



# Optimal Control of Nonlinear Systems with Controlled Transitions

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**Abstract:** This paper studies the optimum stochastic control problem with piecewise deterministic dynamics. The controls enter through the system dynamics as well as the transitions for the underlying Markov chain process, and are allowed to depend on both the continuous state and the current state of the Markov chain. The paper shows that the feedback optimal control relies on the viscosity solutions of a finite set of coupled Hamilton-Jacobi-Bellman (HJB) equations. Explicit control structures are provided by using the concept of subdifferential of a continuous function.

**Keywords:** *Markov process; Hamilton–Jacobi–Bellman equations; viscosity solutions;  $\beta$ -stochastically stabilizable.*

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