A Fredholm Operator and Solution Sets to Evolution Systems

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Abstract: In this paper we deal with the Peano phenomenon for general initial boundary-value problems of quasilinear evolution systems with arbitrary even order space derivatives. The nonlinearity is a continuous or continuously Frechét differentiable function. Qualitative and quantitative structure of solution sets is studied by the theory of proper, Fredholm and Nemitskii operators. These results can be applied to the different technical and natural science models.

Keywords: Evolution systems; an initial boundary-value problem; a linear Fredholm operator; a proper and coercive operator; a bifurcation point; a surjectivity.

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0 Introduction

The Peano phenomenon of the existence of a solution continuum of the initial value problem for ordinary differential systems is well-known. This phenomenon has been studied by many authors in [3–5, 8, 17, 27]. The structure of solution sets for second order partial differential problems was observed in the authors papers [12, 13].

In this paper we shall study generic properties of quasilinear initial boundary-value problems for evolution systems of an even order with the continuous or continuous differentiable nonlinearities and the general boundary value conditions. In special Hölder spaces we use the Nikolskií decomposition theorem from [29, P. 233] for linear Fredholm operators, the global inversion theorem of [9, 6] and [7, PP. 42–43] and the Ambrosetti

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