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Performance Analysis of Communication Networks Based on Conditional Value-at-Risk

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Abstract: In this paper, we present an analysis of optimization and risk calculation in Communication Networks (CNs). The model is proposed for offline traffic engineering optimization with bandwidth allocation and performance analysis. First, we introduce an optimization model in the CN and derive the optimal bandwidth capacity. Then, we analyze the profit shortfall risk in the CN by using a conditional value-at-risk approach for two typical arrival processes of traffic demand: Poisson arrival process and uniform distribution arrival process. Finally, we give numerical results to show the impact of risk averseness and compare how the characteristics of these two arrival processes of traffic demand affect the network performance.

Keywords: *Communication networks; performance analysis; stochastic traffic engineering; conditional value-at-risk; optimization.*

Mathematics Subject Classification (2000): 46N99, 90B15, 90B18.

1 Introduction

As we have presented in [1], traffic engineering in Communication Networks (CNs) is a process of controlling traffic demand in a network so as to optimize resource utilization and network performance [2], [3]. There are two forms of traffic engineering: online planning and offline planning. Online traffic engineering focuses on instantaneous network states and individual connections. Offline traffic engineering simultaneously examines each channel's resource constraints and studies what is needed of each Local Service Provider (LSP) in order to provide global calculations and solutions for the CNs by a centralized view. Traffic engineering has greatly improved network utilization and performance by using advanced technologies such as Multi-Protocol Label Switching (MPLS) and Optical Channel Trails (OCT) [4], [5].

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Previously, the offline traffic engineering optimization problem was formulated as a deterministic Multi-Commodity Flow (MCF) model with the objective to optimize the network total revenue derived from transmitting traffic demand. In the deterministic MCF model, the demand of each channel was assumed to be a fixed quantity and the network revenue was a linearly increasing function of the amount of bandwidth allocated to the network [6]-[8]. This approach may be improper when dealing with the case that the input for off-line optimization was assumed to be stochastic demand. In view of this, recently, Mitra and Wang developed a stochastic traffic engineering framework and proposed new approaches for risk analysis in communication networks.

An important aspect of Mitra and Wang's model is the formulation of the demand and revenue under a two-tier market structure: one is the wholesale market, the other is the retail market. In the wholesale market, the demand is assumed to be deterministic and there is no risk in revenue. In the retail market, the demand is random and there exists the risk of revenue shortfall. The objective includes both the maximization of the mean revenue and the acceptable risk level [8]-[10].

Based on the two-tier market structure for demand and revenue, Mitra and Wang also analyzed the impacts of demand variability on various aspects of traffic engineering design in their numerical studies. They observed significant changes in shadow costs, link utilization, bandwidth provisioning and routing with demand variability, and explained their causes and implications [8].

In [9], Mitra and Wang developed an optimization model to support bandwidth management decision-making based on the mean-risk framework. They discussed the selection of risk indices and proposed the use of standard deviation of total profit. They investigated the service provider's risk averseness on various aspects of bandwidth management. They also discussed profit improvement brought about by the presence of the wholesale market under various market and network conditions [9].

In [10], Mitra and Wang furthered their studies in [8] and [9] and developed the efficient frontier of mean revenue and revenue risk. They discussed three different risk indices including variance, Tail value-at-risk, and standard deviation. They obtained conditions under which the optimization problem was an instance of convex programming and therefore efficiently solvable. They also studied the properties of the solution for the special case of Gaussian distributions of demands. They also analyzed the impact of demand uncertainty on various aspects of traffic engineering, such as link utilization, bandwidth provisioning and total revenue [10].

Based on the analysis framework presented in [8]-[10], Wu, Yue and Wang proposed a stochastic model for macro-level bandwidth management from the viewpoint that emphasizing the randomness and risk averseness and their impacts on the network's performance. First, they treated the whole network as an integrated service provider and did not consider the routing and capacity sharing within the communication network. They also removed the wholesale market with deterministic demand and only focused on the random demand in the retail market, which was also mentioned in [8]. Second, they emphasized the stochastic properties of the demand and the impact of risk averseness on the system performance, such as bandwidth capacity and profit function. Finally, they presented a loss rate constraint to guarantee the network performance.

In their model, the communication network was regarded as a service provider, who can charge revenue from transmitting demand and pay for the cost for bandwidth allocation. Since the profit function obtained by transmitting traffic load was a random variable and thus had the risk of deviation from the desired expected profit. They characterized the risk by use of the variance of the profit function. In [1], Wu, Yue and Wang proposed a stochastic model for optimizing bandwidth allocation with a loss rate constraint. They analyzed the loss rate constraint and risk averseness in the CN optimization model and showed the impact of loss rate constraint and risk averseness on the network performance. Wu, Yue and Wang also introduced a penalty cost based on the model presented in [1] to guarantee the network performance. They analyzed penalty cost and risk averseness in the CN optimization model and showed the impact of penalty cost and risk averseness on the network performance [11].

In regard to the selection of risk index, Mitra and Wang had proposed several approaches including variance, standard deviation and Tail value-at-risk [8]-[10]. However, they also mentioned that they did not use Tail value-at-risk in their analysis because of the optimization computational difficulties [10]. In [1] and [11], the risk index was defined as the deviation from the expected profit and measured by variance of the profit function, which included the upside risk and downside risk.

In this paper, we define the risk to be the downside risk of the profit shortfall and use an equivalent definition of a risk analysis tool named conditional value-at-risk, which is similar to Tail value-at-risk. There are two advantages of using this approach: first, it can avoid the computational difficulties mentioned in [10] and thus obtain an explicit solution for this problem; second, it can avoid the disadvantage of equally penalizing the desirable upside and the undesirable downside outcomes that is inherent in the mean-variance approach.

In this paper, we first describe the basic model proposed in [1] and derive the optimal bandwidth capacity without risk. Then, we analyze the system performance and derive the optimal bandwidth capacity by using CVaR approach and show the impact of risk on the network performance by the analysis. We also compare the characteristics of network performance by using CVaR approach presented in this paper with the characteristics of network performance presented in [1], where the mean-variance approach was used. Finally, numerical results are given to show the impact of risk on network performance.

The rest of this paper is organized as follows. In Section 2, we present the system model, notations and preliminaries. In Section 3, we present the optimization model and derive the optimal bandwidth capacity. In Section 4, we analyze the network profit shortfall risk by using the CVaR approach and also derive the optimal bandwidth capacity for two typical arrival processes of traffic demand: Poisson arrival process and uniform distribution arrival process. In Section 5, we give some numerical results to show the impact of risk averseness on the network performance. Conclusions are given in Section 6.

2 System Model

A Communication Network (CN) is formulated as a collection of nodes and links that should derive its revenue by delivering traffic load to and from its users. A unit cost is charged for unit bandwidth capacity allocated to the network. The objective of this system is to maximize the expected profit of the whole network. To guarantee the optimal network performance, we present performance analyses for the loss rate constraint and the risk of profit shortfall where the model presented in this paper is the same as the one presented in [1].

Similar to the description in [8], let (N, L) denote a CN composed of nodes n_i ($n_i \in N,\ 1 \le i \le N$) and links l ($l \in L$), where N is the total number of nodes and L is the total number of links in the network. Let V denote the set of all node pairs, and $n \in V$ denote an arbitrary node pair where $n = (n_i, n_j)$ and $n_i, n_j \in N$. Let C_l denote the maximal bandwidth capacity of link l, $R(n)$ denote an admissible route set for $n \in V$, ξ_s $(s \in R(n))$ denote the amount of capacity provided on route s, D_n $(n \in V)$ denote the traffic load on an arbitrary node pair $n \in V$, and b_n $(n \in V)$ denote the amount of bandwidth capacity provided to an arbitrary node pair n. Since between two node pairs n_i and n_j, there may be more than one route to be routed, then $b_n = \sum_{s \in R(n)} \xi_s$.

In this paper, we consider the CN to be a whole system. We let b denote the amount of bandwidth capacity provided to the CN, then we have $b = \sum_{n \in V} b_n$. If we let D denote the traffic demand in the CN, then we have $D = \sum_{n \in V} D_n$, which is characterized by a random distribution with its probability density function $f(x)$ and cumulative distribution function $F(x)$. $b \wedge D$ is the actual traffic load transmitted in the CN, where " $\wedge$ " represents the choice of the smaller value between b and D. Let r denote the unit revenue by serving the traffic demand, so the total revenue of the CN is $r \times (b \wedge D)$. Let c denote the unit cost for unit bandwidth capacity allocated in the CN, so the total cost is $c \times b$.

To avoid unrealistic and trivial cases, we make the following assumptions:

(1) Probability density function of the random traffic load is $f(x) \geq 0$.

(2) Cumulative distribution function of the random traffic load is $F(x)$ and $F(x)$ is strictly increasing in x.

(3) Traffic demand D in the CN is assumed to be positive, i.e., $D > 0$.

(4) Total bandwidth capacity b provided to the CN is assumed to be positive, i.e., $b > 0$.

(5) Maximal capacity C_{max} that can be allocated to the CN is assumed to be positive, i.e., $C_{max} > 0$.

(6) System parameters are as follows: unit revenue r and unit cost c satisfy $r > c > 0$.

3 Optimal Bandwidth Capacity without Risk

In this section, we present the model for the network bandwidth allocation problem and derive the optimal bandwidth capacity that can be attained without incurring any risk (see [1]).

Let $\pi(b, D)$ denote the random profit function by transmitting messages in the network, namely,

$$\pi(b, D) = r(b \wedge D) - cb. \tag{1}$$

Let $\Pi(b, D)$ denote the mean profit function as follows:

$$\Pi(b, D) \quad = \quad E[\pi(b, D)] = r\int_0^b xf(x)dx + rb\int_b^{+\infty} f(x)dx - cb. \tag{2}$$

By using the method of integral by parts, Eq. (2) can be obtained as follows:

$$E\left[\pi(b, D)\right] = (r - c)b - r\int_0^b F(x)dx. \tag{3}$$

The objective function of the system is given by

$$\Pi^* = \max_{b>0}\{\Pi(b, D)\}, \tag{4}$$

subject to

$$P(b \geq \alpha D) \geq \beta \tag{5}$$

and

$$b \leq C_{max}, \tag{6}$$

where Π^* is the optimal profit function.

Eq. (5) is the loss rate constraint proposed in [1] adopted from [12]. In the loss rate constraint, α $(0 \leq \alpha \leq 1)$ is the percentage of satisfied users and $1 - \alpha$ is the loss rate. As α increases, the loss rate $1 - \alpha$ deceases. The higher α is, the better the network performance is. β $(0 \leq \beta \leq 1)$ is the confidence level, which represents the probability of $b \geq \alpha D$. As $1 - \beta$ decreases, the confidence level β increases. A higher confidence level β guarantees a higher probability of achieving a better network performance. The loss rate constraint enables us to control the network performance by properly setting the system parameters.

With the above assumptions, we can derive the optimal bandwidth capacity that should be allocated to the CN. First, we analyze the property of the mean profit function $\Pi(b, D)$ without any constraints.

The first order derivative of $\Pi(b, D)$ of Eq. (2) with respect to b is given as follows:

$$\frac{d\Pi(b, D)}{db} = (r - c) - rF(b). \tag{7}$$

The second order derivative of $\Pi(b, D)$ presented in Eq. (2) with respect to b is given as follows:

$$\frac{d^2\Pi(b, D)}{db^2} = -rf(b). \tag{8}$$

From the assumptions in Section 2, we know that $f(b) \geq 0$ and $r > 0$, hence,

$$\frac{d^2\Pi(b, D)}{db^2} \leq 0. \tag{9}$$

Therefore, we can say that $\Pi(b, D)$ is a concave function of b. So, the optimal bandwidth capacity that should be allocated to the CN is given by

$$F^{-1}\left(\frac{r - c}{r}\right), \tag{10}$$

where $F^{-1}(\cdot)$ is the inverse function of $F(\cdot)$.

Next, we analyze the loss rate constraint. Note that the loss rate constraint is equivalent to

$$P\left(b \geq \alpha D\right) = P\left(D \leq \frac{b}{\alpha}\right) \geq \beta. \tag{11}$$

By the definition of cumulative distribution function $F(x)$, Eq. (11) becomes

$$P\left(D \leq \frac{b}{\alpha}\right) = \int_0^{\frac{b}{\alpha}} f(x)dx = F\left(\frac{b}{\alpha}\right) \geq \beta. \tag{12}$$

So the loss rate constraint is equivalent to

$$b \in \left[\alpha F^{-1}(\beta), +\infty\right). \tag{13}$$

Thus, the optimal bandwidth capacity b^* for the CN bandwidth allocation with loss rate constraint is given as follows:

$$b^* = F^{-1}\left(\frac{r-c}{r}\right) \vee \alpha F^{-1}(\beta), \tag{14}$$

where "$\vee$" represents the choice of the larger value between $F^{-1}\left(\frac{r-c}{r}\right)$ and $\alpha F^{-1}(\beta)$.

Finally, if we consider the maximal capacity constraint, then the optimal bandwidth capacity for the network is given as follows:

$$b^* \wedge C_{max}, \tag{15}$$

where "$\wedge$" represents the choice of the smaller value between b^* and C_{max}.

4 Risk Analysis in Communication Networks

The term risk plays an important role in the literature on economic, financial and technological issues. There are various attempts to define and to characterize the risk for descriptive as well as prescriptive purpose. In general, we regard risk as random profit or loss of a position. It can be positive (gains) as well as negative (losses) [13].

In the presence of demand uncertainty, maximizing only the mean revenue in the network, which is implied in the earlier deterministic models, may be incomplete for the random demand case. Mitra and Wang had proposed a broader optimization objectives in financial area to address the issue of risk averseness. In this paper, we use CVaR as the risk measurement which is adopted from financial risk management. In the following, we give a brief introduction on risk management.

The mean-variance analysis, which was first introduced by Markowitz [14], has been a standard tool in risk management. It involves a systematic tradeoff between the mean and the variance [15]. Value-at-Risk (VaR), introduced in 1994, has been extensively used for measuring risk and has become a part of the financial regulations in the world [16]. It allows a manager to specify a confidence level (a certain level of probability) for attaining a certain level of the wealth. Recently, Rockafellar and Uryasev presented an alternative measure of risk with the CVaR approach [17]. It measures the average value of the profit below the γ-quantile $(0.0 < \gamma < 1.0)$ level. Some empirical evidence proposed by [18] showed that the CVaR approach had superior computational characteristics when it is compared with the VaR approach and the mean-variance approach.

In [8]-[10], Mitra and Wang had proposed several approaches including variance, standard deviation and Tail value-at-risk. However, they also mentioned that they did not use Tail value-at-risk in their analysis because of the optimization computationally difficult [10]. In [1] and [11], the risk index was measured by the variance of the profit function. In this paper, we use CVaR as the risk analysis tool, which can transform the original problem into a two step optimization problem.

By using CVaR approach as the risk index, we have the objective function given as follows [19]:

$$\max_{b>0} C_\gamma \left\{\pi(b, D)\right\}, \tag{16}$$

where

$$C_\gamma \left\{\pi(b, D)\right\} = \max_{v\in R} \left\{ v + \frac{1}{\gamma} E\left[(\pi(b, D) - v)^-\right] \right\}, \tag{17}$$

where $(\cdot)^{-1}$ is the negative part of $(\cdot)$, $\pi(b, D)$ is the random profit function given by Eq. (1), v belongs to the real number set R and γ is a risk parameter.

Then, the objective function using the CVaR approach can be obtained as follows:

$$\max_{b>0} \left\{ \max_{v\in R} \left\{ v + \frac{1}{\gamma} E\left[(\pi(b, D) - v)^-\right] \right\} \right\}. \tag{18}$$

We define a jointly concave function as follows:

$$g(v, b) = \left\{ v + \frac{1}{\gamma} E\left[(\pi(b, D) - v)^-\right] \right\}. \tag{19}$$

By using the definition of expectation, we can get that

$$g(v, b) \quad = \quad v + \frac{1}{\gamma} \left\{ \int_0^b (rx - cb - v)^- dF(x) + \int_b^\infty (rb - cb - v)^- dF(x) \right\}, \tag{20}$$

where r, c and b are defined in Section 2.

According to the objective function given by Eq. (18), we know that to find the optimal bandwidth capacity b^* is equivalent to a two-step optimization. The first step is to maximize $g(v, b)$ with $v \in R$ as follows:

$$\max_{v\in R} \left\{g(v, b)\right\}. \tag{21}$$

The second step is to maximize $\max_{v\in R}\{g(v, b)\}$ with $b > 0$ as follows:

$$\max_{b>0} \left\{ \max_{v\in R} \left\{g(v, b)\right\} \right\}. \tag{22}$$

With respect to the first-step optimization, we illustrate four cases to derive the optimal solution v^*.

(1) For $v < -cb$:

In this case, $(rx - cb - v)^- = 0$ and $(rb - cb - v)^- = 0$. Both the two terms in large parenthesis of Eq. (20) vanish. Consequently, $g(v, b) = v$, thus $\dfrac{\partial g(v, b)}{\partial v} = 1 > 0$.

(2) For $-cb < v < rb - cb$:

In this case, $(rb - cb - v)^- = 0$. The second term in large parenthesis of Eq. (20) vanishes while the first term in large parenthesis of Eq. (20) remains, consequently,

$$g(v, b) = v + \frac{1}{\gamma} \int_0^b (rx - cb - v) dF(x). \tag{23}$$

Since $rx - cb - v < 0$, we have

$$x < \frac{v + cb}{r}. \tag{24}$$

So, Eq. (23) becomes as follows:

$$g(v, b) = v + \frac{1}{\gamma}\int_0^{\frac{v+cb}{r}}(rx - cb - v)dF(x). \tag{25}$$

Thus,

$$\begin{aligned}\frac{\partial g(v,b)}{\partial v} &= 1 + \frac{1}{\gamma}\int_0^{\frac{v+cb}{r}}(-1)dF(x) + \frac{1}{\gamma}\cdot\frac{1}{r}\left[(rx - cb - v)f(x)\right]\Big|_{x=0}^{x=\frac{v+cb}{r}} \\ &= 1 - \frac{1}{\gamma}F(\frac{v+cb}{r}) + \frac{1}{\gamma}\cdot\frac{1}{r}(0-0) \\ &= 1 - \frac{1}{\gamma}F(\frac{v+cb}{r}). \end{aligned}\tag{26}$$

(3) For $rb - cb < v$:

In this case, both the two terms in large parenthesis of Eq. (20) remain, consequently,

$$g(v, b) = v + \frac{1}{\gamma}\left\{\int_0^b (rx - cb - v)dF(x) + \int_b^\infty (rb - cb - v)dF(x)\right\}. \tag{27}$$

Thus,

$$\begin{aligned}\frac{\partial g(v,b)}{\partial v} &= 1 + \frac{1}{\gamma}\int_0^b(-1)dF(x) + \frac{1}{\gamma}\int_b^\infty(-1)dF(x) \\ &= 1 - \frac{1}{\gamma} < 0. \end{aligned}\tag{28}$$

(4) Let us consider about what happened when v approaches $-cb$ from the right, and also when v approaches $rb - cb$ from the left as follows:

Let $v = -cb + \Delta$ for a sufficiently small positive number Δ ($\Delta > 0$), then

$$\begin{aligned}\frac{\partial g(v,b)}{\partial v} &= 1 - \frac{1}{\gamma}F\left(\frac{\Delta}{r}\right) \\ &= 1 - \frac{1}{\gamma}\int_0^{\frac{\Delta}{r}} f(x)dx > 0. \end{aligned}\tag{29}$$

In the same way, for $v = rb - cb - \Delta$,

$$\frac{\partial g(v,b)}{\partial v} = 1 - \frac{1}{\gamma}F\left(b - \frac{\Delta}{r}\right). \tag{30}$$

We cannot say whether Eq. (30) is negative or not for a sufficiently small positive number Δ. But the sufficient condition for Eq. (30) is negative to be given by

$$b > F^{-1}(\gamma). \tag{31}$$

Note that b is an unknown value in the first-step optimization. Let $v^*(b)$ denote the optimal solution of the first-step optimization. We illustrate two cases to investigate the optimal bandwidth capacity b^*.

(1) For $b \geq F^{-1}(\gamma)$:

In this case, $v^*(b) = rF^{-1}(\gamma) - cb$. Hence,

$$g(v^*(b), b) = rF^{-1}(\gamma) - cb - \frac{r}{\gamma}\int_0^{F^{-1}(\gamma)} F(x)dx. \tag{32}$$

Thus,

$$\frac{dg(v^*(b), b)}{db} = -c. \tag{33}$$

From Eq. (33), we know that $g(v^*(b), b)$ is a monotone function of b. So, the optimal solution in this case is the boundary value $b = F^{-1}(\gamma)$.

(2) For $b \leq F^{-1}(\gamma)$:

In this case, $v^*(b) = rb - cb$. Hence,

$$g(v^*(b), b) = rb - cb - \frac{r}{\gamma}\int_0^{b} F(x)dx. \tag{34}$$

Thus,

$$\frac{dg(v^*(b), b)}{db} = r - \frac{r}{\gamma}F(b) - c. \tag{35}$$

So, the optimal solution in this case is

$$b^* = F^{-1}\left(\gamma\frac{r-c}{r}\right), \tag{36}$$

which is also the optimal solution of this problem.

From the analysis above, we can say that:

(1) Compared with the model without risk presented in [1], the result presented in this paper enlarges the dimension of the problem without risk and provides more insight for those network managers who has a different risk preference:

- **(i)** When the parameter $\gamma \to 1$, the result of the model with risk in this paper is the same as that of the model without risk presented in [1].
- **(ii)** When the parameter $\gamma \to 0$, it means that there is no bandwidth allocated in the CN, i.e., the network manager is unwilling to provide any service.
- **(iii)** When the parameter $0 < \gamma < 1$, the optimal bandwidth obtained in this paper is less than the optimal bandwidth obtained without risk presented in [1]. It means that the optimal bandwidth capacity that is allocated in a network with risk is always less than that without risk.

(2) Compared with the mean-variance analysis model presented in [1], the result presented in this paper reveals advantages of using the CVaR approach over the mean-variance approach. It avoids the disadvantage of the mean-variance approach, which equally penalizes the desirable upside and the undesirable downside outcomes. It also provides a closed-form solution, $F^{-1}\left(\gamma \cdot \dfrac{r-c}{r}\right)$, which has superior computational characteristics than the mean-variance approach.

In the following, we are going to present two typical arrival processes to express the random arrival processes of traffic demand in a CN. One is the Poisson arrival process; the other is the uniform distribution process.

4.1 Poisson arrival process

In this subsection, we consider a fully distributed communication network, where the traffic demand offered to the whole CN forms a Poisson process with arrival rate $\lambda > 0$, since in most of the system models in CNs, the arrival process of traffic demand is assumed to form a Poisson process.

The interarrival times are exponentially distributed with rate λ. Let X be a random variable representing the time between successive demand arrivals in the Poisson process, then we have the probability distribution function $F_X(x)$ and the probability density function $f_X(x)$ of X as follows:

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0, \end{cases} \tag{37}$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0. \end{cases} \tag{38}$$

The mean and variance of the exponential distribution are $1/\lambda$ and $1/\lambda^2$, respectively.

Based on the assumption of the traffic demand, the optimal bandwidth of Eq. (36) can be obtained as follows:

$$b^* = -\frac{Ln\left[1 - \gamma \dfrac{r-c}{r}\right]}{\lambda}, \tag{39}$$

where $Ln[\cdot]$ is the natural logarithm function based on e. The optimal mean profit of Eq. (2) can be obtained as follows:

$$\Pi^*(b, D) = \frac{r}{\lambda}\left(1 - e^{-\lambda b^*}\right) - cb^*, \tag{40}$$

where b^* is given by Eq. (39).

4.2 Process with uniform distribution

In this subsection, we consider the same fully distributed communication network, but the traffic demand offered to the whole CN forms a uniform distribution on some interval $[m, n]$ $(-\infty < m < n < +\infty)$. Without loss of generality, we choose the interval $[0, 1]$ and the distribution function is denoted as $U[0, 1]$. (This assumption is sometimes used in some of the system models of CNs, such as ATM system). The arrival process of traffic demand is assumed to form a uniform distribution with the probability distribution function $F_X(x)$ and the probability density function $f_X(x)$ given as follows:

$$F_X(x) = \begin{cases} 1, & x > 1 \\ x, & 0 \le x \le 1 \\ 0, & x < 0, \end{cases} , \quad (41)$$

$$f_X(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases} . \quad (42)$$

The mean and variance of the uniform distribution are 1/2 and 1/12, respectively.

Based on the assumption of the traffic demand, the optimal bandwidth of Eq. (36) can be obtained as follows:

$$b^* = \gamma \frac{r-c}{r}. \quad (43)$$

The optimal mean profit of Eq. (2) can be obtained as follows:

$$\Pi^*(b, D) = (r-c)b^* - \frac{r}{2}(b^*)^2, \quad (44)$$

where b^* is given by Eq. (43).

From Eqs. (39) and (43), we can obtain that the value of optimal bandwidth capacity with risk increases linearly to reach the value of the optimal bandwidth capacity without risk as the risk parameter γ increases from 0.0 to 1.0, i.e., the risk parameter has a linear impact on the bandwidth capacity.

5 Numerical Results

With the same system parameters as in [1] and assumptions of arrival processes of traffic demand as presented in Section 4, we give some numerical results to show the impact of loss rate constraint and the impact of risk averseness on the network performance and compare the characters of network performance obtained in [1] and in this paper.

According to the engineering experience, we choose three different arrival rates following Poisson arrival process to represent the different cases of traffic load in the CN as: $\lambda = 0.1, 0.5, 0.9$ where $\lambda = 0.1$ represents the case that the traffic load in the CN is low, $\lambda = 0.5$ represents the case that the traffic load in the CN is normal, and $\lambda = 0.9$ represents the case that the traffic load in the CN is heavy.

5.1 Impact of risk averseness on bandwidth capacity

We give some numerical results to show the impact of risk averseness on the network bandwidth capacity.

Note that the optimal bandwidth capacity without risk, which is presented in [1], is $F^{-1}\left(\frac{r-c}{r}\right)$. However, in this paper the optimal bandwidth capacity with risk is given by Eq. (36). They are different ones.

Figure 5.1 shows the optimal bandwidth b^* as a function of the risk parameter γ with two different arrival processes of the Poisson arrival process and the uniform distribution process.

The ordinate axis b^* of Figure 5.1 corresponds to the optimal bandwidth capacity given by Eq. (36). b^* means the value of optimal bandwidth capacity provisioned in the

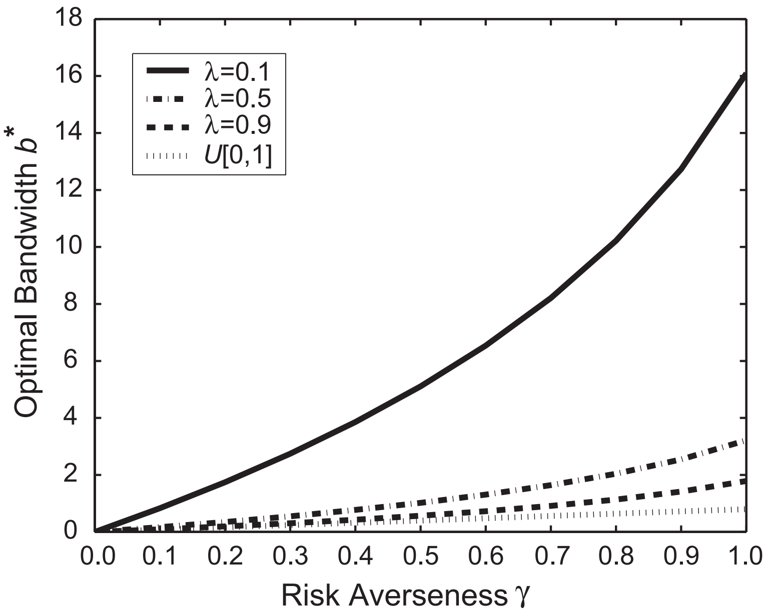

Figure 5.1: Impact of risk on the CN optimal bandwidth.

CN with risk. The smaller the value of b^* is, the less the amount of bandwidth capacity will be. So, it clearly reflects the impact of risk averseness on the network performance.

The horizontal axis γ of Figure 5.1 corresponds to the risk parameter γ taking values from 0.0 to 1.0 by 0.1 each step. $\gamma = 0.0$ denotes the special case of most averse to risk. $\gamma = 1.0$ denotes the special case of risk neutrality. When γ increases from 0.0 to 1.0, it indicates the CN manager's risk attitude changes from risk-averse to risk-neutral.

From Figure 5.1, we can discuss the impact of risk averseness on the network performance. When γ increases from 0.0 to 1.0, the CN manager becomes less risk averse and he is inclined to bear more risk and sacrifice less profit to hedge risk. So, the value of b^* becomes larger as the increase of γ. The less risk averse (with larger values of γ) the CN manager is, the more the optimal bandwidth capacity b^* is.

Our numerical results include the optimal bandwidth capacity obtained without risk averseness presented in [1], which is one point in the curves with the value of $\gamma = 1.0$ in the horizontal axis of Figure 5.1.

From the numerical results in Figure 5.1, we can conclude that:

(1) In all curves, the optimal bandwidths with risk are always less than that without risk and the bandwidth capacity increases as the risk averseness decreases. This is because the values that the risk parameter γ takes are less than 1.0.

(2) The curves with smaller arrival rates as $\lambda = 0.1, 0.5$ have quicker bandwidth capacity increasing speed than the curve with a larger arrival rate $\lambda = 0.9$. This is because the bandwidth b is a decreasing function of arrival rate λ, which can be easily obtained by Eq. (39).

(3) With the same risk averseness, all the curves with the assumption of exponential distribution reveal a larger impact than the curve with the assumption of uniform distribution. This is because the inverse function of the probability cumulative function of the Poisson arrival process is always larger than the inverse function of the probability cumulative function of the arrival process following the uniform distribution in the system, it results in a direct impact on the bandwidth capacity.

Compared with the results presented in [1] without risk, the numerical results in our paper reveal a distinct impact of risk on the network bandwidth capacity.

5.2 Impact of risk averseness on mean profit function

With the same system parameters and assumptions of traffic demand for Figure 5.1, we give some numerical results to show the impact of risk averseness on the network mean profit.

Figure 5.2 shows the optimal mean profit $\Pi^*(b, D)$ as a function of the risk parameter γ with two different arrival processes of the Poisson arrival process and the uniform distribution process.

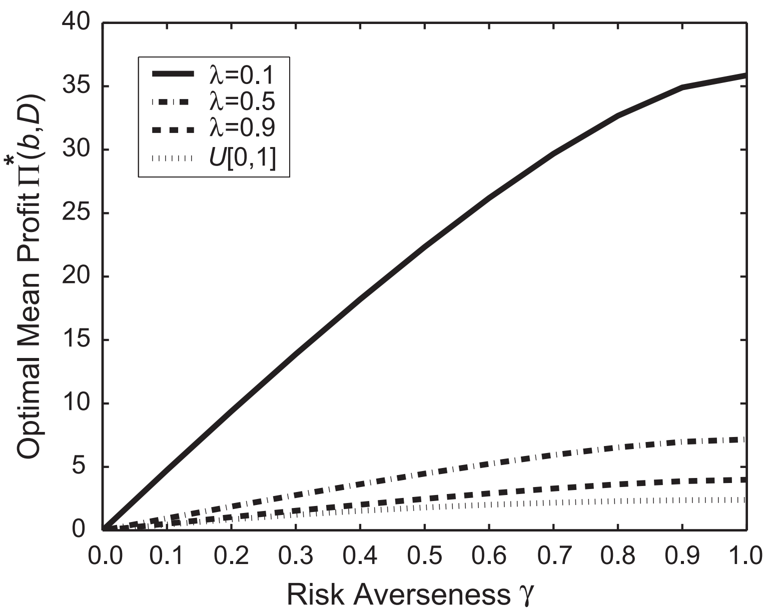

Figure 5.2: Impact of risk on the CN optimal profit.

The horizontal axis γ of Figure 5.3 has the same meaning as that of Figure 5.1. The ordinate axis $\Pi^*(b, D)$ of Figure 5.2 corresponds to the optimal mean profit given by Eq. (2).

$\Pi^*(b, D)$ means the value of optimal profit obtained by the CN with risk. The smaller the value of $\Pi^*(b, D)$ is, the less the amount of optimal profit obtained by the CN will be. So, it clearly reflects the impact of risk averseness on the network performance.

From Figure 5.2, we can discuss the impact of risk averseness on the network performance. When γ increases, the CN manager becomes less risk averse and he is inclined to bear more risk and sacrifice less profit to hedge risk. So, the value of $\Pi^*(b, D)$ becomes larger as the increase of γ. The less risk averse the CN manager is, the more the optimal mean profit $\Pi^*(b, D)$ is.

Our numerical results include the optimal profit obtained without risk averseness presented in [1], which is one point in the curves with the value of $\gamma = 1.0$ in the ordinate axis of Figure 5.2.

Similarly, as we concluded in Subsection 5.1, from the numerical results shown in Figure 5.2, we can conclude that:

(1) In all curves, the profits with risk are always less than the profit without risk and the profit increases as risk averseness decreases. This is because the values that the risk parameter γ takes are less than 1.0.

(2) The curves with smaller arrival rates as $\lambda = 0.1, 0.5$ have quicker mean profit increasing speeds than the curve with a larger arrival rate $\lambda = 0.9$. This is because the profit $\Pi^*(b, D)$ is a decreasing function of arrival rate λ, which can be easily obtained by Eq. (40).

(3) With the same risk averseness, all the curves with the assumption of exponential distribution reveal a larger impact than the curve with the assumption of uniform distribution. This is because the inverse function of the probability cumulative function of the Poisson arrival process is always larger than the inverse function of the probability cumulative function of the arrival process following the uniform distribution in the system, which results in a direct impact on the mean profit.

Compared with the results without risk presented in [1] without risk, the numerical results in our paper reveal a distinct impact of risk on the network profit function.

6 Conclusions

In this paper, we presented a stochastic model for bandwidth allocation and performance analysis in Communication Networks (CNs) with risk analysis included. We have derived the optimal bandwidth allocation capacity with risk averseness. We have analyzed the risk averseness in CNs by using the conditional value-at-risk approach. We have given numerical results to compare our model with the previous model presented in [1] and shown the impact of the risk on the network performance for two arrival processes of traffic demand. We can conclude that risk averseness has a distinct impact on the network performance. The implications presented in this paper provided insight for traffic engineering design and planning.

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