Nonlinear Dynamics and Systems Theory, **6**(3) (2006) 245–256

Thermal Stresses in a Hexagonal Region With an Elliptic Hole

Sukhwinder Kaur Bhullar *

Department of Mathematics, Panjab University Chandigarh–160014, India

Received: April 30, 2005; Revised: June 23, 2006

Abstract: Considering importance of stress concentration around holes and notches of arbitrary shape in a given elastic medium for modern engineering, a two dimensional model for a thermoelastic problem in an hexagon region with an elliptic hole is established. The expressions for the temperature distribution and thermal stresses which have their importance in nuclear engineering are obtained for the model. The five elementary function's method in plane thermoelasticity of multiply connected regions is used to obtain the solutions for temperature distribution and thermal stresses. Numerical calculations are computed assuming a central elliptic hole in the hexagonal region having thermally insulated outer boundary under uniform heat generation. The obtained results are depicted graphically.

Keywords: *Temperature; thermal stress; Lame's constants.*

Mathematics Subject Classification (2000): 74F05, 74A10, 74G50.

1 Introduction

The investigation of stress concentration around holes and notches of arbitrary shape in a given elastic medium is very important for modern engineering. The high stress concentration found at the edge of a hole is of great importance. The heat generating cylinder with a hole are used in the construction of the reactor. The circular cylinder with a square hole is an applicable problem in the construction of support of the bridge. Polygon region with an elliptic hole have been used in nuclear reactor. As an example holes in ships deck may be mentioned. When the hull of a ship is bent, tension or compression is produced in the decks and there is a high stress concentration at the holes. Under the cycles of stress produced by waves, fatigue of the metal at the over

* Corresponding author: sbhullar_2000@hotmail.com

stressed portions may result finally in fatigue cracks. It is often necessary to reduce the stress concentration at holes such as access holes in airplane wings and fuselages. The contribution of several authors in this field is in [1] – [5]. Takeouti et al. studied problem of a thick cylinder having a polygon hole, thermal stress distribution in a triangular, square, hexagonal and octagonal region with a circular hole and theoretical thermal stress distribution in square region with an elliptic hole in [6] – [11]. Deresiewicz [12] calculated thermal stresses in a plate due to disturbance of uniform heat flow by a hole of general shape. Florence et al. [13] studied the problem of an infinite plate under a steady-state temperature distribution with uniform heat due to presence of an insulated ovaloid hole. Chowdhury [14] obtained thermal stresses due to uniform temperature distributed over a band of the cylindrical hole in an infinite body. Verba [15] et al. discussed static problem of thermoelasticity for an infinite plate weakened by a rectangular hole. Pan [16] found stresses in an infinite elastic plate containing two unequal circular holes. Chao et al. [17] considered problems for an anisotropic thermoelastic body containing an elliptic hole boundary.

With above background in this paper, a basic analysis is presented for thermal stress analysis in multiply connected region and the solutions for the temperature and thermal stress in a hexagon regions with an elliptic hole are obtained in the form of the infinite series expressed by the elliptic co-ordinates. The unknown constants are determined so as to satisfy boundary conditions and as they become enormous, therefore, we use point matching technique, as an extension of five elementary function's method in plane thermoelasticity of multiply-connected regions [18], [19].

2 Formulation of problem

Consider a hexagon region as shown in Figure 2.1, with an elliptic hole at the centre.

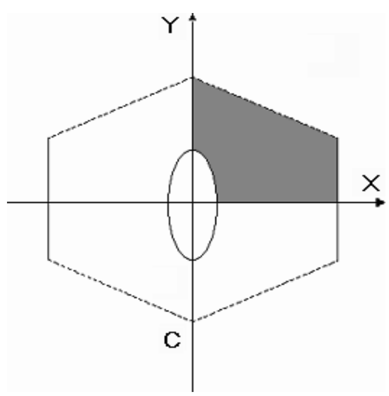

Figure 2.1: Geometry of the Problem.

Assume that the region is thermally insulated at the outer boundary with an internal convective boundary and is free from external forces. The region is made from an isotropic linear elastic material then following Takeuti [18], its behaviour under the influence of in-plane nonuniform temperature distribution which produces infinitesimal displacements is governed by the equation

$$\nabla\nabla\chi_\tau = -k\nabla\tau, \tag{1}$$

where τ is temperature change from reference state, $k =\varepsilon$ E for plane stress problem, ε is coefficient of linear thermal expansion, E is Young's modulus, χ is Airy's stress function, $\nabla = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$. The mean stresses in two dimensions are expressed in terms of stress function χ by the equation

$$\sigma_{ij} = (\nabla\delta_{ij} - \partial_i\partial_j)\chi, \tag{2}$$

where δ_{ij} is Kronecker delta, ∂_i is partial differential with respect to i $(i, j = 1, 2)$.
Steady-state heat conduction with internal heat source is governed by the equation

$$-\lambda\nabla\tau = q, \tag{3}$$

where q is heat generation per unit volume per unit time, λ is thermal conductivity.
As the region is multiply connected, the stress function χ can be expressed in terms of five elementary functions χ_τ, χ_0, χ_{1l}, χ_{2l} and χ_{3l} so that

$$\chi = \chi_\tau + \chi_0 + \sum_{h=1}^{3}\sum_{l=1}^{n} C_{hl}\chi_{hl}, \tag{4}$$

where C_{hl} are constants, $h = 1, 2, 3,$ $l = 1, 2, \ldots, n$ and h, l are not summed. Now, functions given in equation (4) should have to satisfy the following equations

$$\nabla\nabla\chi_\tau = -k\nabla\tau, \tag{5}$$

$$\nabla\nabla(\chi_0, \chi_{hl}) = 0. \tag{6}$$

Boundary conditions: Boundary conditions on the m-th boundary,

$$(\chi_\tau)_{p_m} = (\chi_{\tau,\nu})_{p_m} = 0, \tag{7}$$

$$(\chi_0)_{p_m} = -\int^{pm} dx_1 \int^{pm} \chi_{2\nu} ds + \int^{pm} dx_2 \int^{pm} \chi_{1\nu} ds, \tag{8}$$

$$(\chi_{0,\nu})_{p_m} = -(\nu_1)_{p_m} \int^{pm} \chi_{2\nu} ds + (\nu_2)_{p_m} \int \chi_{1\nu} ds, \tag{9}$$

$$(\chi_{hl})_{p_m} = [(x_h)_{p_m}(\delta_{1h} + \delta_{2h}) + \delta_{3h}]\delta_{lm}, \tag{10}$$

$$(\chi_{hl,\nu})_{p_m} = [(\nu_h)p_m(\delta_{1h} + \delta_{2h})]\delta_{lm}, \tag{11}$$

where ν is outward normal, $\nu_1 = \cos(x_1, \nu)$, $\nu_2 = \cos(x_2, \nu)$, p_m is an arbitrary point on m-th boundary, $m = 1$ corresponds to the elliptic boundary, m=0 corresponds to the hexagon boundary. In the multiply-connected bodies general equations expressed in stress components, are not sufficient for determining stresses and to get a complete solution an additional investigation of displacement is necessary. The first investigation of this kind was made by J.H. Michell [20] which are known as Michell's condition and given by

$$\int_{c_l} [\nabla(\chi_\tau + \chi_0 + \sum_{h=1}^{3}\sum_{l=1}^{n} C_{hl}\chi_{hl}) + k\tau]ds = 0, \tag{12}$$

$$\int_{c_l} [(x_2\partial_\nu - x_1\partial_s)(\chi_\tau + \chi_0 + \sum_{h=1}^{3}\sum_{l=1}^{n} C_{hl}\chi_{hl}) + k\tau]ds = 0, \tag{13}$$

$$\int_{c_l} [(x_1\partial_\nu + x_2\partial_s)(\chi_\tau + \chi_0 + \sum_{h=1}^{3}\sum_{l=1}^{n} C_{hl}\chi_{hl}) + k\tau]ds = 0. \tag{14}$$

The function given in equation (4) should have to satisfy the equations (7)-(14). We consider the resultant force and moment vanish on each boundary. Consequently, for pure thermal problem of zero traction on the boundary gives $\chi_0 = 0$, and we are taking $l = 1$. Thus (4) will take form as

$$\chi = \chi_\tau + \sum_{h=1}^{3} C_{hl}\chi_{hl} = 0. \tag{15}$$

Boundary conditions for the temperature

$$\tau = 0, \quad \textit{on the elliptic region}, \tag{16}$$

$$\tau_{,\nu} = 0, \quad \textit{on the hexagon region}. \tag{17}$$

To discuss thermal stresses and temperature distribution around the elliptic hole, use of elliptic co-ordinates is advantageous,therefore we are introducing the elliptic coordinates as (α, β) are defined for $0 \leq \alpha \leq \infty,\ 0 \leq \beta \leq 2\pi,\ x_1 = c\sinh\alpha\cosh\beta,\ x_2 = c\cosh\alpha\sinh\beta$,

$$\alpha = \sinh^{-1}\sqrt{\frac{x_1^2 + x_2^2 - c^2 + \sqrt{(x_1^2 + x_2^2 - c^2)^2 + 4x_1^2c^2}}{2c^2}},$$
$$\beta = \cosh^{-1}\sqrt{-\frac{x_1^2 + x_2^2 - c^2 - \sqrt{(x_1^2 + x_2^2 - c^2)^2 + 4x_1^2c^2}}{2c^2}}.$$

The coordinate α is constant, and $\alpha = \alpha_1$ on an ellipse of semi axes, $c\sinh\alpha_1$ and taking the semi axes as a and b. Hence c and α_1 are calculated as $c^2 = b^2 - a^2$ and α_1=$\tanh^{-1}\frac{a}{b}$, and

$$x_1 + \imath x_2 = c\sinh(\alpha + \imath\beta). \tag{18}$$

Now any complex quantity can be written in the form, $J\cos\theta + \imath J\sin\theta$, where J and θ are real. This together with equation (18) gives

$$J^2 = c^2(\cosh 2\alpha + \cos 2\beta), \quad \tan\theta = \tanh\alpha\tan\beta.$$

The expressions for thermal stress components given by equation (2), in the elliptic co-ordinates are

$$\sigma_{\alpha\alpha} = J^2 \frac{\partial^2 \chi}{\partial \beta^2} - J \frac{\partial J \partial \chi}{\partial \alpha \partial \alpha} + J \frac{\partial J \partial \chi}{\partial \beta \partial \beta}, \tag{19}$$

$$\sigma_{\beta\beta} = J^2 \frac{\partial^2 \chi}{\partial \alpha^2} - J \frac{\partial J \partial \chi}{\partial \alpha \partial \alpha} - J \frac{\partial J \partial \chi}{\partial \beta \partial \beta}, \tag{20}$$

$$\sigma_{\alpha\beta} = -J^2 \frac{\partial^2 \chi}{\partial \alpha \partial \beta} - J \frac{\partial J \partial \chi}{\partial \alpha \partial \beta} + J \frac{\partial J \partial \chi}{\partial \alpha \partial \beta}. \tag{21}$$

The expression for steady heat conduction with a constant heat generation given by (3) in elliptic coordinate will become

$$J^2 \nabla^\star \tau = -\frac{q}{\lambda}. \tag{22}$$

The displacement equations given by (5)-(6) in elliptic co-ordinates will be written as

$$J^2 \nabla^\star J^2 \nabla^\star \chi_\tau = k J^2 \nabla^\star \tau. \tag{23}$$

$$J^2 \nabla^\star J^2 \nabla^\star \chi_{hl} = 0. \tag{24}$$

Boundary equations given by (7)-(11) in elliptic co-ordinates on the boundary (m=1,0) are

$$(\chi_\tau)_{p_m} = (\frac{\partial}{\partial n} \chi_\tau)_{p_m} = \frac{\partial}{\partial \alpha} \chi_\tau (\nabla \alpha \cdot n) = 0, \tag{25}$$

$$(\chi_{11}, \chi_{21}, \chi_{31})_{p_m} = (c \sinh \alpha \cos \beta, c \cosh \alpha \sin \beta, 1)\, \delta_{1m}, \tag{26}$$

$$(\frac{\partial}{\partial n} \chi_{11}, \frac{\partial}{\partial n} \chi_{21}, \frac{\partial}{\partial n} \chi_{31})_{p_m} = (c \cosh \alpha \cos \beta, c \sinh \alpha \sin \beta, 1)\, \delta_{1m}. \tag{27}$$

Michell's conditions given by (12)-(14) in elliptic co-ordinates will become

$$\int_{\alpha=\alpha_1} \frac{\partial}{\partial \alpha} [J^2 \nabla^\star (\chi_\tau + \sum_{h=1}^{3} C_{hl} \chi_{hl}) + k\tau] d\beta = 0, \tag{28}$$

$$\int_{\alpha=\alpha_1} (\cosh \alpha \sin \beta \frac{\partial}{\partial \alpha} - \sinh \alpha \cos \beta \frac{\partial}{\partial \beta}) [J^2 \nabla^\star (\chi_\tau + \sum_{h=1}^{3} C_{hl} \chi_{hl}) + k\tau] d\beta = 0, \tag{29}$$

$$\int_{\alpha=\alpha_1} (\cosh \alpha \sin \beta \frac{\partial}{\partial \beta} + \sinh \alpha \cos \beta \frac{\partial}{\partial \alpha}) [J^2 \nabla^\star (\chi_\tau + \sum_{h=1}^{3} C_{hl} \chi_{hl}) + k\tau] d\beta = 0, \tag{30}$$

and boundary conditions given by (16)-(17) will become

$$\tau = 0, \quad on \quad the \quad elliptic\, (inner) \quad region, \tag{31}$$

$$\frac{\partial \tau}{\partial n} = \frac{\partial \tau}{\partial \alpha} (\nabla \alpha \cdot \hat{n}) = 0, \quad on \quad the \quad hexagon\, (outer) \quad region, \tag{32}$$

$$\nabla = J^2 \nabla^\star, \qquad \nabla^\star = \frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2}.$$

3 Solution of the problem

Introducing a new variable τ_r and τ_s such that

$$\tau = \tau_r + \tau_s$$

together with equation (22) gives

$$\nabla^\star \tau_s = 0, \tag{33}$$

$$J^2 \nabla^\star \tau_r = -\frac{q}{\lambda}. \tag{34}$$

The general plane harmonic temperature distribution in elliptic co-ordinates is expressed in the following series

$$\phi = \bar{A}_0 + \bar{B}_0 \alpha + \sum_{n=1}^{\infty} (\bar{A}_{2n} \cosh 2n\alpha + \bar{B}_{2n} \sinh 2n\alpha) \cos n\beta +$$

$$\sum_{n=1}^{\infty} (\bar{C}_{2n} \cosh 2n\alpha + \bar{D}_{2n} \sinh 2n\alpha) \sin n\beta. \tag{35}$$

Assuming symmetry of the region about x_1 and x_2-axis, the solution for τ_s is given as follows

$$\tau_s = \bar{A}_0 + \bar{B}_0 \alpha + \sum_{n=1}^{\infty} (\bar{A}_{2n} \cosh 2n\alpha + \bar{B}_{2n} \sinh 2n\alpha) \cos n\beta, \tag{36}$$

where $\bar{A}_0, \bar{B}_0, \bar{A}_{2n}$ and $\bar{B}_{2n}$ are unknown constants. From equation (22) particular solution is

$$\tau_r = -\frac{q^2}{8\lambda} (\cosh 2\alpha - \cos 2\beta), \tag{37}$$

Therefore

$$\tau = -\frac{q^2}{8\lambda} (\cosh 2\alpha - \cos 2\beta) + \bar{A}_0 + \bar{B}_0 \alpha + \sum_{n=1}^{\infty} (\bar{A}_{2n} \cosh 2n\alpha + \bar{B}_{2n} \sinh 2n\alpha) \cos 2n\beta. \tag{38}$$

From the consideration for Michell's conditions, the coefficients $\bar{A}_0$, $\bar{A}_{2n}$ and $\bar{B}_{2n}$ vanish in the integration of the equation as to continuity of the displacement on the boundary of the hole. These coefficients do not appear in the expressions of stress components. In our problem coefficients appearing in expression of temperature distribution are of less importance, except $\bar{B}_0$.

As the outer hexagon boundary is thermally insulated under the steady state conditions, the amount of heat generation must carry away by inner elliptic boundary. The condition of thermal insulation on outer boundary is

$$\lambda \int \frac{\partial \tau}{\partial n} ds = q \int (p \tan \frac{\pi}{p} - \pi ab), \tag{39}$$

where p represents the sides of polygon. Equation (39) together with (38) solved to obtain the value of $\bar{B}_0$ as follows

$$\bar{B}_0 = \frac{\sqrt{3}}{\lambda\pi}q. \tag{40}$$

Now to calculate stress function we are introducing a new stress function

$$\chi_\tau = \chi_{\tau r} + \chi_{\tau s}. \tag{41}$$

The equation (1) together with equation (41) will become

$$J^2\nabla^\star J^2\nabla^\star(\chi_{\tau r} + \chi_{\tau s}) = \frac{kq}{\lambda}. \tag{42}$$

By solving the equation (42), we get particular solution as

$$\chi_{\tau r} = \frac{kqc^4}{512\lambda}(\cosh 4\alpha + \cos 4\beta) \tag{43}$$

and

$$\nabla\nabla\chi_{\tau s} = 0. \tag{44}$$

We consider the symmetry of the region about x_1 and x_2- axis. The general solution for $\chi_{\tau s}$ is given by

$$\begin{aligned}\chi_{\tau s} &= A_{00} + B_{00}\alpha + C_{00}(\cosh 2\alpha - \cos 2\beta) + D_{00}(\alpha\cosh 2\alpha - \alpha\cos 2\beta - \sinh 2\alpha) + \\ &\sum_{n=1}^{\infty}[A_{2n0}\cosh 2n\alpha\cos 2n\beta + B_{2n0}\sinh 2n\alpha\cos 2n\beta + C_{2n0}(\cosh(2n+2)\alpha\cos 2n\beta + \\ &B_{2n0}\cosh(2n+2)\alpha\cos 2n\beta) + D_{2n0}(\sinh(2n+2)\alpha\cos 2n\beta + \\ &B_{2n0}\sinh(2n+2)\alpha\cos 2n\beta)].\end{aligned} \tag{45}$$

Equations (43) and (45) together give the expression for the stress function χ_τ as

$$\begin{aligned}\chi_{\tau} &= A_{00} + B_{00}\alpha + C_{00}(\cosh 2\alpha - \cos 2\beta) + D_{00}(\alpha\cosh 2\alpha - \alpha\cos 2\beta - \sinh 2\alpha) + \\ &\sum_{n=1}^{\infty}[A_{2n0}\cosh 2n\alpha\cos 2n\beta + B_{2n0}\sinh 2n\alpha\cos 2n\beta + C_{2n0}(\cosh(2n+2)\alpha\cos 2n\beta + \\ &B_{2n0}\cosh(2n+2)\alpha\cos 2n\beta) + D_{2n0}(\sinh(2n+2)\alpha\cos 2n\beta + \\ &B_{2n0}\sinh(2n+2)\alpha\cos 2n\beta)] + \frac{kqc^4}{512\lambda}(\cosh 4\alpha + \cos 4\beta).\end{aligned} \tag{46}$$

The remaining three constants C_{hl} in equation (4) are to be determined so as to satisfy the three relations of Michell's conditions. Symmetry of the region about x_1 and x_2 axis, temperature distribution of the body and Michell's conditions give

$$C_{11} = C_{12} = 0, \quad C_{31} = -\frac{8D_{00} + kB_0c^2}{8D_{03}}. \tag{47}$$

Similarly, symmetry of the region about x_1 and x_2 axis, the general solution for χ_{hl}, formed by the terms which satisfy the biharmonic equation in elliptic co-ordinates are given respectively as

$$\chi_{11} = A_{01} + B_{01} + C_{01}(\cosh 2\alpha - \cos 2\beta) + D_{01}(\alpha \cosh 2\alpha - \alpha \cos 2\beta - \sinh 2\alpha) +$$
$$A_{11}(\cosh 3\alpha \cos \alpha + B_{11}\alpha \sinh \alpha \cos \beta) + D_{11}(\sinh 3\alpha \cos \beta - \alpha \sinh \alpha \cos 3\beta) +$$
$$\sum_{n=1}^{\infty}[A_{n1} \cosh n\alpha \cos n\beta + B_{n1} \sinh n\alpha \cos n\beta + C_{n1}(\cosh(n+2)\alpha \cos n\beta -$$
$$\cosh n\alpha \cos(n+2)\beta) + D_{n1}(\sinh(n+2)\alpha \cos n\beta - \sinh n\alpha \cos(n+2)\beta)], \tag{48}$$

$$\chi_{21} = E_{12}\alpha \sinh \alpha \sin \beta + F_{12}\alpha \cosh \alpha \sin \beta +$$
$$G_{12}(\cosh 3\alpha \sin \beta - \cosh \alpha \sin 3\beta) + H_{12}(\sinh 3\alpha \sin \beta - \alpha \sinh \alpha \sin 3\beta) +$$
$$\sum_{n=1}^{\infty}[E_{n2} \cosh n\alpha \sin n\beta + G_{n2}(\cosh(n+2)\alpha \sin n\beta - \cosh n\alpha \cos(n+2)\beta) +$$
$$H_{n2}(\sinh(n+2)\alpha \sin n\beta - \sinh n\alpha \sin(n+2)\beta)], \tag{49}$$

$$\chi_{31} = A_{03} + B_{03} + C_{03}(\cosh 2\alpha - \cos 2\beta) + D_{03}(\alpha \cosh 2\alpha - \alpha \cos 2\beta - \sinh 2\alpha) +$$
$$\sum_{n=1}^{\infty}[A_{2n3} \cosh 2n\alpha \cos 2n\beta + C_{2n3}(\cosh(2n+2)\alpha \cos 2n\beta - \cosh 2n\alpha \cos(2n+2)\beta) +$$
$$D_{2n3}(\sinh(2n+2)\alpha \cos 2n\beta - \sinh 2n\alpha \cos(2n+2)\beta)]. \tag{50}$$

Therefore from relation (15), we have

$$\chi = \chi_{\tau} + C_{31}\chi_{31}, \tag{51}$$

which gives expression for χ as

$$\chi = A_{00} + B_{00}\alpha + C_{00}(\cosh 2\alpha - \cos 2\beta) + D_{00}(\alpha \cosh 2\alpha - \alpha \cos 2\beta - \sinh 2\alpha) +$$
$$\sum_{n=1}^{\infty}[A_{2n0} \cosh 2n\alpha \cos 2n\beta + B_{2n0} \sinh 2n\alpha \cos 2n\beta + C_{2n0}(\cosh(2n+2)\alpha \cos 2n\beta +$$
$$B_{2n0} \cosh(2n+2)\alpha \cos 2n\beta) + D_{2n0}(\sinh(2n+2)\alpha \cos 2n\beta +$$
$$B_{2n0} \sinh(2n+2)\alpha \cos 2n\beta)] + \frac{kqc^4}{512\lambda}(\cosh 4\alpha + \cos 4\beta) - \frac{8D_{00} + kB_0c^2}{8D_{03}} \times$$
$$[A_{03} + B_{03} + C_{03}(\cosh 2\alpha - \cos 2\beta) + D_{03}(\alpha \cosh 2\alpha - \alpha \cos 2\beta - \sinh 2\alpha) +$$
$$\sum_{n=1}^{\infty}[A_{2n3} \cosh 2n\alpha \cos 2n\beta + C_{2n3}(\cosh(2n+2)\alpha \cos 2n\beta - \cosh 2n\alpha \cos(2n+2)\beta) +$$

$$D_{2n3}(\sinh(2n+2)\alpha\cos 2n\beta - \sinh 2n\alpha\cos(2n+2)\beta)]], \tag{52}$$

where $A_{ij}, B_{ij}, C_{ij}, D_{ij}$, $(i = 0, 1, 2, ...n, j = 0, 3)$, are constants appearing in thermal stress function. Substituting (45) and (48) into (25)-(27), we show that χ_{31} and χ_τ must satisfy the boundary conditions around elliptic hole (perimeter of elliptic hole) and outer edge of hexagon. As these functions have been derived in order to satisfy the requirements of symmetry of the region about both x_1 and x_2- axes it is only necessary to consider the conditions of one quadrant of the region and as these functions are expressed in the forms of infinite series, the conditional equations to get the unknown constants become infinite. For this purpose the numerical calculations performed to get the unknown constants $A_{ij}, B_{ij}, C_{ij}, D_{ij}$, $(i = 0, 1, 2, ...n, j = 0, 3)$ become enormous. Therefore, we use the point-matching technique to satisfy the boundary conditions. That is, if we replace $\sum_{n=1}^{\infty}$ in equation (45) and (48) by $\sum_{n=1}^{n}$ approximately, the temprature and stress functions contain $4(N+1)$ unknown constants. Hence we have to solve $4(N+1)$ simultaneous equations.

We have obtained numerical values for unknown constants as follows

$$A_{00} = -4.47137\times10^{15}, B_{00} = -4.47137\times10^{15}, C_{00} = 2.6966\times10^{14}, D_{00} = -9.43448\times10^{13},$$

$$A_{20} = 7.12448\times10^{13}, B_{20} = -7.1724\times10^{13}, C_{20} = -0.463993, D_{20} = 0.490715,$$

$$A_{03} = 1, B_{03} = C_{03} = D_{03} = A_{23} = B_{23} = C_{23} = D_{23} = 0.$$

The expressions for stress components $\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$ and $\sigma_{\alpha\beta}$ are obtained substituting from (52) into (19)-(21).

4 Numerical calculations and conclusion

To analyze the results given here, we consider a numerical example. The results depict isothermals for the distributions of temperature and thermal stresses. For this purpose, we take steel as thermoelastic material. The values for the different physical parameters arising in the analysis in SI units are:

$Thermal\ \ Conductivity,\ \ \lambda = 19.5W/m^oC,$
$Specific\ \ heat\ \ at\ \ constant\ \ volume,\ \ q = 0.560\times10^3 J/kg^oC,$
$Linear\ \ thermal\ \ expansion,\ \ \varepsilon = 17.7\times10^{-6}\ \ ^oC,$
$Young's\ \ modulus,\ \ E = 195\times10^9 Pa.$

Figure 4.2 exhibits the isothermals for the temperature distributions. The region $OABC$ in Figure 4.2, represents a quadrant of hexagon region with an elliptic hole. The distribution of temperature is shown around an elliptic hole of semi-axes $a = 0.495$ and $b = 0.505$. We see that contours are moving with the increase in distance.

Figure 4.3 depicts the variation of tangential stress $\sigma_{\alpha\beta}$ around elliptic hole with same semi-axes i.e. $a = 0.495$ and $b = 0.505$ and variation in thermal stresses in this case also occur with distance. It is observed contour lines are moving with the variation in distance.

Figure 4.4 depicts variation of principal stress $\sigma_{\alpha\alpha}$ with distance along x_1-axis while Figure 4.5 depicts the variation of principal stress $\sigma_{\beta\beta}$ with respect to distance along x_2-axis from elliptic hole. It can be seen from Figure 4.4 and Figure 4.5 that principal stresses increase with distance but in opposite fashion.

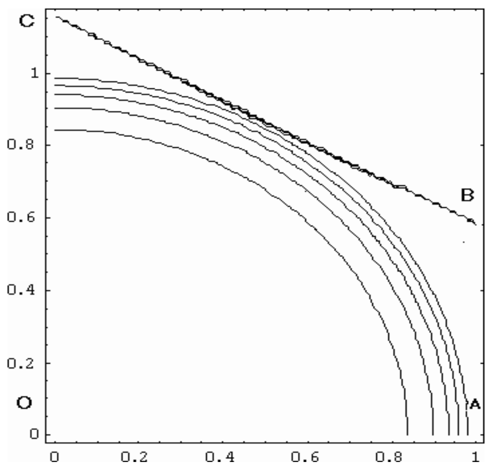

Figure 4.2: Isothermals for the elliptic hole.

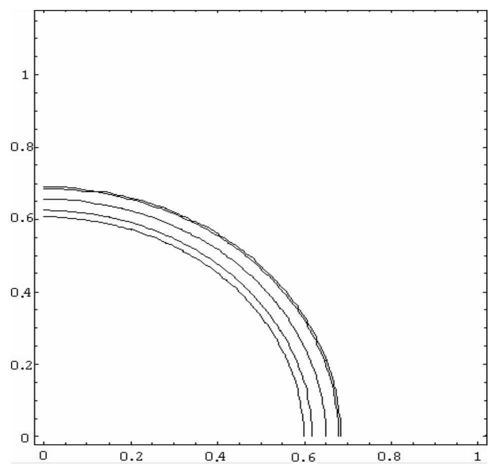

Figure 4.3: Variation of tangential stress $\sigma_{\alpha\beta}$.

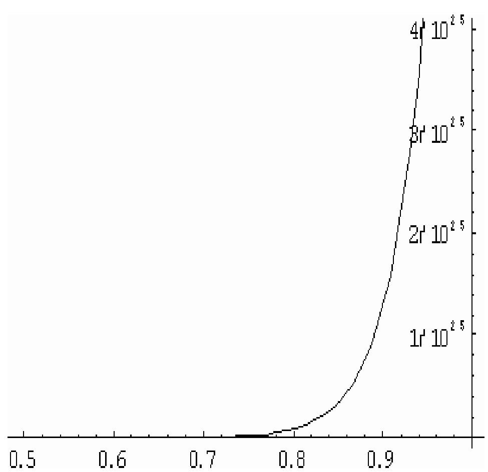

Figure 4.4: Variation of principal stress $\sigma_{\alpha\alpha}$.

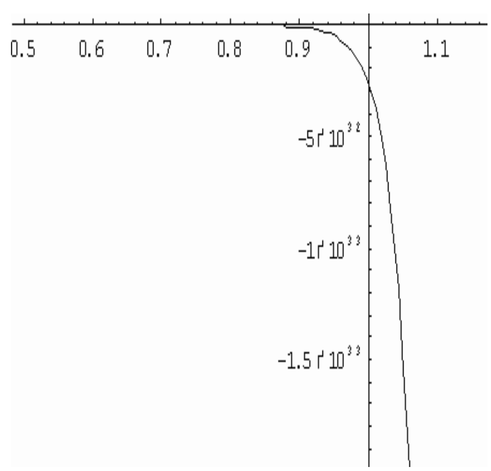

Figure 4.5: Variation of principal stress $\sigma_{\beta\beta}$.

We conclude that the isothermals of the temperature distribution around an elliptic hole within a hexagon region under constant heat generation shows that the variation in temperature occurs with distance and the pattern of variation is the same in the temperature and tangential stress $\sigma_{\alpha\beta}$ around elliptic hole. The variation in principal stress $\sigma_{\alpha\alpha}$ on x_1-axis follows the same behaviour as $\sigma_{\beta\beta}$ on x_2-axis but in opposite direction.

Acknowledgement

I express my sincere gratitude to Professor Harinder Singh, Panjab University Chandigarh, for guidance and encouragement during the preparation of this paper.

References

[1] Jeffery, G.B. Plane Stress and Strain in bipolar co-ordinates. *Trans. Roy. Soc., Series A* **221**(1921) p. 265.

[2] Neuber, H. *Kerbspannungslehre. 2. Aufl.* Springer, Berlin, 1958.

[3] Savin, G.N. *Spannungserhohung am Rande von Lochern*. VEB Verlag Technik, Berlin, 1956.

[4] Milne-Thomson, L.M. *Plane elastic systems.* Springer, Berlin, 1968.

[5] Buchwaid, V.T. and Davies, G.A.O. Plane elastostatic boundary value problems of doubly connected regions I, II. *Quart. Journ. Mech. and Appl. Math.* **17** (1964) 1-15 and 271–277.

[6] Takeouti, Y., Sumi, N. and Sugano, Y. Thermal stresses in a square region with an elliptic hole under a heat generation. In: *Structural Mechanics in Reactor Technology.* Proceedings of 2-nd International Conference, Berlin, Germany, Vol. V, Part L (1973) 10-14.

[7] Takeouti, Y. Thermal stresses in circular disc due to instantaneous line heat source. *ZAMM* **45:8** (1965) 177–184.

[8] Takeouti, Y. Thermal stresses in circular disc due to instantaneous point heat source. *Bulletin of JSME* **10** (1967) 423–429.

[9] Flynn, P.D. and Durelli, J. Discussion of stress distribution around hyderostatically loaded circular holesin the neighbourhood corners. *Trans. of ASME* **E 26** (1959) 150–152.

[10] Takeouti, Y. and Noda, N. Thermal stresses in heat generating cylinder containing a polygon hole. *ZAMM* **50**(1970) p. 587.

[11] Sekiya, T. and Sumi, N. The use of analytic continuation in solutions of plane thermoelastic problem on doubly connected regions. *Proceedings of the First International Conference on Structural Mechanics in Reactor Technology.* Berlin, M 7/3 (1971) p. 503.

[12] Deresiewicz, H. Thermal stress in a plate due to disturbance of uniform heat flow by a hole of general shape. *J. Appl. Mech.* **28** (1961) 147–149.

[13] Florence, A. L. and Goodier, J. N. Thermal stresses due to disturbance of uniform heat flow by an insulated ovaloid hole. *J. Appl. Mech.* **27** (1960) 635–639.

[14] Chowdhury, P. Thermal stresses due to uniform temperature distributed over a band of the cylindrical hole in an infinite body. *Appl. Sci.* **8** (1959) 474–478.

[15] Verba, I. I. and Kolyano, Yu. M. State of thermal stress and strain of a plate weakened by a rectangular hole. *J. Appl. Math. Mech.* **51**(3) (1987) 365–370.

[16] Pan, S. K. Thermal stresses in an infinite elastic plate containing two unequal circular holes the boundaries of which are kept at different temperatures. *Bull. Calcutta Math. Soc.* **55** (1963) 63–77.

[17] Chao, Ching-Kong, Gao, B. Mixed boundary-value problems of two-dimensional anisotropic thermoelasticity with elliptic boundaries. *Internat. J. Solids Structures* **38** (2001) 5975–5994.

[18] Takeouti, Y. and Tsuyoshi Sekiya. Thermal stresses in a polygon cylinder with a circular hole under internal heat generation. *ZAMM* **48**(1968) 237–246.

[19] Sekiya, T. and Takeouti, Y. Approximate solution of thermal stress problem in plane elasticity. *Proc. 8-th Japan Natl. Congr. App. Mech.* (1958) p. 119.

[20] Michell, J.H. *Proc. London Math Society* (1921) p. 305.