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PERSONAGE IN SCIENCE

Alexander Mikhaylovich Liapunov

Yu.A. Mitropolskii [1], P. Borne [2] and A.A. Martynyuk [3*]

[1] *Institute of Mathematics, National Academy of Sciences of Ukraine, Tereshchenkivska Str., 3, Kyiv, 04604, Ukraine*
[2] *Ecole Centrale de Lille, F-59651 Villeneuve d'Ascq cedex, France,*
[3] *Institute of Mechanics, National Academy of Sciences of Ukraine, Nesterov Str., 3, Kyiv, 03057, MSP-680, Ukraine*

The sixth of June, 2007 is the 150-th birthday anniversary of the outstanding Russian mathematician and mechanical scientist, Academician Liapunov. Taking into account great significance of Liapunov's works for modern development of nonlinear dynamics and systems theory, the Editorial Board of the journal is publishing a sketch of his life, a brief survey of main directions of his scientific activity and a list of his works published to date.

1 Biographical sketch

Alexander Mikhaylovich Liapunov was born on the 6th of June, 1857 in the family of the prominent astronomer Mikhail Vasilievich Liapunov in the town of Yaroslavl.

The grandfather of Alexander Mikhaylovich, Vasiliy Alexandrovich Liapunov, served for the Kazan University. The elder son of Vasiliy Alexandrovich was the grandfather of Academician Krylov and his younger daughter Ekaterina was the wife of R.M. Sechenov, the brother of the prominent Russian physiologist I.M.Sechenov. Their daughter Nataliya Rafailovna Sechenova was the cousin of Alexander Mikhaylovich. She became his wife in 1886.

In the family of Mikhail Vasilievich and Sofiya Alexandrovna Shipilova there were seven children born but only three sons survived: Alexander (1857–1918), Sergey (1859–1924), and Boris (1864–1942). Four other children died in infancy.

In 1863 Alexander Mikhaylovich's father resigned and settled in the family estate of his parents but soon he moved to the family estate of his wife Sofiya Alexandrovna in the village of Bolobonovo, Simbirskaya province.

Sofiya Alexandrovna was the first to raise the Liapunov's sons until they reached the age of 7 and then Mikhail Alexandrovich took over. He trained his children on his own, applying deliberately developed technique to practicing fast calculation and stirring the

* Corresponding author: anmart@stability.kiev.ua

interest to geography by encouraging the children's games with the elements of travelling around the world.

After the death of Mikhail Vasilievich in 1868 Alexander Mikhaylovich was brought up in the family of his uncle R.M.Sechenov. In 1870 together with his mother Alexander Mikhaylovich moved to Nizhniy Novgorod, where he was admitted to the third class of Nizhegorodskaya gymnasium. At gymnasium he was a bright student and read a lot of books on Russian and European literature, history and natural science. He finished gymnasium with a golden medal in 1876 and entered the Natural Science Department of Physico-Mathematical Faculty of the St Petersburg University where Professor Mendeleyev delivered his lectures, but soon changed it for the Mathematical Department. At that time professors of the Mathematical Department were P.L. Chebyshev, A.N. Korkin and E.I. Zolotariov. In 1880 Alexander Mikhaylovich finished his education and joined the Chair of Mechanics of the University to be prepared for professor rank, as Professor Bobylev had proposed.

In 1882 Alexander Mikhaylovich passed the master's examinations and got down to his master's thesis. P.L. Chebyshev proposed him to investigate the loss of ellipsoidal equilibrium forms of rotating fluid. He was to find out if in this case they would turn into other forms of equilibrium that under slight increase of angular velocity would be little different from ellipsoids. This problem proved to be quite difficult, but its statement involved the other one, namely, the problem on stability of ellipsoidal equilibrium forms. Alexander Mikhaylovich solved this problem in 1885 and defended his master's thesis "On stability of ellipsoidal equilibrium forms of rotating fluid". His opponents were Professors Bobylev and Budayev.

In August, 1885 Alexander Mihaylovich moved to Kharkov on invitation of the Kharkov University to deliver lectures on mechanics at the University and the Technological Institute.

In January, 1886 he went to St Petersburg to get married to Natalia Rafailovna Sechenova. Natalia Rafailovna was a highly educated woman, very sophisticated, she had a profound knowledge of Slavonic philology and was good at painting.

In June–July, 1886 Alexander Mikhaylovich and his family went on a trip to Germany, Switzerland, Austria and also to Serbia for his wife doing research in philology.

Since 1888 Alexander Mikhaylovich began publication of his works on motion stability of mechanical systems with finite number of degrees of freedom. In 1892 the Kharkov Mathematical Society published Liaponov's work "A general problem on stability of motion". This work was defended by Alexander Mikhaylovich as the doctor thesis at Moscow University in 1892. His opponents were Professors Zhukovskii and Mlodzeevskii. Soon A.M. Liapunov was assigned as ordinary professor. In December, 1900 A.M. Liapunov was elected a corresponding member of the St Petersburg Academy of Sciences and in October, 1901 he became an ordinary academician of the Academy.

In spring, 1902 A.M. Liapunov returned to St Petersburg to take up exclusively the scientific work. At the St Petersburg University he headed the Chair of Applied Mathematics Department, this position being vacant since P.L. Chebyshev's death in 1884.

At this period of his scientific activity Alexander Mikhaylovich turned back to the problem on equilibrium figures of rotating fluid. In 1908 A.M. Liapunov took part in the work of the 4th International Mathematical Congress in Rome. In November, 1907 Alexander Mikhaylovich was elected a member of the Palermo Mathematical Society and in September, 1908 — a member of the Academy of Sciences dei Lincei in Rome. Since

1909 Alexander Mikhaylovich was involved in publication of the collected works by Euler who once had commented with humour that it would take quite a bit of time for the Academy of Sciences to publish his papers after his death.

In June, 1917 on doctors' request Alexander Mikhaylovich took his wife away from starving St Petersburg to settle in Odessa where at that time his brother Boris Mikhaylovich lived and worked. In spring, 1918 Natalia Rafailovna suffered from a severe cold which caused an acute attack of a pulmonary tuberculosis. In the end of summer her state became critical.

By that time the wave of revolutionary transformation had reached the Liapunovs' family estate. The house was destroyed and the unique library was burnt. The "Ghost of communism" had strayed to the Russian empire from the West and warmed itself by bonfires made of libraries of intellectuals which were not been in demand of revolutionary crowd.

On the 31st of October, 1918 Natalia Rafailovna died. On the same day Alexander Mikhaylovich was brought to Professor Sapezhko's surgical clinics with a gun-shot wound of his head. Three days later on the 3rd of November, 1918 Alexander Mikhaylovich passed away. His ante-mortem note expressed his last will to be buried in his wife's grave and it was executed.

So was the tragic end of the life of a mathematical genius of the 19th century who under other circumstances would have done a lot of good for his country and world science.

2 Main Directions of Scientific Activity

Being the closest disciple of P.L. Chebyshev Alexander Mikhaylovich upheld the best traditions of the St Petersburg mathematical school founded by Chebyshev. Hence, the fundamental importance of problems, accuracy of statement and strictness of solutions are the characteristics of Liapunov's research. Now we shall briefly outline the main directions of his scientific activity.

2.1 Stability of equilibrium and motion of mechanical systems with a finite number of degrees of freedom

The problem on stability of equilibrium and motion which is traced back to the ancient times had remained unsolved until A.M. Liapunov undertook his research in this direction. The strict definition of stability was given by Liapunov in 1892 and was the completion of his intensive work during 1889–1892. The notion "stability by Liapunov" accepted nowadays defines the stability of solutions with respect to perturbations of initial data on infinite time interval. The accurate formulation of the notion of stability was of great importance for further searching for the criteria of equilibrium stability and/or motion of mechanical systems.

A.M. Liapunov considers differential equations of perturbed motion of a general type to discover two general methods for analyzing their solutions. The first method is based on integration of the equations considered by means of special form series. The second is based on application of a certain auxiliary function whose properties together with the properties of its total time derivative along solutions of the system under study allow one to draw a conclusion about the system dynamic behavior. Along with these two methods A.M. Liapunov introduces a new concept of a function characteristic number to apply it to analyzing the stability of solutions of linear systems of differential equations

with variable coefficients. A.M. Liapunov completely solved the problem on the first approximation stability and investigated the problem on solutions stability in certain critical cases.

2.2 Equilibrium figures of uniformly rotating fluid

A.M. Liapunov dedicated the last 15 years of his life to this field of research to obtain results of utmost importance. No strict theory existed before Liapunov. His precursors, including Newton, Makloren, Jacobi, Darvin, and Laplace failed to develop a faultless theory, the convergence matters being involved. It was Liapunov, who succeeded. In his work dated by 1903 he and established the existence of figures of equilibrium close to a sphere in the case of heterogeneous fluid slowly rotating around its axis. In a series of works dated by 1905–1914 he studied a more complex problem on existence of equilibrium figures close to known ellipsoidal figures in the case of homogeneous fluid. The subsequent works published in 1915–1917 investigated the problems on equilibrium figures of weakly heterogeneous fluids close to the Macloren or Jacobi ellipsoids. Moreover it was proved that any Macloren or Jacobi ellipsoid different from the bifurcation one generated a series of new equilibrium figures of almost the same shape as the initial ellipsoid, the new figures being also similar to the initial ellipsoid in heterogeneity of density and angular velocity of rotation. To solve the problem considered A.M. Liapunov applied various means of mathematical analysis required for obtaining the result.

2.3 Stability of equilibrium figures of rotating fluid

Works of Liouville and Riemann preceded A.M. Lipunov's research in this field. The first work of Alexander Mikhaylovich on this problem was his master's thesis. Of principle importance was the formulation of definition of equilibrium stability of rotating fluid. Having done this, Alexander Mikhaylovich reduced the problem considered to purely mathematical problem on minimum of a certain expression representing a potential energy of fluid. Analyzing the expression obtained, Liapunov established stability conditions of Macloren and Jacobi ellipsoids as well as instability of pear-shaped figures. In so doing the erroneous Darvin's result on stability of pear-shaped figures was corrected.

As far as the viscous fluid is concerned A.M. Liapunov noticed the following "According to this principle (principle of energy minimum), if the fluid considered is viscous, then the equilibrium figure will be stable or unstable depending on the complete energy corresponding to this figure having minimum or not having minimum provided invariability of momentum with respect to center of gravity. Although this principle has never been proved satisfactorily, there is a good reason to believe it to be valid." This principle for the ideal fluid was proved by Liapunov yet in 1884.

2.4 Equations of mathematical physics

The results obtained by Liapunov in this area of research were of great value both for substantiation of the methods of mathematical physics (Green, Neyman, Roben methods) and for Alexander Mikhaylovich to gain a foothold in the international mathematical community. While investigating properties of simple and double layer potentials the fundamentals of the potential theory and harmonic function theory were first established. In 1899 he found sufficient condition for existence and equality of limiting values of double

layer normal derivative. The results obtained caused a sensation since they either refuted or justified the methods of mathematical physics the versions of which belonged to the famous authors such as Poincare, Shvarts and others.

2.5 Probability theory

In this field A.M. Liapunov published two papers in Russian and two in French in 1900 and 1901 correspondingly. These works deal with the problem on probability limit of a sum of infinitely growing number of magnitudes, dependent on random causes, being in the given limits. To prove the limit theorem for this case A.M. Liapunov developed a new method referred to as the method of characteristic functions. It is one of the basic methods in the modern probability theory.

2.6 Lecture courses on theoretical mechanics

During Kharkov period of his activity (1885–1902) A.M. Liapunov prepared the courses of lectures on theoretical mechanics which he delivered at the University and the Technological Institute. On the occasion of the 125-th birthday of A.M. Liapunov these lectures were issued by "Naukova Dumka" Publisher in 1982. The book begins with the sketches about life and activity of A.M. Liapunov and about his lectures on theoretical mechanics written by Boris Liapunov and A.N. Krylov. The first section of the book contains the course of theoretical mechanics which was delivered by Liapunov at the Technological Institute. The second, third and fourth sections contain the university course of lectures on analytical mechanics including "bases of deformable bodies and hydrostatics" and "attraction theory".

These lectures, as Academician Steklov pointed out, had been written by Liapunov himself and were a valuable contribution to theoretical mechanics.

3 A.M. Liapunov's credo

It is generally acknowledged that in many branches of mathematics and mechanics A.M. Liapunov established a certain level of consideration accuracy and proof strictness that raised the mathematical sciences to the state of the art and made them classical.

The characteristic feature of A.M.Liaponov's creative work is his interest to the most difficult problems of mechanics arised due to the development of science and whose solution was of great importance for applications. In all his scientific work Alexander Mikhaylovich adheared to the rule:

> "It is not appropriate to make use of ambiguous means in solving a certain problem no matter if it refers to mechanics or physics but is definitely stated from the mathematical point of view. It becomes a pure analysis problem and must be dealt with as it is."

In the end of our survey of scientific and pedagogical activity of A.M. Liapunov we note the permanent effect of his works on further development of mathematics and mechanics in the 20-th century. In all cases when for a real process or a natural phenomenon there was constructed a mathematical model in the form of differential or other type equation or system of such equations the application of Liapunov methods provides the possibility to carry out dynamic analysis of the phenomenon considered no matter whether this phenomenon occurs in biology or astrodynamics.

Entirely devoted to science, a person of perfect integrity Alexander Mikhaylovich actively supported democratic transformations in Russia, freedom of press and opposed reactionaries' impact on secondary and high school. V.A.Steklov wrote that "his spiritual values matched each other so well and nobly that Russia can do be proud of her son".

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