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A Parametrization Approach for Solving the Hamilton–Jacobi Equation and Application to the $\mathcal{A}_2$-Toda Lattice

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Abstract: Hamilton–Jacobi (HJ)-theory is an extension of Lagrangian mechanics and concerns itself with a directed search for a coordinate transformation in which the equations of motion can be easily integrated. Hamilton (1838) has developed the method for obtaining the desired transformation equations by finding a smooth function S called a *generating function* or *Hamilton's principal function*, which satisfies a certain nonlinear first-order partial-differential equation (PDE) also known as the *Hamilton–Jacobi equation* (HJE).

Unfortunately, the HJE being nonlinear is very difficult to solve; and thus, except for the case in which the variables in the equation are separable, its application remains limited. It is thus our aim in this paper to present a new approach for solving the Hamilton–Jacobi equation for a fairly large class of Hamiltonian systems and to apply it in particular to the $\mathcal{A}_2$-Toda lattice.

Keywords: *Lagrangian mechanics; Hamiltonian system; contact transformation; generating function; Hamilton–Jacobi equation.*

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