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Satellite Maneuvers Using the Hénon's Orbit Transfer Problem: Application to Geostationary Satellites

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Abstract: The main objective of the present paper is to study minimum fuel maneuvers to change the position of a spacecraft in orbit around the Earth. The control used is a bi-impulsive maneuver, where the first impulse is applied in the initial position of the satellite to send it to a transfer orbit that will cross the desired final position of the spacecraft. Both initial and final position of the satellite belongs to the same Keplerian orbit. The goal is to find the transfer that has the minimum total increment in velocity and that performs the desired maneuver.

Keywords: *Astrodynamics; orbital maneuvers; bi-impulsive control.*

Mathematics Subject Classification (2000): 70M20, 70H12.

1 Introduction

In this paper, the problem of transfer orbits from one body back to the same body (known in the literature as the Henon's problem) is used to study maneuvers that has the goal of changing the position of a satellite, in the sense of sending it to a different point (true anomaly) of the same orbit. The net result is a relocation of the satellite in the same orbit. The problem of transfer orbits from one body back to the same body has been under investigation for a long time. Hénon [6] originally developed a timing condition for orbits that allow a spacecraft to leave a massless body M_2, go in an orbit around the primary M_1 and meet M_2 again, after a certain time. This was treated as the problem of consecutive collision orbits in the restricted three body problem. Several authors then worked on improvements of this problem. Hitzl [7] and Hitzl and Hénon [8, 9] studied stability and critical orbits. Perko [12] derived a proof of existence and a timing condition

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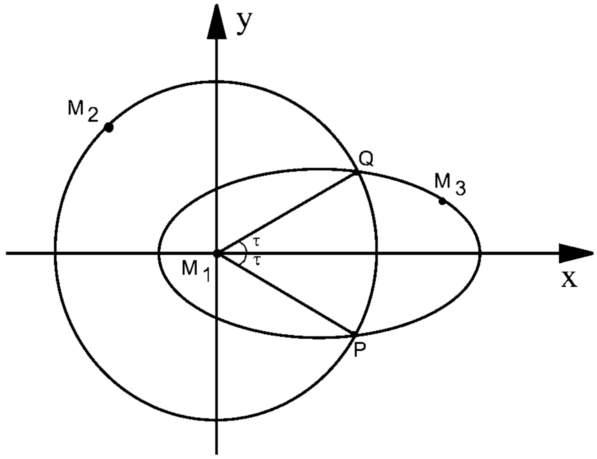

Figure 2.1. Orbit transfer from M_2 back to M_2.

for what was shown later to be a special case of Hénon's work. Results for the perturbed case where μ (mass of M_2 divided by the mass of M_1) is not zero also appeared in the literature. Some examples are the papers published by Gomez and Olle [3, 4] and Bruno [1]. Howell [10] and Howell and Marsh [11] extended Hénon's results for the case where the orbit of M_2 is elliptic.

In the present research this problem is formulated as that of an orbit transfer, as done previously in Prado [13], which can be solved with Gooding's implementation of the Lambert's problem [5]. In the approach used here, the second body M_2 is a fixed point in the orbit of the spacecraft and not a real body, but this nomenclature is used to facilitate the comparison with the results obtained from the consecutive collision orbits problem approach. Both cases, with the circular or elliptic orbits for the spacecraft are considered in the present research. The implementation developed here is generic with respect to the angle that the spacecraft has to be shifted. These transfer orbits are studied in terms of the ΔV and the time required for the transfer. The ΔVs are plotted against the transfer time for several cases and a family of transfer orbits with very small ΔV (on the order of 0.001 in canonical units, a system of units where the gravitational constant of M_1, the angular velocity of the spacecraft and the distance between M_1 and the spacecraft are all unity) is shown to exist in almost all cases studied. These orbits are studied in detail. They consist of a family of slightly different orbits (when compared to the orbit of M_2) that meet all the requirements to provide the transfer desired. A relocation of a geostationary satellite is shown as an example of a practical application of this theory.

2 Formulation of the Problem

Let M_1 be the main body of the system (the Earth, in the example used here) and M_2 be a fixed point in a circular or elliptic orbit around M_1. The massless spacecraft M_3 leaves the point M_2 from a position denoted by P $(t = -\tau\ )$, follows an orbit around M_1 and meets again with M_2 at a point Q $(t = \tau)$. The basic equations of the Kepler problem apply. The canonical system of units is used. Figure 2.1 shows a sketch of the transfer.

The solution to be found is the coordinate of the point P as a function of the transfer time. The solution is not unique, and a graph including many solutions was published by Hénon [6]. He plotted $\underline{\eta}/\pi$ (where $\underline{\eta}$ is the redefined "eccentric anomaly" of the point P) against τ/π (where τ is half of the transfer time). Another problem that is considered in the present research is the calculation of the ΔV and the time required for each of these transfers, in a search for transfer orbits with small ΔV. The solution consists of plots of the ΔV against the time required for the transfer (both in canonical units). A detailed study of the transfer orbits with small ΔV is included.

2.1 Lambert's problem formulation

A different approach used in the present research formulates Hénon's problem as a Lambert's problem. The Lambert's problem can be defined as [5]:

"An (unperturbed) orbit, about a given inverse-square-law center of force is to be found connecting two given points, P and Q, with a flight time Δt $(= t_2 - t_1)$ that has been specified. The problem must always have at least one solution and the actual number, which is denoted by N, depends on the geometry of the problem — it is assumed, for convenience and with no loss of generality, that t is positive."

Using this formulation, Hénon's problem can be defined in the following way: "Find an unperturbed orbit for M_3, around M_1, which leaves the point P at $t = -\tau$ and goes to point Q at $t = \tau$". Since M_2 is assumed to have zero mass, it has no participation in the equations of motion of the system. Its only use is to relate the time τ with the eccentric anomaly η, in such a way that M_3 has the same position as M_2 at P and Q at the times $t = -\tau$ and $t = \tau$, respectively.

3 Mathematical Formulation

In terms of mathematical formulation, Hénon's problem formulated as a Lambert's problem can be described as follows. The following information is available:

1. The position of M_3 at $t = -\tau$ (point P). It can be specified by the radius vector R_1 and the angle $-\tau$. R_1 can be related to $-\tau$ by using the equation $R_1 = a(1-\mathrm{e}^2)/(1+\mathrm{e}\cos(-\tau))$ for the orbit of M_2, since M_2 and M_3 occupy the same position at $t = -\tau$.

2. The position of M_3 at $t = \tau$ (point Q). It can be specified by the radius vector R_2 and the angle τ. R_2 can be related to τ by using the same equation used in the above paragraph.

3. The total time for the transfer, $\Delta t = 2\tau$. Remember that the angular velocity of the system is unity, so τ can be considered to be the time as well as the angle.

4. The total angle the spacecraft must travel to go from P to Q, that is called ϕ. For the case where the orbit of M_3 is elliptic this variable has several possible values. First of all, there are two possible choices for the transfer: the one that uses the direction of the shortest possible angle between P and Q (that is called the "short way"), and the one that uses the direction of the longest possible angle between these two points (that is called the "long way"). Which one is the shortest or the longest depends on the value of τ. After considering these two choices, it is also necessary to consider the possibilities of multi-revolution transfers. In this case, the spacecraft leaves P, makes one or more complete revolutions around M_1, and then goes to Q. Then, by combining

these two factors, the possible values for ϕ are: $2\tau + 2m\pi$ and $2(\pi - \tau) + 2m\pi$, where m is an integer that represents the number of complete revolutions during the transfer. There is no upper limit for m, and this problem has an infinite number of solutions. In the case where the orbit of M_3 is parabolic or hyperbolic, ϕ has a unique value. The multi-revolution transfer does not exist anymore (the orbit is not closed), and the only direction of transfer that has a solution is the one that makes the spacecraft goes in a retrograde orbit passing by periapse at $t = 0$.

The information needed (the solution of the Lambert's problem) is the Keplerian orbit that contains the points P and Q and requires the given transfer time $\Delta t = 2\pi$ for a spacecraft to travel between these two points. This solution can be specified in several ways. The velocity vectors at P or Q are two possible choices, since the corresponding position vectors are available. The Keplerian elements of the transfer orbit is also another possible set of coordinates to express the solution of this problem. In the implementation developed here, all three sets of coordinates are obtained, since all of them are useful later.

To obtain the ΔVs, the following steps are taken:

1. Find the radial and transverse velocity components of M_2 at P and Q. They are also the velocity components of M_3 just before the first impulse and just after the second impulse, respectively, since they match their orbits at these points. They are obtained from the equations [2]:

$$V_r = \frac{\mathrm{e}\sin(\nu)}{\sqrt{a(1-\mathrm{e}^2)}}, \tag{1}$$

$$V_t = \frac{1+\mathrm{e}\cos(\nu)}{\sqrt{a(1-\mathrm{e}^2)}}, \tag{2}$$

where V_r and V_t are the radial and transverse components of the velocity vector, a and e are the semi-major axis and the eccentricity of the transfer orbit and ν is the true anomaly of the spacecraft.

2. Find an unperturbed orbit for M_3 that allows it to leave the point P at $t = -\tau$ and arrive at point Q at $t = \tau$. This orbit is found by solving the associate Lambert's problem, as explained in the next section. At this point the total time for this transfer, 2τ is already known.

3. Find the velocity components at these points (P and Q) in the transfer orbit determined above. They are the velocity components for M_3 just after the first impulse and just before the second impulse. They are provided by Gooding's Lambert routine [5].

4. With the velocity components just after and just before both impulses it is possible to calculate the magnitude of both impulses (ΔV_1 and ΔV_2) and add them together to get the total impulse required (ΔV) for the transfer.

4 Gooding's Implementation of the Lambert's Problem

The solution of the Lambert's problem, as defined in the previous paragraphs, has been under investigation for a long time. The approach to solve this problem is to set up a set of non-linear equations (from the two-body problem) and start an iterative process to find an orbit that satisfies all the requirements. There is no closed-form solution available for this problem. The major difficulty is to choose the best set of equations and parameters for iterations to guarantee that convergence occurs in all cases. The routine used in this research is due to Gooding [5]. He chooses $\pm\sqrt{1 - s/2a}$ as the parameter for convergence, where a is the semi-major axis of the transfer orbit and s the semi-perimeter of the triangle formed by P, Q and M_1. He also makes several substitutions of variables, trying to find the best set of equations to guarantee convergence in all cases. His implementation is able to find all the possible solutions of the Lambert's problem, including "long way", "short way" and "multi-revolution" transfers. He gives the velocity vectors at P and Q and the Keplerian elements of the transfer orbit in his solution.

Including all phases of the present research, Gooding's routine has been called about 3 million times with no failure detected.

5 Results

In this section some results are shown in the problem of finding the ΔVs required for the transfers to be able to get the transfers with the minimum consumption. Plots of $(\Delta V) \times (\tau/\pi)$ were made for thousands of possible transfer orbits. Five orbits for M_2 around M_1 are used:

(1) The circular orbit with $a = 1$.
(2) The elliptic orbit with e $= 0.4$ and $a = 1$, with M_2 passing by periapse at $t = 0$.
(3) The elliptic orbit with e $= 0.4$ and $a = 1$, with M_2 passing by apoapse at $t = 0$.
(4) The elliptic orbit with e $= 0.97$ and $a = 1$, with M_2 passing by periapse at $t = 0$.
(5) The elliptic orbit with e $= 0.97$ and $a = 1$, with M_2 passing by apoapse at $t = 0$.

The results for orbits 1, 2 and 4 are shown in Figures 5.1 – 5.3. The vertical axis shows the total ΔV in canonical units and the horizontal axis shows τ/π, where τ is half of the transfer time. Only elliptic transfer orbits are included in these plots, since the hyperbolic or parabolic transfer orbits are too expensive, in terms of ΔV (always more than 1.6), to be useful. In these figures, τ/π varies from 0 to 14 and the maximum number of complete revolutions allowed for M_3, while in its transfer orbit, is also 14. This means that we restrict ourselves to the orbits contained in a square region with side 14 $(0 \leq \tau/\pi \leq 14$ and $0 \leq \nu/\pi \leq 14)$.

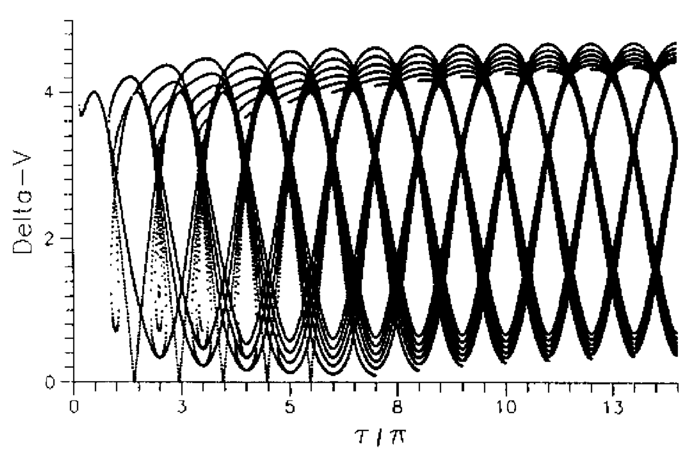

Figure 5.1. (ΔV) *vs* (τ/π) for Orbit 1 for M_2.

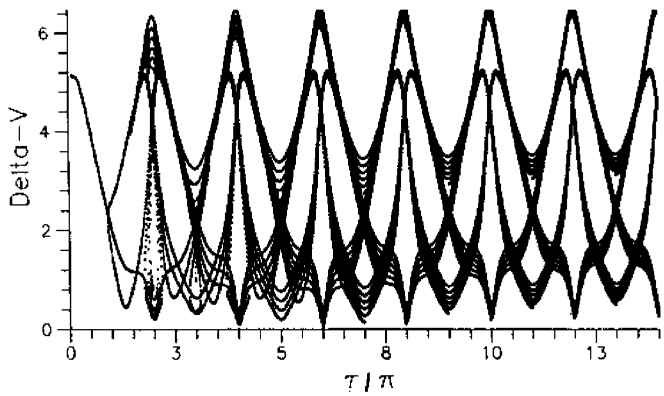

Figure 5.2. (ΔV) *vs* (τ/π) for Orbit 2 for M_2.

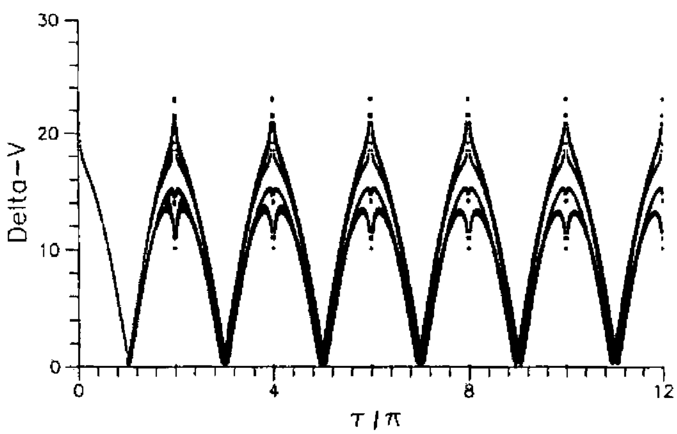

Figure 5.3. (ΔV) *vs* (τ/π) for Orbit 4 for M_2.

An examination of those figures shows the existence of points (orbits) with very small ΔV. They appear in several locations in the plot and they reveal a whole family of small ΔV transfer orbits. In all cases studied in this research, this family appears in the "short transfer time" part of the graph (small τ). A more detailed plot of $(\Delta V) vs (\tau/\pi)$ is shown in Figure 5.4. It includes only the orbits where $\Delta V \leq 0.5$ and it is restricted to orbit 1 (circular orbit) only. Plots for the orbits 3 and 5 are similar to the plots for orbits 2 and 4, respectively, and are omitted in the present text to save space. It is possible to see that the local minimums increase with time after $\tau/\pi = 6$. An investigation for τ/π varying from zero to 200 (and with the maximum number of complete revolutions for M_3 equal to 200) was done, and no more orbits with $\Delta V \leq 0.1$ were found.

Table 5.1 shows the main characteristics of the orbits with $\Delta V \leq 0.1$ found in the circular and elliptic cases. It is interesting to see that for the circular case (see the part $\mathrm{e} = 0$ in Table 5.1) most of the orbits appear in pairs, with almost identical values of τ/π. A good example is the pair formed by the first two orbits in Table 5.1: $\tau/\pi = 1.400$ and $\tau/\pi = 1.410$. In each pair one orbit has the periapse in a positive abscissa and the other one has the periapse in a negative abscissa. In this Table the orbit of M_2 is assumed to

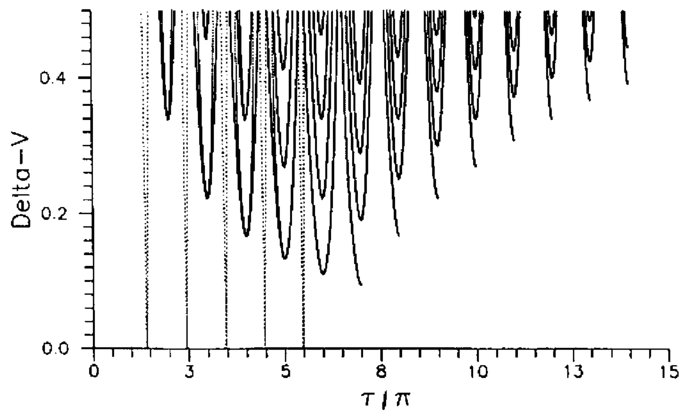

Figure 5.4. (ΔV) *vs* (τ/π) for $\Delta V \leq 0.5$ (Orbit 1 for M_2).

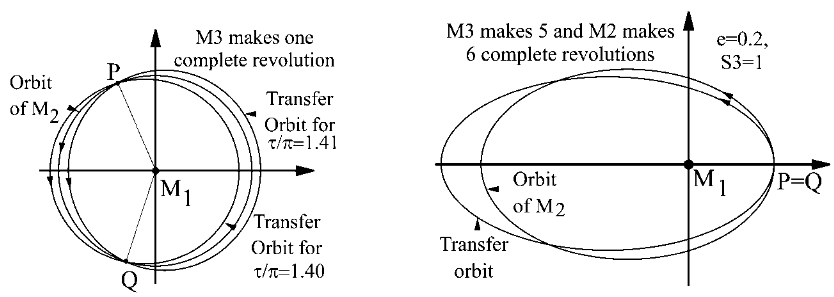

Figure 5.5. Some transfer orbits with small ΔV.

be elliptic with several values for the eccentricity. Both cases, M_2 at periapse at $t = 0$ and M_2 at apoapse at $t = 0$ are considered. Figure 5.5 shows some of those orbits.

Table 5.1 and Figure 5.5 show the mechanism of the majority of these transfer orbits. They consist of orbits with slightly different semi-major axis and eccentricity (compared with the orbit of M_2) and they have a periapse coincident with the periapse of the orbit of M_2. They have mean angular velocity (n) such that $2\tau(1-n) = \pm 2\pi$. Then, after M_3 makes m complete revolutions in its transfer orbit, M_2 makes $m+1$ or $m-1$ complete revolutions in its own orbit and they can meet each other at the common periapse, after the time 2τ.

Here τ is half of the transfer time in canonical units, $\underline{\nu}$ is redefined true anomaly, $\underline{\eta}$ is redefined eccentric anomaly, a is semi-major axis of the transfer orbit, e is eccentricity of the transfer orbit, $S3 = 1$ if M_2 is at periapse at $t = 0$ and -1 if it is at apoapse, $L = 1$ for "short way" transfer, 0 for "long way" transfer, $P = 1$ if periapse is in a positive abscissa, 0 if in a negative abscissa, $S = 1$ if transfer is direct, 0 if transfer is retrograde, $A = 1$ if M_3 pass by the periapse at $t = 0$, 0 if it pass by the apoapse, ΔV is Velocity increment in meters/second.

Table 5.1. Transfer orbits with $\Delta V \leq 0.1$ for the circular and elliptic case.

	τ/π	a	e	η/π	ν/π	L	P	S	A	ΔV
e=0	1.400	0.993	0.0216	1.406	1.400	1	1	1	1	0.0417
	1.410	1.003	0.0105	1.406	1.410	1	0	1	0	0.0204
	2.440	0.997	0.0167	2.445	2.440	1	0	1	0	0.0331
	2.450	1.002	0.0149	2.445	2.450	1	1	1	1	0.0295
	3.460	0.999	0.0036	3.461	3.460	1	1	1	1	0.0072
	3.470	1.003	0.0279	3.461	3.470	1	0	1	0	0.0555
	4.460	0.997	0.0310	4.469	4.460	1	0	1	0	0.0618
	4.470	1.000	0.0005	4.469	4.470	1	1	1	1	0.0010
	5.470	0.998	0.0169	5.475	5.470	1	1	1	1	0.0336
	5.480	1.001	0.0146	5.475	5.480	1	0	1	0	0.0292
	6.990	1.108	0.9777	5.991	6.990	0	0	1	1	0.0955
e2=0.1 S3=-1	1.410	1.4386	1.0025	0.1085	1.4729	1	0	1	0	0.0453
	2.440	2.4133	0.9979	0.1125	2.3793	1	0	1	0	0.0435
	3.460	3.4930	0.9995	0.0962	3.5238	0	0	1	0	0.0404
	4.470	4.4380	1.0002	0.0975	4.4078	1	0	1	0	0.0398
	5.480	5.5072	1.0011	0.1142	5.5436	0	0	1	0	0.0500
	7.000	6.0000	1.1082	0.1879	6.0000	0	0	1	1	0.0869
e2=0.1 S3=+1	1.400	1.3747	0.9962	0.1132	1.3420	1	1	1	1	0.0411
	2.440	2.4772	0.9970	0.0829	2.5036	0	1	1	1	0.0503
	3.460	3.4293	0.9999	0.1009	3.3982	1	1	1	1	0.0389
	4.470	4.5018	1.0000	0.1003	4.5337	0	1	1	1	0.0402
	5.470	5.4435	0.9989	0.1148	5.4078	1	1	1	1	0.0479
e2=0.2,S3=-1	7.000	6.0000	1.1082	0.2782	6.0000	0	0	1	1	0.0793
e2=0.2, S3=1	6.000	5.0000	1.1292	0.2916	5.0000	1	1	1	0	0.0917
e2=0.5, S3=-1	5.000	4.0000	1.1604	0.5691	4.0000	1	0	1	1	0.0789
e2=0.5 S3=+1	4.000	3.0000	1.2114	0.5873	3.0000	1	1	1	0	0.0993
	4.000	5.0000	0.8618	0.4198	5.0000	1	1	1	0	0.0939
	6.000	5.0000	1.1292	0.5572	5.0000	1	1	1	0	0.0655
e2=0.6 S3=+1	4.000	3.0000	1.2114	0.6698	3.0000	1	1	1	0	0.0863
	4.000	5.0000	0.8618	0.5358	5.0000	1	1	1	0	0.0810
	6.000	5.0000	1.1292	0.6458	5.0000	1	1	1	0	0.0568
e2=0.7 S3=-1	3.000	2.0000	1.3104	0.7711	2.0000	1	0	1	1	0.0985
	3.000	4.0000	0.8255	0.6366	4.0000	1	0	1	1	0.0897
	5.000	4.0000	1.1604	0.7415	4.0000	1	0	1	1	0.0577
	7.000	5.0000	1.2515	0.7603	5.0000	1	0	1	0	0.0837
e2=0.7 S3=+1	4.000	3.0000	1.2114	0.7524	3.0000	1	1	1	0	0.0728
	6.000	4.0000	1.3104	0.7711	4.0000	1	1	1	1	0.0985
	4.000	5.0000	0.8618	0.6519	5.0000	1	1	1	0	0.0679
	6.000	5.0000	1.1292	0.7343	5.0000	1	1	1	0	0.0478

6 Practical Applications

To show one possible practical application for these orbits, this theory is applied in a transfer for a satellite from one point in a circular geostationary orbit to another point in the same orbit (a point 180 degrees ahead of the initial point is used as an example, but the scheme proposed here can be used for any transfer angle desired). This problem is very important nowadays. Its solution can be used to transfer a geosynchronous satellite, to use it above a point with different longitude on Earth. Figure 6.1 shows this situation.

Figure 6.2 shows the $(\Delta V) vs (\tau/\pi)$ for elliptic transfer orbits. Hyperbolic transfer

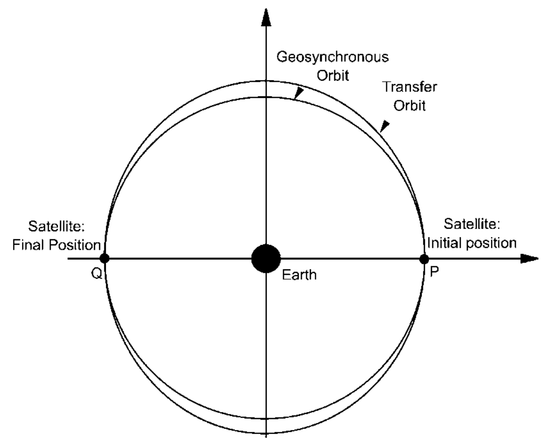

Figure 6.1. Orbit transfer for a geosynchronous satellite.

orbits are also available, but they have ΔV too large to be useful. It is assumed that the change in longitude desired for the satellite is 180 degrees. Table 6.1 shows the whole family of small ΔV orbits. Under the assumption that the orbital velocity of the satellite is 3075 m/s [14] and its orbital period is 1 day, Table 6.1 shows the real values of ΔV and 2τ (total time required for the transfer). The mechanism used by these transfers is to insert M_3 in an elliptic transfer orbit that have a periapse coincident with the periapse of the orbit of M_2. These transfer orbits have a mean angular velocity (n) smaller than 1, such that $(1-n)2\tau = \pi$. Then, in the same time that M_3 makes m revolutions in its transfer orbit, M_2 makes $m + (1/2)$ revolutions in its own orbit and M_3 meets with a point 180 degrees ahead of its initial point at Q.

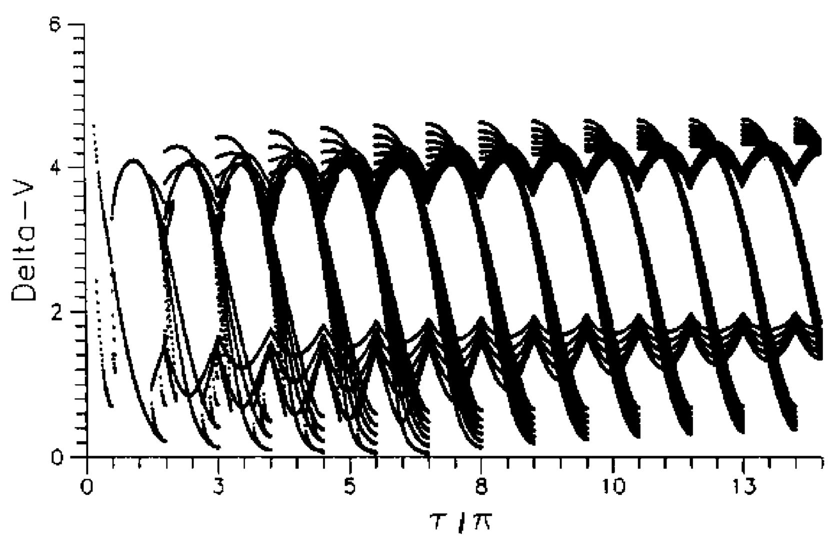

Figure 6.2. (ΔV) *vs* (τ/π) to transfer a geosynchronous satellite (Elliptic Transfer Orbits).

The same comment about other multi-revolution possible transfer orbits with a lower ΔV made in the previous cases are valid here. In this case M_2 does not exist as a real body. It is only a reference point in orbit and, in consequence, its mass is really zero. For this reason, this example fits very well the model used and the results found here are expected to be in close agreement with the real world.

Table 6.1 Transfer orbits with $\Delta V \leq 0.1$ for the transfer in the geosynchronous orbit.

τ/π	$\underline{\eta}/\pi$	a	e	$\underline{\nu}/\pi$	L	P	S	A	ΔV_c	ΔT	ΔV
3.500	3.0000	1.1081	0.0976	3.0000	0	1	1	0	0.095	3.49	292
3.500	4.0000	0.9149	0.0931	4.0000	0	0	1	0	0.095	3.49	292
4.500	4.0000	1.0816	0.0755	4.0000	0	0	1	1	0.074	4.49	228
4.500	5.0000	0.9322	0.0727	5.0000	0	1	1	1	0.074	4.49	228
5.500	5.0000	1.0656	0.0616	5.0000	0	1	1	0	0.061	5.49	188
5.500	6.0000	0.9437	0.0597	6.0000	0	0	1	0	0.061	5.49	188
6.500	6.0000	1.0548	0.0520	6.0000	0	0	1	1	0.051	6.49	157

The symbols are the same ones used in the previous tables.

7 Conclusions

The problem previously called "consecutive collision orbits" in the three-body problem is formulated as a problem of transfer orbits from one body back to the same body. Using this approach, Hénon's problem became a special case of the Lambert's problem.

Gooding's implementation of the Lambert's problem [5] is used to solve this problem with great success.

The ΔVs and the transfer time required for these transfers are calculated. Among a large number of transfer orbits, a small family is found, such that the ΔV required for the transfer is very small. These orbits and their properties are shown in detail.

A practical applications for these orbits are studied in detail: a transfer for a satellite from a point in a circular geosynchronous orbit to another point in this same orbit, 180 degrees ahead of its initial point.

The possibilities of transfers like this one is open for several types of missions and the algorithm developed here can be used to relocate a satellite to a different position in one orbit.

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