\Pi}_6 = R^{\mathrm{T}}(\lambda) + \epsilon R^{\mathrm{T}}(\lambda)A(\lambda) + \epsilon\bar{B}_F C(\lambda), \quad \bar{\Pi}_7 = S^{\mathrm{T}}(\lambda) + \epsilon S^{\mathrm{T}}(\lambda)A(\lambda) + \epsilon\bar{B}_F C(\lambda),$$
$$\bar{\Pi}_8 = \sqrt{\epsilon}R^{\mathrm{T}}(\lambda)B(\lambda) + \sqrt{\epsilon}\bar{B}_F D(\lambda), \quad \bar{\Pi}_9 = \sqrt{\epsilon}S^{\mathrm{T}}(\lambda)B(\lambda) + \sqrt{\epsilon}\bar{B}_F D(\lambda).$$

Moreover, under the above condition, the matrices for an admissible robust stochastic $\mathcal{H}_\infty$ filter are given by

$$\begin{bmatrix} A_F & B_F \\ C_F & 0 \end{bmatrix} = \begin{bmatrix} T^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_F & \bar{B}_F \\ \bar{C}_F & 0 \end{bmatrix}. \tag{35}$$

Based on Proposition 4.1, we readily have the main robust filtering result.

Theorem 4.1 (Robust Stochastic $\mathcal{H}_\infty$ Filtering) *Given system $\mathcal{S}$ in (1), an admissible robust stochastic $\mathcal{H}_\infty$ filter in the form of $\mathcal{F}_R$ in (2) exists if for a sufficiently small scalar $\epsilon > 0$, there exist matrices $\bar{Q}_i \triangleq \begin{bmatrix} \bar{Q}_{1i} & \bar{Q}_{2i} \\ * & \bar{Q}_{3i} \end{bmatrix} > 0$, R_i, S_i, T, $\bar{A}_F$, $\bar{B}_F$,*

$\bar{C}_F$ *satisfying*

$$\Delta_{ii} < 0, \quad i = 1, \ldots, s, \tag{36}$$
$$\Delta_{ij} + \Delta_{ji} \le 0, \quad 1 \le i < j \le s, \tag{37}$$

where

$$\Delta_{ij} = \begin{bmatrix} \Lambda_1 & \Lambda_2 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}R_i^{\mathrm{T}}E_j & 0 & 0 & 0 \\ * & \Lambda_3 & 0 & 0 & 0 & 0 & \sqrt{\epsilon}\bar{\alpha}S_i^{\mathrm{T}}E_j & 0 & 0 & 0 \\ * & * & \Lambda_1 & \Lambda_2 & 0 & 0 & \sqrt{\epsilon}\alpha R_i^{\mathrm{T}}E_j + \bar{B}_F F_j & 0 & 0 & 0 \\ * & * & * & \Lambda_3 & 0 & 0 & \sqrt{\epsilon}\alpha S_i^{\mathrm{T}}E_j + \bar{B}_F F_j & 0 & 0 & 0 \\ * & * & * & * & \Lambda_1 & \Lambda_2 & \Lambda_4 & T + \epsilon\bar{A}_F & \begin{matrix}\sqrt{\epsilon}R_i^{\mathrm{T}}B_j + \\ \sqrt{\epsilon}\bar{B}_F D_j\end{matrix} & 0 \\ * & * & * & * & * & \Lambda_3 & \Lambda_5 & T + \epsilon\bar{A}_F & \begin{matrix}\sqrt{\epsilon}S_i^{\mathrm{T}}B_j + \\ \sqrt{\epsilon}\bar{B}_F D_j\end{matrix} & 0 \\ * & * & * & * & * & * & -\bar{Q}_{1i} & -\bar{Q}_{2i} & 0 & \sqrt{\epsilon}L_j^{\mathrm{T}} \\ * & * & * & * & * & * & * & -\bar{Q}_{3i} & 0 & -\sqrt{\epsilon}\bar{C}_F^{\mathrm{T}} \\ * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned} &\Lambda_1 = \bar{Q}_{1i} - R_i^{\mathrm{T}} - R_i, \quad \Lambda_2 = \bar{Q}_{2i} - T - S_i, \quad \Lambda_3 = \bar{Q}_{3i} - T - T^{\mathrm{T}}, \\ &\Lambda_4 = R_i^{\mathrm{T}} + \epsilon R_i^{\mathrm{T}}A_j + \epsilon\bar{B}_F C_j, \quad \Lambda_5 = S_i^{\mathrm{T}} + \epsilon S_i^{\mathrm{T}}A_j + \epsilon\bar{B}_F C_j. \end{aligned} \tag{38}$$

Moreover, under the above conditions, the matrices for an admissible robust stochastic $\mathcal{H}_\infty$ filter in the form of (2) are given by (35).

The theorem can be proved by following similar lines as in the proof of Theorem 3.1 and thus omitted.

With Theorem 4.1, the minimum (in terms of the feasibility of Theorem 4.1) attenuation level of robust stochastic $\mathcal{H}_\infty$ filters can be readily found by solving the following convex optimization problem:

$$\text{Minimize } \gamma \text{ subject to (36) and (37) for sufficiently small } \epsilon > 0.$$

5 Illustrative Example

Consider the following numerical example:

$$\begin{aligned} dx(t) &= \left\{ \begin{bmatrix} -0.6 & 4+a \\ -4 & -0.6 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1.5 & 0 \end{bmatrix} w(t) \right\} dt + \begin{bmatrix} -0.4 & 0.2 \\ 0.3 & 0.5 \end{bmatrix} x(t)d\beta(t), \\ y(t) &= \left\{ \begin{bmatrix} 0 & -1.2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \end{bmatrix} w(t) \right\} + \begin{bmatrix} 0.3 & 0.4+0.1a \end{bmatrix} x(t)d\beta(t), \\ z(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t), \end{aligned} \tag{39}$$

where a represents an uncertain parameter satisfying $|a| \le \bar{a}$. This uncertain system can be modeled with a two-vertex polytope.

First assume $\bar{a} = 0.5$, we solve the filtering problem for this system by several approaches described as follows:

1. By Theorem 4.1, the obtained minimum $\mathcal{H}_\infty$ performance of robust stochastic filters is $\gamma = 1.6988$ for $(\epsilon = 0.001)$, and the associated matrices for filter $\mathcal{F}_R$ in (2) are given by

$$A_F = \begin{bmatrix} -7.2213 & 6.8684 \\ -5.4021 & -0.1494 \end{bmatrix}, \quad B_F = \begin{bmatrix} 0.0024 \\ -0.0066 \end{bmatrix}, \quad C_F = \begin{bmatrix} 0.0000 & -1.0000 \end{bmatrix}.$$

The actual calculated $\mathcal{H}_\infty$ performance of the filtering error system for different a by connecting the above filter to the original system is depicted in Figure 5.1. From this figures, we can see that the $\mathcal{H}_\infty$ performances for the entire uncertainty domain are below the prescribed value $\gamma = 1.6988$.

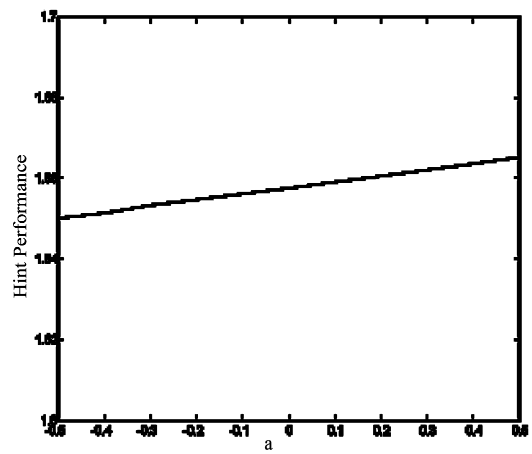

Figure 5.1: $\mathcal{H}_\infty$ performance of robust stochastic filter for entire uncertainty domain.

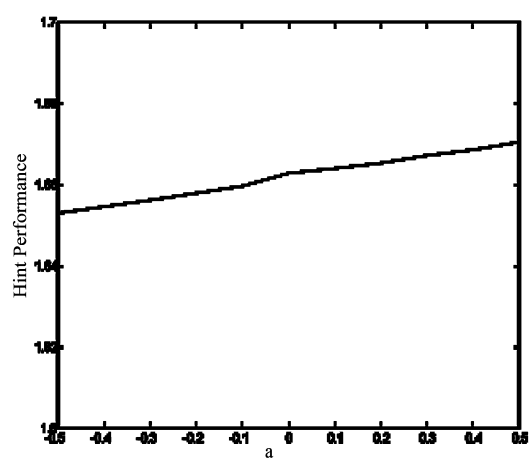

Figure 5.2: $\mathcal{H}_\infty$ performance of parameter-dependent stochastic filter for entire uncertainty domain.

2. By Theorem 3.1, the obtained minimum $\mathcal{H}_\infty$ performance of parameter-dependent stochastic filters is $\gamma = 1.6900$ for ($\epsilon = 0.001$), and the associated matrices needed for the calculation of (29) are given by

$$T_1 = \begin{bmatrix} 0.6035 & -0.1113 \\ -0.1113 & 0.5390 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.6251 & -0.1128 \\ -0.1129 & 0.7156 \end{bmatrix},$$
$$\bar{A}_{F1} = \begin{bmatrix} -0.1019 & 2.1424 \\ -2.0922 & -0.7071 \end{bmatrix}, \quad \bar{A}_{F2} = \begin{bmatrix} -0.0988 & 2.8630 \\ -2.7663 & -0.7701 \end{bmatrix},$$
$$\bar{B}_{F1} = \begin{bmatrix} 0.0009 \\ -0.00331 \end{bmatrix}, \quad \bar{B}_{F2} = \begin{bmatrix} 0.0031 \\ -0.0054 \end{bmatrix},$$
$$\bar{C}_{F1} = \begin{bmatrix} 0.0001 & -1.0000 \end{bmatrix}, \quad \bar{C}_{F2} = \begin{bmatrix} 0.0001 & -1.0003 \end{bmatrix}.$$

The actual calculated $\mathcal{H}_\infty$ performance of the filtering error system for different a by connecting the above filter to the original system is depicted in Figure 5.2. It can be seen that the $\mathcal{H}_\infty$ performances for the entire uncertainty domain are below the prescribed value $\gamma = 1.6900$.

3. By Corollary 1 of [14], the obtained minimum $\mathcal{H}_\infty$ performance of robust stochastic filters is $\gamma = 2.0472$, and the associated matrices for filter $\mathcal{F}_R$ in (2) are given

$$A_F = \begin{bmatrix} -0.1735 & 4.0691 \\ -4.0141 & -1.9794 \end{bmatrix}, \quad B_F = \begin{bmatrix} 0.0874 \\ -1.4642 \end{bmatrix}, \quad C_F = \begin{bmatrix} 0.0000 & 1.0000 \end{bmatrix}.$$

The above calculated results show that for this example, the robust filtering result in the quadratic framework [14] is conservative than the approaches presented in this paper. In addition, since the parameter-dependent stochastic filter design makes use of information of the uncertain parameter, it is reasonable to obtain less conservative filter designs than the robust filtering approach.

Finally, Table 5.1 presents a comparison of minimum $\mathcal{H}_\infty$ performance obtained by using Theorem 4.1, Theorem 3.1 and Corollary 1 of [14] for different cases. This table shows again the reduced conservativeness of the filtering approaches proposed in this paper. Notably for $1.0 \leq \bar{a} \leq 4$ where Corollary 1 of [14] fails to find feasible solutions, the parameter-dependent and robust approach presented here are still able to provide desired filters.

	$\bar{a} = 0.5$	$\bar{a} = 0.8$	$\bar{a} = 1.0$	$\bar{a} = 3$	$\bar{a} = 4$
Minumum γ by Theorem 4.1	1.6900	1.7102	1.7280	2.5071	21.5990
Minumum γ by Theorem 3.1	1.6988	1.7189	1.7399	2.5293	22.5724
Minumum γ by [14]	2.0472	6.0166	infeasible	infeasible	infeasible

Table 5.1: Minimum $\mathcal{H}_\infty$ performance for different cases.

6 Conclusions

The problem of $\mathcal{H}_\infty$ filtering for continuous-time stochastic systems with parameter uncertainties residing in a polytope has been investigated in this paper. Two approaches, namely robust stochastic $\mathcal{H}_\infty$ filtering and parameter-dependent stochastic $\mathcal{H}_\infty$ filtering, have been proposed according to the online availability of the information on the uncertain parameters. Sufficient conditions are derived based on an improved bounded real lemma for stochastic systems and formulated in terms of linear matrix inequalities, upon which desired filters can be obtained by solving convex optimization problems. Since the filter designs make full use of the parameter-dependent stability idea, the obtained results are less conservative than the existing one in the quadratic framework, which has been illustrated via a numerical example.

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