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Fault Detection Filter for Linear Time-Delay Systems

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Abstract: By extension of a fault detection optimization approach to linear time invariant (LTI) systems, this short paper deals with the fault detection filter (FDF) problem for linear time-delay systems with L_2-norm bounded unknown inputs. The basic idea is first to introduce a new FDF as the residual generator; and then based on an objective function to formulate the FDF design as an optimization problem. Through appropriate choice of the filter gain matrix and a post-filter, the convergence of the residual generator and satisfactory FDF performance can be achieved. A numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: *Fault detection; filter; robustness; sensitivity; time delay.*

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1 Introduction

Many significant approaches to the problems of robust fault detection and isolation (FDI) have been developed during the past two decades, for instance unknown input observer (UIO), parity space, H_∞ optimization, eigenstructure assignment, and H_∞ filtering [1, 5, 6, 9, 12]. However, most of these aforementioned works are about delay-free systems. Time delay is an inherent characteristic of many physical systems, such as rolling mills, chemical processes, water resources, biological, economic and traffic control systems. To the best of our knowledge, only few researches on FDI have been carried out

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for time-delay systems [4, 7, 8, 10]. Note that [7] did not consider the influence of unknown inputs; [10] formulated the fault detection filter (FDF) design problem as a two-objective nonlinear programing problem where no analytic solution can be constructed in general; [8] extended the results of [10] to the discrete-time case. The authors' earlier work in [4] developed an LMI approach to FDF design for linear time invariant (LTI) time-delay systems, but the selection of weighting transfer function matrix has strong influence on FDF performance. Research on fault detection (FD) of time-delay system is as yet an open and important issue.

The main objective of this short paper is to deal with the FDF design problem for linear systems with L_2-norm bounded unknown input and multiple time delays. An FDF will be developed such that a robustness/sensitivity based objective function is minimized. The core of this study is the introduction of a new FDF as a residual generator and an extension of the optimization FDI method for LTI systems in [2, 3] to time-delay systems. A sufficient condition to the solvability of FDF is derived and a solution can be obtained by appropriate choice of a filter gain matrix and post-filter. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Notations. Throughout this paper, the superscript T stands for the matrix transposition, R^n denotes the n dimensional Euclidean space. $R^{n\times m}$ is the set of all $n\times m$ real matrices. I is the identity matrix with appropriate dimensions. L_2 denotes the space of square integrable vector functions over $[0,\infty)$. For $h(t)\in L_2$, $\|h\|_2$ denotes the L_2-norm of $h(t)$. For a real matrix P, $P>0$ (respectively, $P<0$), means that P is real symmetric and positive definite (respectively, negative definite). $\mathbf{RH}_\infty$ denotes the set of rational transfer functions analytic in closed right half plane. For $G(s)\in\mathbf{RH}_\infty$, $\|G(s)\|_\infty$ denotes the H_∞ norm of transfer function matrix $G(s)$.

2 Preliminaries and Problem Formulation

2.1 Brief review of related FD approach

Consider LTI systems described by

$$\dot{x}(t) = Ax(t) + Bu(t) + B_f f(t) + B_d d(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) + D_f f(t) + D_d d(t) \tag{2}$$

where $x(t)\in R^n$, $u(t)\in R^p$, $y(t)\in R^q$ are the state vector, control input and measurement output respectively. $d(t)\in R^m$ denotes the L_2-norm bounded unknown input, $f(t)\in R^l$ is the fault to be detected. A, B, B_f, B_d, C, D, D_f and D_d are known matrices with appropriate dimensions. It has been shown by Ding and Frank [3] that the dynamics of observer-based residual generator for systems (1) − (2) can be expressed as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H(y(t) - \hat{y}(t)), \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t), \quad r(s) = R(s)(y(s) - \hat{y}(s)) \tag{4}$$

or the frequency domain description

$$\begin{aligned} r(s) &= R(s)[(C(sI - A + HC)^{-1}(B_d - HD_d) + D_d)d(s) \\ &\quad + (C(sI - A + HC)^{-1}(B_f - HD_f) + D_f)f(s)] \\ &= R(s)G_{\varepsilon d}(s)d(s) + R(s)G_{\varepsilon f}(s)f(s) = G_{rd}(s)d(s) + G_{rf}(s)f(s), \end{aligned}$$

where $\hat{x}(t) \in R^n$ and $\hat{y}(t) \in R^q$ represent the state and output estimation vectors respectively, r is the so-called residual signal. The transfer function matrix $R(s) \in \mathbf{RH}_\infty$, also called a post-filter, and observer gain matrix H are parameters to be determined. In the case of a full decoupling of unknown input being not achievable, the main task of FDF design is to find a suitable H and $R(s)$ such that the H_∞ norm of $G_{rd}(s)$ is minimized by guaranteeing a desired sensitivity to fault. One widely accepted way is to formulate the FDF problem as the following optimal problem

$$J = \min_{R(s),H} \frac{\|R(s)G_{\varepsilon d}(s)\|_\infty}{\|R(s)G_{\varepsilon f}(s)\|_\infty}. \tag{5}$$

Under some assumptions, [2, 3] has developed an optimization method to solve the problem (5).

Lemma 1 [2, 3] *Consider system (1) – (2) and suppose the assumptions*

(A1) *system (1) – (2) is asymptotically stable when* $u(t) = 0$, $d(t) = 0$ *and* $f(t) = 0$ *for* $t \geqslant 0$;
(A2) (C, A) *is detectable;*
(A3) $\begin{bmatrix} A - j\omega I & B_d \\ C & D_d \end{bmatrix}$ *is of full row rank for* $\omega \in [0, \infty)$

hold, then

$$R^*(s) = Q^{-1/2}, \quad H^* = (B_d D_d^{\mathrm{T}} + Y C^{\mathrm{T}}) Q^{-1}$$

solve the optimal problem (5), where $Q = D_d D_d^{\mathrm{T}}$ *and* $Y \geq 0$ *is a solution of the algebraic Riccati equation*

$$Y(A - B_d D_d^{\mathrm{T}} Q^{-1} C)^{\mathrm{T}} + (A - B_d D_d^{\mathrm{T}} Q^{-1} C)Y - Y C^{\mathrm{T}} Q^{-1} C Y + B_d (I - D_d^{\mathrm{T}} Q^{-1} D_d) B_d^{\mathrm{T}} = 0$$

Moreover, $G^*_{rd}(s)$ *is a co-inner matrix, where*

$$G^*_{rd}(s) = R^*(s) \left[C(sI - A + H^* C)^{-1} (B_d - H^* D_d) + D_d \right].$$

Remark 1 From the view point of FDI, Assumptions A1 and A2 are trivial and do not lead to a loss of generality. The results in Lemma 1 are true only under the assumptions made, in particular, Assumption A3. Upon removing it, the lemma will lose its validity [3].

2.2 Problem formulation

In this short paper, we consider the FDF problem for a class of linear time-delay systems described by

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_i x(t - \tau_i) + Bu(t) + \sum_{j=1}^{L} B_j u(t - \mu_j) + B_f f(t) + B_d d(t), \tag{6}$$

$$y(t) = Cx(t) + Du(t) + D_f f(t) + D_d d(t), \tag{7}$$

$$x(-t) = 0, \quad u(-t) = 0, \quad t \geqslant 0, \tag{8}$$

where $x(t) \in R^n$, $u(t) \in R^p$, $y(t) \in R^q$, $d(t) \in R^m$, $f(t) \in R^l$ and matrices A, B, B_f, B_d, C, D, D_f and D_d are defined as in system (1) – (2). A_i $(i = 1, 2, \dots, N)$ and B_j $(j = 1, 2, \dots, L)$ are known matrices with appropriate dimensions. τ_i and μ_j denote known constant time delays. Throughout this work, Assumptions A1 to A3 corresponding to system (6) – (8) are also made, that is

(A4) system (6) – (8) is asymptotically stable when $u(t) = 0$, $d(t) = 0$ and $f(t) = 0$ for $t \geqslant 0$;

(A5) (C, A) is detectable;

(A6) $\begin{bmatrix} A - j\omega I & B_d \\ C & D_d \end{bmatrix}$ is of full row rank for $\omega \in [0, \infty)$.

The type of filter considered in this paper is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \sum_{i=1}^{N} A_i x_u(t - \tau_i) + Bu(t) + \sum_{j=1}^{L} B_j u(t - \mu_j) + H(y(t) - \hat{y}(t)), \quad (9)$$

$$\dot{x}_u(t) = Ax_u(t) + \sum_{i=1}^{N} A_i x_u(t - \tau_i) + Bu(t) + \sum_{j=1}^{L} B_j u(t - \mu_j), \quad (10)$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t), \quad \varepsilon(t) = y(t) - \hat{y}(t), \quad (11)$$

$$r(s) = R(s)\varepsilon(s), \quad (12)$$

$$\hat{x}(-t) = 0, \quad x_u(-t) = 0, \quad t \geqslant 0, \quad (13)$$

where $\hat{x}(t) \in R^n$, $\hat{y}(t) \in R^q$ and $x_u(t) \in R^n$ are vectors, $R(s) \in \mathbf{RH}_\infty$ is a so-called post-filter, H is the filter gain matrix, r is the generated residual. H and $R(s)$ are parameters to be determined for achieving perfect FD performance. Especially, in the case of unknown input full decoupling being not achievable, the main task of FDF design is to determine H and $R(s)$ such that

(i) When $d(t) = 0$ and $f(t) = 0$ for all t, the generated residual r asymptotically decays to zero for any $u(t)$.

(ii) The residual r achieves best compromise between sensitivity to faults and robustness to known input.

By denoting $e(t) = x(t) - \hat{x}(t)$ and $x_{df}(t) = x(t) - x_u(t)$, the overall dynamics of the residual generator are governed by

$$\dot{e}(t) = (A - HC)e(t) + \sum_{i=1}^{N} A_i x_{df}(t - \tau_i) + (B_d - HD_d)d(t) + (B_f - HD_f)f(t), \quad (14)$$

$$\dot{x}_{df}(t) = Ax_{df}(t) + \sum_{i=1}^{N} A_i x_{df}(t - \tau_i) + B_d d(t) + B_f f(t), \quad (15)$$

$$\varepsilon(t) = Ce(t) + D_d d(t) + D_f f(t), \quad (16)$$

$$r(s) = R(s)\varepsilon(s). \quad (17)$$

It can be seen from the above that $u(t)$ has no influence on the residual r. The main problem of FDF can be formulated as to determine H and $R(s)$ such that system (14) – (17) is asymptotically stable, while an FDF designing performance index as in (5) is satisfied.

Remark 2 Compared with the residual generator used in [4, 7, 8, 10], here $x_{df}(t-\tau_i)$ $(i=1,2,\dots,N)$ in equation (14) is used instead of the time-delay state estimate error $e(t-\tau_i)$ in [4, 7, 8, 10]. Notice that $x_{df}(t)$, which describes the effect of d and f in state x, is independent of filter gain matrix H. Especially, under the assumptions on system (6) – (8) being asymptotically stable and d, f being L_2-norm bounded, $x_{df}(t)$ is also L_2-norm bounded. Finally, the FDF problem for time-delay system can be solved by an extension of the optimization FD approach in [2, 3].

3 Design of FDF

In this section, an extension of the FD approach presented in [2, 3] will be performed for the FDF problem of time-delay system (6) – (8).

3.1 Basic idea of our study

Notice that if system (14) – (17) is asymptotically stable, then residual $r(t)$ is convergent to zero when $d(t)=0$ and $f(t)=0$. To express clearly the influences of past unknown input $d(t-\tau_i)$ and fault signal $f(t-\tau_i)$ on residual $r(t)$, we first separate $x_{df}(t)$ into $x_d(t)$ and $x_f(t)$,

$$\dot{x}_d(t) = Ax_d(t) + \sum_{i=1}^{N} A_i x_d(t-\tau_i) + B_d d(t), \tag{18}$$

$$\dot{x}_f(t) = Ax_f(t) + \sum_{i=1}^{N} A_i x_f(t-\tau_i) + B_f f(t) \tag{19}$$

and denote

$$\begin{aligned}
\theta_d(t) &= \begin{bmatrix} x_d^{\mathrm{T}}(t-\tau_1) & x_d^{\mathrm{T}}(t-\tau_2) & \cdots & x_d^{\mathrm{T}}(t-\tau_N) \end{bmatrix}^{\mathrm{T}}, \\
\theta_f(t) &= \begin{bmatrix} x_f^{\mathrm{T}}(t-\tau_1) & x_f^{\mathrm{T}}(t-\tau_2) & \cdots & x_f^{\mathrm{T}}(t-\tau_N) \end{bmatrix}^{\mathrm{T}}, \\
A_\theta &= \begin{bmatrix} A_1 & A_2 & \cdots & A_N \end{bmatrix}.
\end{aligned}$$

It is obvious that $\theta_d(t)$ and $\theta_f(t)$ respectively describe the influences of past unknown input $d(t-\tau_i)$ and fault signal $f(t-\tau_i)$ $(i=1,2,\dots,N)$, while $\theta_d(t)$ and $\theta_f(t)$ are independent of H. Recall that for L_2-norm bounded d and f, the asymptotic stability of system (6) – (8) ensures that $x_d(t)$, $x_f(t)$ and, furthermore, $\theta_d(t)$ and $\theta_f(t)$ are also L_2-norm bounded. Introduce vector $w(t)=\begin{bmatrix} d^{\mathrm{T}}(t) & \theta_d^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ to describe both the present and past unknown input, and let $B_w \triangleq \begin{bmatrix} B_d & A_\theta \end{bmatrix}$, $D_w \triangleq \begin{bmatrix} D_d & 0 \end{bmatrix}$. From the above definitions, we have

$$\dot{e}(t) = (A-HC)e(t) + (B_w - HD_w)w(t) + (B_f - HD_f)f(t) + A_\theta\theta_f(t), \tag{20}$$

$$\varepsilon(t) = Ce(t) + D_w w(t) + D_f f(t), \tag{21}$$

$$r(s) = R(s)\varepsilon(s) \tag{22}$$

and

$$r(s) = G_{rw}(s)w(s) + G_{rf}(s)f(s), \tag{23}$$

where

$$G_{rw}(s) = R(s)G_{\varepsilon w}(s), \qquad G_{\varepsilon w}(s) = [C(sI - A + HC)^{-1}(B_w - HD_w) + D_w], \tag{24}$$

$$G_{rf}(s) = R(s)[G_{\varepsilon\theta_f}(s)G_{\theta f}(s) + G_{\varepsilon f}(s)], \qquad G_{\varepsilon\theta_f}(s) = C(sI - A + HC)^{-1}A_\theta, \tag{25}$$

$$G_{\theta f}(s) = \begin{bmatrix} e^{-s\tau_1}I & e^{-s\tau_2}I & \cdots & e^{-s\tau_N}I \end{bmatrix}^{\mathrm{T}} \left(sI - A + \sum_{i=1}^{N} A_i e^{-s\tau_i}\right)^{-1} B_f, \tag{26}$$

$$G_{\varepsilon f}(s) = C(sI - A + HC)^{-1}(B_f - HD_f) + D_f. \tag{27}$$

As in [3], we use $\|G_{rw}(s)\|_\infty$ to measure the robustness of residual against unknown inputs, while the sensitivity of residual to faults is represented by $\|G_{rf}(s)\|_\infty$. Then the FDF problem for time-delay system (6) – (8) can be further formulated as to find H and $R(s)$ such that system (14) – (17) is asymptotically stable on one hand, while on the other hand solves the following optimization problem

$$J = \min_{R(s),H} \frac{\|G_{rw}(s)\|_\infty}{\|G_{rf}(s)\|_\infty}. \tag{28}$$

The procedure to solve the FDF problem is made of two steps, namely (a) the choice of filter gain matrix H to ensure the asymptotic stability of system (14) – (17), and (b) the derivation of $R(s)$ so that $(H, R(s))$ is an optimal solution of the problem (28).

Remark 3 By solving the above formulated FDF problem, not only the convergence of the residual but also the satisfactory robustness and sensitivity criterion of FD system defined in (28) are achieved.

3.2 Main results

The following Lemmas are required to solve the FDF problem.

Lemma 2 [11] *System*

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{N} A_i x(t - \tau_i), \\ x(t) &= 0 \quad \textit{for} \quad t \leqslant 0, \end{aligned}$$

is asymptotically stable, if there exist matrices $P > 0$ *and* $R_i > 0$, $(i = 1, 2, \ldots, N)$ *such that LMI*

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + \sum\limits_{i=1}^{N} R_i & PA_1 & \cdots & PA_N \\ A_1^{\mathrm{T}}P & -R_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ A_N^{\mathrm{T}}P & \cdots & 0 & -R_N \end{bmatrix} < 0$$

holds.

Lemma 3 [2] *Given*

$$\widehat{M}_1(s) = V_1 - V_1 C(sI - A + H_1 C)^{-1} H_1,$$
$$\widehat{M}_2(s) = V_2 - V_2 C(sI - A + H_2 C)^{-1} H_2,$$

where H_1 and H_2 are selected such that $A - H_1 C$ and $A - H_2 C$ are Hurwitz, V_1 and V_2 are invertible, there exists a stable solution $Q(s)$ for the equation

$$Q(s)\widehat{M}_1(s) = \widehat{M}_2(s).$$

Furthermore, the solution can be expressed by

$$Q(s) = V_2[I + C(sI - A + H_2 C)^{-1}(H_1 - H_2)]V_1^{-1}.$$

Now we are ready to present the main results of this short paper, which give a sufficient condition to solve H and parameterize FDF using the obtained solutions of H. By applying Lemma 2, we first present the determination of filter gain matrix H ensuring the asymptotic stability of system (14) – (17) (with proof omitted).

Theorem 1 *If there exist matrices $P_1 > 0$, $P_2 > 0$, $R_i > 0$, $S_i > 0$ $(i = 1, 2, \dots, N)$ and Y such that LMI*

$$\begin{bmatrix} A^{\mathrm{T}}P_1 + P_1 A - C^{\mathrm{T}}Y^{\mathrm{T}} - YC + \sum\limits_{i=1}^{N} R_i & 0 & P_1 A_1 & \cdots & P_1 A_N \\ 0 & A^{\mathrm{T}}P_2 + P_2 A + \sum\limits_{i=1}^{N} S_i & P_2 A_1 & \cdots & P_2 A_N \\ A_1^{\mathrm{T}} P_1 & A_1^{\mathrm{T}} P_2 & -S_1 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ A_N^{\mathrm{T}} P_1 & A_N^{\mathrm{T}} P_2 & 0 & 0 & -S_N \end{bmatrix} < 0$$

holds, then system (14) – (17) is asymptotically stable. Moreover, the observer gain matrix is determined by

$$H = P_1^{-1} Y.$$

After designing the filter gain matrix H, the remained important task for FDF design is the determination of a post-filter $R(s)$. Following studies show that under Assumptions of A4 to A6, for all H ensuring the stability of system (14) – (17), there exists an $R(s) \in \mathbf{RH}_\infty$ such that $(H, R(s))$ is an optimal solution of the problem (28).

Theorem 2 *Given system (6) – (8) with Assumptions of A4 to A6, there exists $R_h(s) \in \mathbf{RH}_\infty$ such that $(H, R_h(s))$ is an optimal solution of (28), where $R_h(s)$ is given by*

$$R_h(s) = Q^{-1/2}(I + C(sI - A + H^* C)^{-1}(H - H^*)), \tag{29}$$
$$H^* = (B_w D_w^{\mathrm{T}} + YC^{\mathrm{T}})Q^{-1}, \quad Q = D_w D_w^{\mathrm{T}}, \tag{30}$$

and $Y \geq 0$ is a solution of the following algebraic Riccati equation

$$\begin{aligned} Y(A - B_w D_w^{\mathrm{T}} Q^{-1} C)^{\mathrm{T}} + (A - B_w D_w^{\mathrm{T}} Q^{-1} C)Y - YC^{\mathrm{T}} Q^{-1} CY \\ + B_w(I - D_w^{\mathrm{T}} Q^{-1} D_w) B_w^{\mathrm{T}} = 0. \end{aligned} \tag{31}$$

Proof Considering system (6) – (8) and the residual generator (20) – (22), define $G_{rw}(s)$, $G_{\epsilon w}(s)$, $G_{rf}(s)$, $G_{\epsilon f}(s)$, $G_{\epsilon\theta_f}(s)$, $G_{\theta f}(s)$ as in (24) – (27), and

$$\begin{aligned}
G_{yw}(s) &= C(sI-A)^{-1}B_w + D_w,\\
G^*_{\epsilon w}(s) &= C(sI-A+H^*C)^{-1}(B_w - H^*D_w) + D_w,\\
G^*_{rw}(s) &= R^*(s)G^*_{\epsilon w}(s),\\
G^*_{\epsilon f}(s) &= C(sI-A+H^*C)^{-1}(B_f - H^*D_f) + D_f,\\
G^*_{\varepsilon\theta_f}(s) &= C(sI-A+H^*C)^{-1}A_\theta,\\
G^*_{rf}(s) &= R^*(s)[G^*_{\varepsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\varepsilon f}(s)],\\
\widehat{N}_w(s) &= G_{\epsilon w}(s),\\
\widehat{N}^*_w(s) &= G^*_{\epsilon w}(s),\\
\widehat{M}(s) &= I - C(sI-A+HC)^{-1}H,\\
\widehat{M}^*(s) &= I - C(sI-A+H^*C)^{-1}H^*.
\end{aligned}$$

Based on the left coprime factorization of $G_{yw}(s)$, it is easy to get

$$G_{yw}(s) = \widehat{M}^{-1}(s)\widehat{N}_w(s) = (\widehat{M}^*(s))^{-1}\widehat{N}^*_w(s).$$

For any available H ensuring the asymptotic stability of system (14) – (17), we then have

$$\begin{aligned}
G_{rw}(s) = R(s)G_{\varepsilon w}(s) = R(s)\widehat{N}_w(s) &= R(s)\widehat{M}(s)(\hat{M}^*(s))^{-1}\widehat{N}^*_w(s)\\
&= R(s)\widehat{M}(s)(\widehat{M}^*(s))^{-1}G^*_{\epsilon w}(s).
\end{aligned} \tag{32}$$

Moreover, from Lemma 3, it is easy to verify that, for $R^*(s) = Q^{-1/2}$ and the above defined $\widehat{M}(s)$ and $\widehat{M}^*(s)$, there exists a matrix $\Gamma(s)$,

$$\Gamma(s) = [I + C(sI-A+HC)^{-1}(H^* - H)]Q^{1/2} \tag{33}$$

such that

$$\widehat{M}(s) = \Gamma(s)R^*(s)\widehat{M}^*(s). \tag{34}$$

It follows from (32) – (34) that

$$G_{rw}(s) = R(s)\Gamma(s)R^*(s)G^*_{\epsilon w}(s) = R(s)\Gamma(s)G^*_{rw}(s). \tag{35}$$

Also, from Lemma 3, $R_h(s)$ in (29) and $\Gamma(s)$ in (33) satisfy

$$R_h(s)[I - C(sI-A+HC)^{-1}H] = Q^{-1/2}[I - C(sI-A+H^*C)^{-1}H^*], \tag{36}$$

$$\Gamma(s)(Q^{-1/2})(I - C(sI-A+H^*C)^{-1}H^*) = I - C(sI-A+HC)^{-1}H. \tag{37}$$

It is obtained from (36) – (37) that

$$\begin{aligned}
R_h(s)\Gamma(s)(Q^{-1/2})(I - C(sI-A+H^*C)^{-1}H^*) &= (Q^{-1/2})[I - C(sI-A+H^*C)^{-1}H^*]\\
\Rightarrow \quad R_h(s)\Gamma(s) &= I.
\end{aligned}$$

Thus, for $R(s) = R_h(s)$, we have

$$G_{rw}(s) = G^*_{rw}(s).$$

In the same way, we can get

$$\begin{aligned} G_{\epsilon\theta_f}(s) &= \Gamma(s)R^*(s)G^*_{\epsilon\theta_f}(s), \\ G_{\epsilon f}(s) &= \Gamma(s)R^*(s)G^*_{\epsilon f}(s), \\ G_{rf}(s) &= R(s)[G_{\epsilon\theta_f}(s)G_{\theta f}(s) + G_{\epsilon f}(s)] \\ &= R(s)\Gamma(s)R^*(s)[G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s)], \end{aligned} \tag{38}$$

and for $R(s) = R_h(s)$, we have

$$G_{rf}(s) = G^*_{rf}(s).$$

Under Assumptions of A4 to A6, from Lemma 1 we know that $R^*(s) = Q^{-1/2}$ and H^* given in (30)–(31) is an optimal solution of the problem (28) and, in this case, $G^*_{rw}(s)$ is a co-inner matrix. Therefore,

$$\begin{gathered} \|R^*(s)G^*_{\epsilon w}(s)\|_\infty = 1, \qquad \|R_h(s)G_{\epsilon w}(s)\|_\infty = 1, \\ \|R^*(s)(G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s))\|_\infty = \|R_h(s)(G_{\epsilon\theta_f}(s)G_{\theta f}(s) + G_{\epsilon f}(s))\|_\infty. \end{gathered}$$

On the other hand, for co-inner matrix $G^*_{rw}(s) = R^*(s)G^*_{\epsilon w}(s)$ and for all $R(s) \in \mathbf{RH}_\infty$, from (35) and (38) it is easy to get

$$\begin{aligned} \|G_{rw}(s)\|_\infty = \|R(s)G_{\epsilon w}(s)\|_\infty &= \|R(s)\Gamma(s)G^*_{rw}(s)\|_\infty = \|R(s)\Gamma(s)\|_\infty \\ \big\|R(s)(G_{\epsilon\theta_f}(s)G_{\theta f}(s) + G_{\epsilon f}(s))\big\|_\infty &= \big\|R(s)\Gamma(s)R^*(s)[G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s)]\big\|_\infty \\ &\leqslant \big\|R(s)\Gamma(s)\big\|_\infty \ \big\|R^*(s)[G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s)]\big\|_\infty, \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\|R_h(s)G_{\epsilon w}(s)\|_\infty}{\|R_h(s)(G_{\epsilon\theta_f}(s)G_{\theta f}(s) + G_{\epsilon f}(s))\|_\infty} &= \frac{\|R^*(s)G^*_{\epsilon w}(s)\|_\infty}{\|R^*(s)(G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s))\|_\infty} \\ &= \frac{1}{\|R^*(s)(G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s))\|_\infty}, \end{aligned} \tag{39}$$

$$\begin{aligned} \frac{\|R(s)G_{\epsilon w}(s)\|_\infty}{\|R(s)(G_{\epsilon\theta_f}(s)G_{\theta f}(s) + G_{\epsilon f}(s))\|_\infty} &\geqslant \frac{\|R(s)\Gamma(s)\|_\infty}{\|R(s)\Gamma(s)t\|_\infty\|R^*(s)(G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s))\|_\infty} \\ &= \frac{1}{\|R^*(s)(G^*_{\epsilon\theta_f}(s)G_{\theta f}(s) + G^*_{\epsilon f}(s))\|_\infty}, \qquad \forall\, R(s) \in \mathbf{RH}_\infty. \end{aligned} \tag{40}$$

It concludes from (39)–(40) that both $(H^*, R^*(s))$ and $(H, R_h(s))$ are the optimal solutions of problem (28).

Remark 4 The convergence of residual r is guaranteed by a suitable selection of filter gain matrix H, while the selection of stable post-filter $R_h(s)$ in (29) delivers an optimal residual vector. Results in Theorem 2 also show that, for all H ensuring the asymptotic stability of system (14) – (17), $(H, R_h(s))$ is one of the optimal solutions of the FDF problem.

4 Numerical Example

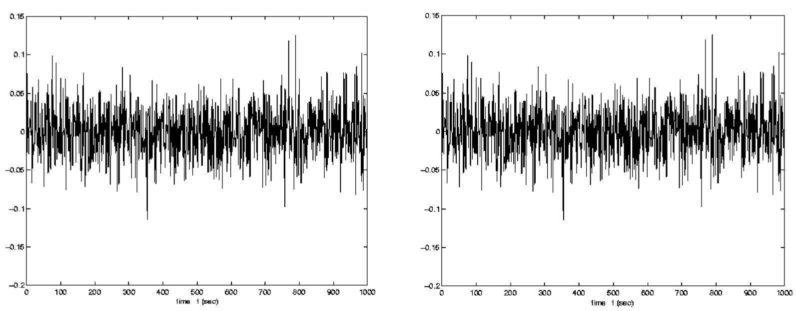
To illustrate the proposed FDF design method, a numerical example is given in this section. Consider a time-delay system of (6) – (8) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C = [\,1 \quad 1\,], \quad D = 0, \quad D_f = 0, \quad D_d = [\,0 \quad 0.1\,], \quad N = 1, \quad L = 0, \quad \tau = 1.$$

By using the proposed approach, we obtain one solution as follows:

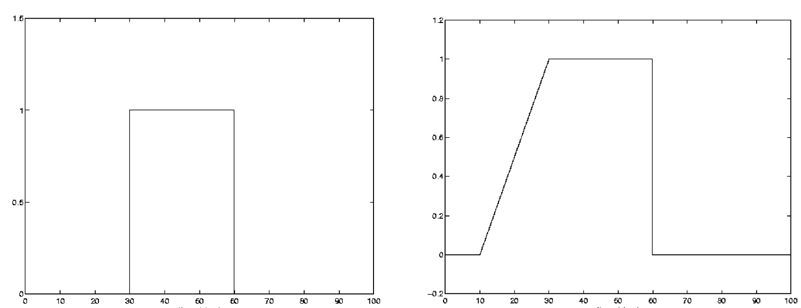
$$H^* = \begin{bmatrix} 1 \\ 1.6056 \end{bmatrix}, \quad H = \begin{bmatrix} 1.0026 \\ -0.9212 \end{bmatrix}, \quad Q = 100,$$
$$R_h(s) = Q^{-1/2}(I + C(sI - A + H^*C)^{-1}(H - H^*)).$$

Over evaluation time window $[0, 100]$ sec, suppose the unknown input is $d(t) = [\,d_1(t) \quad d_2(t)\,]^{\mathrm{T}}$, and $d_1(t)$, $d_2(t)$ are band-limited white noise as in Figure 4.1 (a) and (b). Two faulty cases are considered, where the fault signals are respectively given in Figure 4.2 (a) and (b). Figure 4.3 (a) and (b) show the two cases of residual signal whatever the control input $u(t)$.



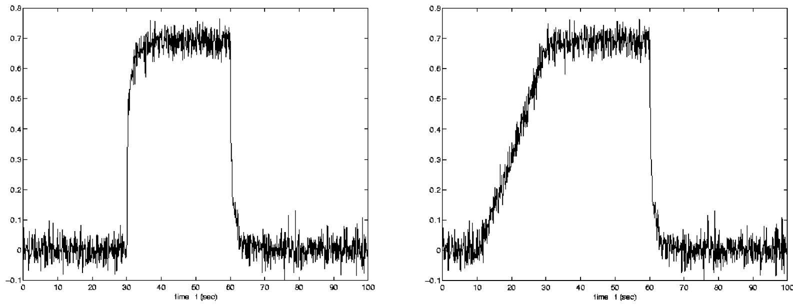
a) b)

Figure 4.1. a) Unknown input signal $d_1(t)$; b) Unknown input signal $d_2(t)$.



a) b)

Figure 4.2. a) Fault signal $f(t)$: case I; b) Fault signal $f(t)$: case II.



a) *b)*

Figure 4.3. a) Residual signal $r(t)$: case I; b) Residual signal $r(t)$: case II.

5 Conclusion

In this short paper, the FDF design problem for linear time-delay systems with unknown input is studied. The main contributions of this work are the introduction of a new FDF, the formulation of an optimization problem based on a performance index, and the extension of the FD optimization approach for LTI systems to the time-delay systems. The convergence of the residual generator is ensured by suitabe choice of the filter gain matrix, while the FDF performance can be guaranteed by the selection of a corresponding stable post-filter in terms of a Riccati equation. A simulation example is given to show the effectiveness of the proposed method.

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