Continuous-Time Optimal Portfolio Selection Using Mean-CaR Models

Zhong-Fei Li\textsuperscript{1*}, Kai W. Ng\textsuperscript{2} and Xiao-Tie Deng\textsuperscript{3}

\textsuperscript{1} Department of Risk Management and Insurance, Lingnan (University) College, Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
\textsuperscript{2} Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong
\textsuperscript{3} Department of Computer Science, City University of Hong Kong, Kowloon, Hong Kong

Received: July 18, 2005; Revised: October 22, 2006

Abstract: This paper studies continuous-time optimal portfolio selection under the setting of Black-Scholes financial markets and constant re-balanced portfolio (CRP) investment strategies. Three mean-CaR models are formulated, which minimize the risk measured by capital-at-risk (CaR) under the constraint that the expected terminal wealth is not lower than a pre-assigned level. These models are converted into bi-level optimization problems by virtue of a decomposition of the feasible solution set and, as a result, explicit optimal strategies and efficient frontiers are obtained in closed-form. A comparison of the three mean-CaR models and a numerical example illustrating the results are presented. Some economic implications of the results are also examined.

Keywords: Continuous-time portfolio selection; Capital-at-Risk (CaR); Black-Scholes financial markets; constant-rebalanced portfolios (CRP); mean-CaR models.