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PERSONAGE IN SCIENCE

Academician Yu.A. Mitropolskii

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On January 3, 2007, Professor Yurii Alexeevich Mitropolskii, the Member of the National Academy of Sciences of Ukraine and Member of Russian Academy of Sciences, will turn 90. The Editorial Board of the International Scientific Journal "Nonlinear Dynamics and Systems Theory" wishes him many happy returns of this day and a great health and prosperity. To honor Professor Mitropolskii, the Editorial Board of "Nonlinear Dynamics and Systems Theory" presents here a biographical sketch highlighting Mitropolskii's research and scholarly activities.

1 Brief Outline of Mitropolskii's Life

Yurii Alexeevich Mitropolskii was born on January 3, 1917 in the Charnysh's estate located in Kobelyakskiy district of Poltava province. Shortly thereafter he was baptized in Shishaki village, Kobelyakskiy district of the same province where his birth certificate was issued. This is why Shishaki village is erroneously referred to as his place of birth.

Yurii Alexeevich's grandfather on his father's side, Savva Alexeevich Mitropolskii, graduated from St. Petersburg Military Medical Academy and then served at Michailovskoye Artillery Academy. In 1906 he retired at the rank of a general.

Yurii Alexeevich's father, Alexey Savvich, enrolled St. Petersburg University, department of physics and mathematics, but later changed his major to law and graduated 1906 with a law degree. Yurii Alexeevich's mother, Vera Vasilievna (whose maiden name was Charnysh) was born to a noble family. Her great-grandfather, Ivan Vasilievich Charnysh, was the chairman of the noble community in Poltava province. His grandson, Vasiliy Nikolaevich Charnysh, the father of Vera Vasilievna, at the age of 17, voluntarily joined the Russian Imperial army during the Russian-Turkish war to liberate Bulgaria from the Ottoman oppression. In 1878, in the battle at Plevna (Bulgaria), he was severely wounded. He died in 1906 at the age of 46.

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Alexey Savvich served in the Russian Army during the First World War. Later he joined the Red Army and continued to serve there until his retirement in 1926.

During the civil war (in 1918), the Charnysh's residence was completely destroyed. It was common for that time. Consequently, the Mitropolskii's moved to Kiev. In 1932, Yurii Alexeevich finished a 7-years school in Kiev followed by his employment at a cannery. In 1938, Yurii Alexeevich graduated from a high school with honors. In the same year, he was accepted at Kiev University in the department of mathematics and physics. During that time, lectures were taught by B.Ya. Bukreev, G.F. Pfeiffer, and V.E. Dyachenko, while N.N.Bogoliubov and M.A.Lavrentiev were among young instructors.

Upon completion of his third year at Kiev University, when on the day of June 22, 1941, the fascist Germany attacked the Soviet Union, Yurii Alexeevich married his university mate Alexandra Ivanovna to live together happily for more than 60 years. All this time, Alexandra Ivanovna has been his loyal friend and a guardian angel. Yurii Alexeevich and Alexandra Ivanovna have two children: son Alexey (born in 1942) and daughter Nadezhda (born in 1948).

On July 7, 1941 Yurii Alexeevich joined the Soviet Army and was stationed in an armor division in the city of Chuguyev. In October of 1941, according to the decree issued by the Defense Secretary S.K. Timoshenko, all fourth and fifth years college students were eligible to continue their degrees at the corresponding universities, with forthcoming appointments at military academies. Yurii Alexeevich was sent to the city of Kzyl-Orda in Kazakhstan where Kiev University was evacuated to. In March of 1942, Yurii Alexeevich successfully passed all exams and graduated from Kazakh University and then was sent to Ryazan Artillery Academy in the city of Talgar, which he graduated from in March of 1943 in the rank of a lieutenant. He was sent to the Stepnoy battlefield thereafter.

In 1946, after being discharged from the army, Yurii Alexeevich joined the Ukrainian Academy of Sciences in Kiev in the capacity of a Junior Scientist. In 1948 Yurii Alexeevich received his Candidate of Science degree (the Western equivalent of a Ph.D. degree). His thesis was titled "Investigation of resonance phenomena in nonlinear systems with variable frequencies." In the same year he joined the Institute of Constructive Mechanics of the Ukrainian Academy of Sciences (now S.P. Timoshenko's Institute of Mechanics of the National Academy of Sciences of Ukraine) in the capacity of a Senior Scientist under the supervision of N.N. Bogoliubov.

In 1951 he was awarded a Doctor of Science degree (the Western and Eastern European equivalent of Habilitation Degree). His thesis was titled "Slow processes in nonlinear oscillatory systems with many degrees of freedom." Earlier he moved to the Institute of Mathematics of the Ukrainian Academy of Sciences where he was appointed a Senior Scientist. In 1953 Yurii Alexeevich was promoted to the rank of Professor and Department Head in the same Institute. In 1956 he became the Associate Provost of Science of this Institute and in 1958 he was appointed its Director. He remained in this capacity up until 1988. Since1988 he has served as the Honorary Director of the Institute of Mathematics.

In 1958, Yurii Alexeevich was elected the Corresponding Member of the Academy of Sciences of Ukrainian SSR and in 1961 he was elected the Full Member of the Academy of Sciences of the USSR (now the Russian Academy of Sciences), then the most prestigious academic title in the USSR.

2 Basic Trends of His Scientific Work

During the years of his scientific activity, Yurii Alexeevich Mitropolskii has obtained numerous fundamental results in nonlinear mechanics and differential equations. The results of his prolific research were manifested in more than 700 articles and 50 monographs, of which most essential are "Nonstationary Processes in Nonlinear Oscillating Systems" (1955), "Asymptotic Methods in the Theory of Nonlinear Oscillations" (1964), "Averaging Method in Nonlinear Mechanics" (1971), and "Nonlinear Mechanics. Single-Frequency Oscillations" (1997).

We present an overview of his most significant work.

2.1 Development of Asymptotic Methods in Nonlinear Mechanics

Along with N.M. Krylov and N.N. Bogoliubov, Mitropolskii was one of the first to develop asymptotic methods in nonlinear mechanics. More specifically, he studied nonstationary processes under variations of frequency, mass and other parameters of nonlinear systems.

Imposing the condition on the system parameters to be of slow variation relative to the characteristic period of oscillations, Mitropolskii created a very efficient approach. In fact, this condition turned out to be very practical and it often applies to various problems of physics and engineering. By means of this approach, he obtained significant results in many real world situations for models with one and many degrees of freedom that pass through a resonance.

Due to the successful applications of this method, a number of phenomena in nonlinear oscillating systems (for example, amplitude delay, breakdowns and abrupt changes of amplitude, beats, etc.) were finally explained.

An important application of this method was the calculation of resonance. This method has also led to the description of the formation of a noise in a cyclotron built in the United Institute of Nuclear Research located in Dubna (Russia). Calculation of turbo-engine rotor oscillations and of centrifuge oscillations were among other notable applications of the method.

A historical development of this method was described in his monographs [1-3, 12, 14, 15, 26, 30, 36, 38, 40, 47, 48] written in Russian. Some of them were translated into many languages worldwide (see [6, 7, 10, 11, 17, 43, 46]).

2.2 The Development of the Single-Frequency Method

In 1948, when investigating an autonomous system with many degrees of freedom, N.N. Bogoliubov suggested a scheme of partial solution for equations describing single-frequency oscillations. The proposed scheme was based on the averaging method. Having used the same averaging method, Mitropolskii developed a technique of an asymptotic solution in the form of a series. A remarkable advantage of the series method is that one can construct a differential equation to define the amplitude and phase with no need of precise motion equations.

Mitropolskii extended the single-frequency method to distributed parameter systems and systems with gyroscopic terms. A version of one-frequency method developed by him for equations in a symbolic form turned out to be effective in the investigation of nonstationary oscillations of crank-shafts, systems of transmissions, and electrical circuits.

The method is well established in a series of papers by Mitropolskii as well as in his monograph [42].

2.3 Contribution to the Method of Integral Manifolds

Another major area of Mitropolskii's research was initiated in his work on the method of integral manifolds. In the early 50th, he proposed and laid the foundation to the method involving the construction of a two-parametric family of partial solutions to systems with many degrees of freedom and slowly varying parameters.

His most important contribution to this subject of study includes establishing an existence criteria for integral manifolds in systems of nonlinear differential equations with variable coefficients and in resonance systems. In particular, these existence criteria were related to the regions of parametric resonance in resonance systems. Furthermore, Mitropolskii came across the quasisyncronization phenomenon. He also extended the method of integral manifolds from finite- to infinite-dimensional systems, distributed parameter systems, and singularly perturbed systems, to name a few.

The results obtained along this topic have been presented in monographs [13, 19, 24].

2.4 The Method of Accelerated Convergence

His work on accelerated of convergence corresponds to yet another direction in Mitropolskii's research. It was initially suggested by N.N. Bogoliubov, and it was based on a combination of the accelerated convergence method and the method of integral manifolds. This approach was essentially developed by Mitropolskii and recently applied to numerous problems in nonlinear mechanics.

Let us mention some of them: the problem of reducibility of a nonlinear system of differential equations to a linear system with constant coefficients, the problem on reducibility of linear systems with quasiperiodical coefficients to linear systems with constant coefficients, investigation of trajectories on tora, and others. The basic results obtained along this subject have been summarized in monograph [22] (see also [27]).

2.5 The Averaging Method

Mitropolskii's studies on the averaging method also generated a new direction in his intense research activities. The initiation of a rigorous theory of the averaging method was due to N.N. Bogoliubov. In Mitropolskii's work, the Bogoliubov's technique became further enhanced and adopted to new classes of differential equations containing small and large parameters, as well as to equations in functional spaces, equations with deviating arguments, and to integro-differential and stochastic differential equations. For the investigation of partial differential equations of a quasi-hyperbolic type he developed a particular version of averaging and shortening technique for infinite-dimensional systems. This begot an investigation of distributed parameter systems and systems with slowly varying parameters.

Numerous results obtained along this line have been brought together in monograph [23] (see also [31, 32, 35, 44] in Russian and [25, 37] in other languages).

2.6 Asymptotic Methods and Averaging Method for Distributed Parameter Systems

Another notable direction in Mitropolskii's research includes his studies on distributed parameter systems. Based on N.M. Krylov's and N.N. Bogoliubov's suggestions to use

asymptotic expansion method for distributed parameter systems, Mitropolskii explored their ideas beginning with a rigorous mathematical formalism. In a combination of the single-frequency method with Fourier method of separation of variables and the averaging method, Mitropolskii developed a very efficient and innovative approach.

This enabled him and his followers to construct approximate solutions for distributed parameter systems under nonlinearity, random perturbations, delays, nonlinearity with boundary conditions, and slowly varying parameters.

All of these led to the creation of the energy method, which is based on the construction of first and second approximation equations for the amplitude and phase of a single-frequency oscillating process. Instead of some preliminary construction of a precise quasi-hyperbolic partial differential equation, the use of the new approach enabled one to proceed immediately to an expression for potential and kinetic energy.

All related results have appeared in Mitropolskii's numerous papers and they were summarized in his monographs [34, 37, 41].

2.7 Contribution to the Theory of Systems with Delay and Small Parameter

The development of the theory of systems with delay and small parameter proves a unique versatility of Mitropolskii's analytical mind.

In this area he established existence conditions for periodic and quasiperiodic solutions for various classes of equations, he rendered stability analysis of these solutions, constructed toroidal manifolds and investigated the path behavior on them. He also solved problems of reducibility of difference equations with quasiperiodic coefficients to a linear system of differential equations, studied quasi-hyperbolic partial differential equations with delay, and constructed periodic solutions for neutral systems.

The results obtained in this area have been the subject to some special courses at Kiev University and they have appeared in monographs [21, 31].

2.8 Development of the Theory of Random Oscillating Processes

Mitropolskii developed asymptotic methods of nonlinear mechanics in the area of oscillating stochastic processes. He investigated the white noise effect on autonomous and nonautonomous quasilinear oscillating systems described by various equations and found various characteristics of oscillating stochastic processes.

The main results obtained along this line have been presented in monograph [36].

2.9 Contribution to the Theory of Decomposition of Systems

Recently, under the guidance of Mitropolskii and with his direct participation, a group of his students and collaborators developed the theory of multi-frequence oscillating processes and theory of decomposition for a wide class of large scale systems of ordinary differential equations. Mitropolskii and his research team developed a group theory approach when studying the solution structure of systems of ordinary differential equations. They constructed an enveloping Lie algebra generated by an initial system and its associated enveloping pseudogroup. Furthermore, they studied systems of linear and nonlinear differential equations with constant and variable coefficients, established conditions for their decompositions, and developed algorithms of some specific decompositions. The results obtained in the direction have been applied in physics and engineering.

The results obtained along this line were summarized in monograph [32] (see also [39]).

At the end of our survey on the main directions of investigations by Mitropolskii it should be noted that one of the characteristic features of his profound and prolific research is his versatility starting with a rigorous problem setting, continuing with analytical solution, and ending with an algorithm including numerical illustrative examples.

3 Research-Organizational and Pedagogical Activity

Since 1958, Mitropolskii has focused his attention on the development of the Institute of Mathematics of Academy of Science of the USSR. He initiated new departments setting up to facilitate research in the areas of algebra, probability theory, real and functional analysis, and mechanics of special systems.

During this period of time, the post-graduate enrollment was substantially expanded. As the result of Mitropolskii's efforts, the Institute produced about 500 candidates of science (equivalent to Ph.D. degrees in the US) and more than 80 doctors of science (equivalent to Habilitation Degree in Eastern and Western Europe) for their further employments at national universities and research labs in Ukraine, Russia, and other countries.

As a consequences of Mitropolskii's colossal scholarly activity, the Institute of Mathematics of Academy of Science of Ukranian SSR has become the leading scientific center of mathematical research in Ukraine.

Mitropolskii began his pedagogical activity in 1948 at Kiev university to extend it up to 1989. During all these years, along with the regular courses at the department of mechanics and mathematics he taught a variety of special courses in nonlinear mechanics, mathematical physics, and differential equations. Under Mitropolskii's supervision the Institute of Mathematics offered seminars, summer mathematical schools, and organized international conferences which all have had an enormous impact upon youth of all ages, including high school students.

Mitropolskii himself supervised and directed 100 Ph.D. and 25 Habilitation theses in physical and mathematical sciences.

From 1961 until 1992 Mitropolskii had been the Head of the Department of mathematics, mechanics and cybernetics at Academy of Sciences of Ukrainian SSR. Over this period of time he paid much attention to the development of mathematical schools in various Ukrainian districts where new scientific research institutes were opened and new programs were launched, all majoring in mathematics and physics.

In 1992 Mitropolskii was appointed the director of the International mathematical center of National Academy of Science of Ukraine and Counselor of Presidium of National Academy of Science of Ukraine. This position he continues to hold to this day.

4 Editorial Activity

Mitropolskii has been much involved in editorial work. He has initiated the publication of works by Academician N.M. Krylov and selected works by Academician N.N. Bogoliubov. In the years 1961 through 1968, the Institute of Mathematics of Academy of Science of Ukrainian SSR copyrighted and published Proceedings of seminars held at the Institute of Mathematics under Mitropolskii's direct editorship.

Since 1967, Mitropolskii has been the Editor-in Chief of the "Ukrainian Mathematical Journal" whose English translation is regularly published in the US. Since 1961, he has been an editorial board member of three Russian and three international journals.

Mitropolskii was among main contributors to a 12-volume selected works by N.N. Bogoliubov in the area of mathematics and nonlinear mechanics. Three volumes of this edition has been already published by the Russian Academy of Sciences.

Mitropolskii dedicated much of his time and efforts to popularize mathematics to the general public. He gave popular lectures, talks, and wrote articles on various topics in mathematics for newspapers and popular magazines. He also held the city hearings on urgent mathematical problems.

5 International Scientific Activity

The first international talk Mitropolskii gave in 1956 at the International congress of mathematicians in Bucharest, Romania. Since 1958, he has been an invited speaker to the International Mathematical Congresses held in Edinburgh, Scotland (1958), Stockholm Sweden (1962), Moscow, Russia (1966), Niece, France (1970), Vancouver, Canada (1974), Warsaw, Poland (1983), Berkeley, USA (1986), and Kyoto, Japan (1990).

In 1960, Mitropolskii spoke at the plenary meeting of 10th International Congress on theoretical and applied mechanics at Streza, Italy, to present the main achievements and unsolved problems in asymptotic methods of nonlinear mechanics. In 1970 he took part in the 1st Pan-African Mathematical Congress.

In 1961, Mitropolskii gave a plenary talk on the method of integral manifolds in nonlinear mechanics at the International Conference held in Colorado Springs, USA. His talk was synchronously translated by Professor S.A. Lefshets. A series of lectures and talks on individual problems in nonlinear mechanics was given by Mitropolskii at various universities in the USA, China, Vietnam, Czechoslovakia, Poland, Mexico, Canada, Italy, and Yugoslavia and numerous international conferences.

His active cooperation over the past two decades with Vietnamese scientists in the area of nonlinear mechanics and theory of differential equations is worth mentioning. Due to this cooperation, they have opened an active scientific school of nonlinear mechanics in Ukraine.

6 Awards

Mitropolskii has been one of the most celebrated scientists who has ever lived in Ukraine and Russia. Consequently, his research, scholarly, pedagogical activities and public service have been highly revered. He was awarded by almost all known highest and most prestigious prizes ever given to a Soviet citizen. Here is the list of some of them:

Hero of the Socialist Labor;
Honored Activist of Science of UkrSSR;
Lenin Prize Laureate;
State Prize Laureate of Ukraine;
Federal Prize Laureate of Soviet Union;
Lyapunov Golden Medal;
Certificate of the Soviet Supreme Presidium;
Certificate of Presidium of Supreme Soviet of UkrSSR;
Lenin Golden Medal;
Two Red Star Orders;
October Revolution Medal;
Labor Red Banner Medal;

Second-Degree Great Patriotic War Medal;

The Fifth Degree Yaroslav Mudryi Order;

Bogdan Hmelnitskiy Medal;

N.M. Krylov, N.N. Bogoliubov and M.A. Lavrentiev prizes of Presidium of the Academy of Sciences of Ukrainian SSR.

Also, outside Ukraine and Russia, Mitropolskii has been treated with a high respect. In 1971, he was elected the foreign member of Bologna Academy of Sciences (Italy) and awarded with a Silver Medal of Czechoslovak Academy of Sciences "For Achievements in Science and Deeds for the Mankind". The government of Vietnam awarded him with the "Friendship" Medals in 1987 and 2001.

Completing our survey of scientific, scientific-organizational and pedagogical activities of Mitropolskii we acknowledge his undisputable achievements in mathematics, his remarkable versatility, novelty and depth of his mathematical thinking, his profound contributions to the development of nonlinear mechanics and theory of differential equations, his loyalty and tireless efforts towards mathematical sciences, that all qualifies him as one of the most distinguished mathematicians of the twentieth century.

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