Existence of Nonoscillatory Solution of High-Order Nonlinear Difference Equation

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Abstract: In this paper, the existence of the nonoscillatory solution to the equation of a class of high-order nonlinear neutral delay difference is investigated. By using fixed point theorem, a sufficient condition is proposed for the existence of eventually positive solution.

Keywords: Difference equation; oscillation; positive solution.
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1 Introduction

In the computer designing and the ecomodeling, a class of neutral difference equation is proposed. In recent years, the oscillatory behavior of neutral difference equations was intensively studied, and some good results were obtained [1–4]. Now we consider the nonlinear high-order difference equation

\[ \Delta^m(x_n - px_{n-\tau}) + q_nf(x_{n-\sigma}) = 0, \quad (1) \]

where \( m \) is a positive odd number; \( n \in N = \{0,1,2,\ldots\} \), \( p \in R \); for \( n \in N \), \( q_n \in R^+ \), \( \sigma \in N \), \( \tau \in N\backslash\{0\} \), \( \mu = \max\{\tau,\sigma\} \), \( f \in C(R,R) \) satisfying that \( xf(x) > 0 \) for \( x \neq 0 \) and for \( \forall x, y \in R \),

\[ |f(x) - f(y)| \leq L|x - y| \quad (2) \]

where \( L \) is a positive constant. The case of \( p = 1 \) was studied in [5], the case of the equation (1) of even order was studied in [6]. In this paper, by using fixed point theorem, the case of the equation (1) of odd order is studied under the condition of \( p \neq \pm 1 \), and

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