A Study of Nonlocal History-Valued Retarded Differential Equations Using Analytic Semigroups

D. Bahuguna* and M. Muslim

Department of Mathematics Indian Institute of Technology,
Kanpur, 208 016, India

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Abstract: In this paper we study a semi-linear retarded differential equation with a nonlocal history condition considered in an arbitrary Banach space. Using the theory of analytic semigroups we establish the existence, uniqueness and regularity of solutions. We also give an example to illustrate the applications of the abstract results.

Keywords: Retarded differential equation; analytic semigroup; mild solution; regularity of a mild solution.

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1 Introduction

Let $X$ be a Banach space with its norm denoted by $\| \cdot \|$ and for $t \in [0, T]$, $0 < T < \infty$, let $C_t = C([-\tau, t]; X)$, $0 < \tau < \infty$, be the Banach space of all continuous functions from $[-\tau, t]$ into $X$ endowed with the supremum norm

$$\| \psi \|_t = \sup_{-\tau \leq \theta \leq t} \| \psi(\theta) \|.$$ 

Let $A$ be a linear operator defined from $D(A) \subset X$ into $X$ be such that $-A$ is the infinitesimal generator of an analytic semigroup $\{S(t): t \geq 0\}$ of bounded linear operators in $X$. It follows that the fractional power $A^\alpha$ of $A$ is defined for $0 \leq \alpha \leq 1$ and $D(A^\alpha)$ is a Banach space endowed with the graph norm of $A^\alpha$. Let $X_\alpha$ be the Banach space $D(A^\alpha)$ endowed with the norm

$$\| x \|_\alpha = \| A^\alpha x \|, \quad x \in D(A^\alpha),$$

*Corresponding author: d_bahuguna@yahoo.com

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