Partial Functional Differential Equations and Applications to Population Dynamics

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Abstract: In this paper we consider a semilinear partial functional differential equation with a nonlocal history condition arising in the study of problems in population dynamics. We reformulate it as a functional differential equation in a Banach space. Using the theory of strongly continuous and analytic semigroups we analyze the existence, uniqueness of mild, strong and classical solutions. Finally, we study the finite dimensional approximation of solutions.

Keywords: Partial functional differential equation; strongly continuous and analytic semigroups; mild, strong and classical solutions; projections; finite dimensional approximations.

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1 Introduction

Of concern is the following nonlocal history-valued boundary value problem for a partial functional differential equation,

\[
\frac{\partial w}{\partial t}(x, t) = a \frac{\partial^2 w}{\partial x^2}(x, t) + f(w(x, t), w(x, t - \tau)),
\]
\[
t > 0, \quad 0 < x < \pi,
\]
\[
w(0, t) = w(\pi, t) = 0, \quad t > 0,
\]
\[
h(w)|_{-\tau,0}(x, t) = \phi(x, t), \quad -\tau \leq t \leq 0, \quad \tau > 0, \quad 0 \leq x \leq \pi,
\]

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