On the Bounded Oscillation of Certain Fourth Order Functional Differential Equations

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Abstract: Some new criteria for the bounded oscillation of a fourth order functional differential equation are established. Comparison results with first/second order equations as well as necessary and sufficient conditions are developed.

Keywords: Oscillation; nonoscillation; half-linear; comparison; necessary conditions.

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1 Introduction

In this paper we are concerned with the oscillatory behavior of the fourth order functional differential equations of the type

\[
\frac{d}{dt} \left( \frac{1}{a_3(t)} \left( \frac{d}{dt} \left( \frac{1}{a_2(t)} \left( \frac{d}{dt} \left( \frac{1}{a_1(t)} \left( \frac{d}{dt} x(t) \right)^{\alpha_1} \right)^{\alpha_2} \right)^{\alpha_3} \right) \right) \right) + q(t)f(x[g(t)]) = 0,
\]

or, written more compactly as

\[
L_4 x(t) + q(t)f(x[g(t)]) = 0, \quad (1.1)
\]

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