Convergence of Solutions to a Class of Systems of Delay Differential Equations

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Abstract: This paper is concerned with a delay differential system which can be regarded as a mathematical model of compartmental system with pipes and time delays. It is shown that every solution of such a differential system tends to a constant vector as \( t \to \infty \). The obtained results improve and extend some existing ones in the literature.

Keywords: Convergence; delay differential equation; compartmental system.

Mathematics Subject Classification (2000): 34C12, 39A11.

1 Introduction

Recently, there has been much attention in the study of the asymptotic behavior of solutions for the following scalar delay differential equation

\[
\frac{dx(t)}{dt} = -F(x(t)) + F(x(t-r)), \quad (1.1)
\]

where \( r > 0 \) is a constant, and \( F: \mathbb{R} \to \mathbb{R} \) is continuous. System (1.1), which has been used to model a variety of phenomena such as some population growth, the spread of epidemics, the dynamics of capital stocks, etc. has been discussed extensively in the literature (see, for example, [2–5, 7, 8, 10, 12–14, 17]), in which various approaches including the first integral, invariance principle of Lyapunov–Razumikhin type, etc. have been applied to conclude that every solution of system (1.1) tends to a constant. However, most of the study deals with the problem of convergence of solutions of system (1.1) under the assumption that \( F \) is either strictly increasing or locally Lipschitz continuous and nondecreasing. To the best of our knowledge, there exist no results for the asymptotic behavior of system (1.1) with \( F \) only assumed to be nondecreasing. Meanwhile, we stress

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