On The Dependence of Fixed Point Sets of Pseudo-Contractive Multifunctions. Application to Differential Inclusions

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Abstract: A weakened notion of multivalued contraction mapping is introduced. Some fixed point results relying on this notion are presented. The associated fixed points sets are shown to enjoy a Lipschitzian behaviour with respect to the graphs of the multifunctions. Applications are given to the dependence of solutions of differential inclusions of the form \( \dot{x}(t) \in R(t, x(t)) \) on initial values or on the right-hand sides or on parameters.

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1 Introduction

Studies about the behavior of fixed points are far less abundant than existence results (let us mention [23–25, 29]). However such studies are important; for instance they can be used to describe the dependence of solutions to differential inclusions or partial differential equations on some parameters or on boundary data.

Since in general the fixed points are not unique, one is led to use concepts of convergence of sets. Such concepts abound (see [1, 6, 8, 21, 31] for instance). But since we are interested in quantitative estimates and not only in qualitative results, we are led to use a recent variant of the Pompeiu–Hausdorff distance or hemi-metric (see [2, 3, 7, 20, 26, 27, 31]). In these developments, briefly recalled below, the stringent convergence relying on the Pompeiu–Hausdorff hemi-metric is replaced by a convergence relativized

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