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Bi-Impulsive Control to Build a Satellite Constellation*

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Abstract: This paper considers the problem of optimal maneuvers to insert a satellite in a constellation. The main idea is to assume that a satellite constellation is given, with all the Keplerian elements of the satellite members having known values. Then, it is necessary to maneuver a new satellite from a parking orbit until its position in the constellation. The control available to perform this maneuver is the application of two impulses (instantaneous change in the velocity of the spacecraft) to the satellite and the objective is to perform this maneuver with minimum fuel consumption. The maneuver that changes the angular position of a satellite keeping all the other Keplerian elements constant is also considered.

Keywords: *Orbital maneuver; astrodynamics; impulsive control; satellite constelation.*

Mathematics Subject Classification (2000): 70F15, 70M20, 93C99.

1 Introduction

To solve the problem of optimal maneuvers to insert a satellite in a constellation, two basic types of maneuvers are simulated: the planar ones, where the initial and final orbits belong to the same plane, and the three-dimensional ones, where they belong to different planes. The initial conditions to solve this problem are the orbits of the spacecraft in the parking and in the final orbits, including the information required to specify its positions in the orbits (the true anomaly or any other equivalent quantity) and the minimum and

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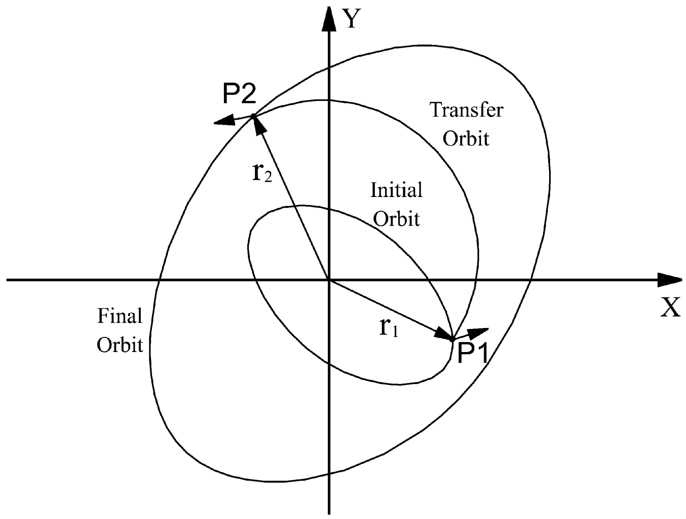

Figure 2.1. Scheme of the transfer.

maximum time of flight for the maneuver. The solution that is searched is the transfer orbit that satisfies all the initial conditions and that require the minimum total impulse (the addition of the magnitudes of the two impulses applied). To obtain the solution of this problem, the Lambert Problem associated with each particular transfer is formulated and solved. This approach is similar to the one used in Prado [1] to study the rendezvous maneuver. The Lambert Problem can be defined has the problem of finding a Keplerian orbit that passes through two given points and that requires a specified time of flight for a spacecraft to travel between those two given points. This problem is then solved using the algorithm developed by Gooding [2]. With the solutions given by this routine, it is possible to calculate the magnitude of both impulses that have to be applied. Several test cases (planar and non-planar) are solved to verify the algorithm developed. The total impulse required is then plotted as a function of the time specified for the transfer. Single and multi-revolution maneuvers are simulated. With those plots it is possible to choose transfer orbits that can satisfy both requirements of minimization of fuel expenditure and constraints on the time for the duration of the maneuver.

2 Statement of the Problem

In this section, the problem and the method of solution used are clearly defined. Figure 2.1 shows a sketch of the maneuver. A spacecraft is in an initial Keplerian orbit. The objective of the maneuver is to move this spacecraft from its initial orbit to a transfer orbit that intercepts the final orbit desired for the spacecraft, that is a second (final) Keplerian orbit. From the moment of the interception the spacecraft has to follow the final orbit.

There are many alternatives to solve this problem. For the present research, the model assumed for the control of the spacecraft is a bi-impulsive thrust, where the first impulse is applied at a time t_0, in a such way that the spacecraft will meet the target point at the time t_f; and the second impulse is applied at the time t_f, to put the spacecraft in its final orbit. Figure 2.1 shows an example of a direct single revolution transfer, where the spacecraft meets the target before making a complete revolution around the attracting body. Transfers where one or more revolutions are completed by the spacecraft before arriving at the destination are also possible and are considered in the present paper. The question that is considered here is how (magnitude and direction) and when to perform those two impulses to obtain the maneuver that has the minimum fuel consumption (minimum total ΔV). To answer this question, the following procedure was developed. The initial and final orbits are given, as an input of the procedure. The information on the position of the spacecraft in both initial and final orbits (the true anomaly or some equivalent quantity) is also required and given. Then, the following parameters are specified: the initial time t_0 of the maneuver, a value for the lower limit for the transfer time $(t_f - t_0)$, a value for the upper limit for the transfer time, a value for the increment of the transfer time, the number of revolutions of the spacecraft before meeting the desired point of the final orbit. With those parameters, an algorithm with the following steps is applied:

i) the lower limit for the transfer time is taken as the transfer time Δt;

ii) the Cartesian elements of the spacecraft at the initial time of the maneuver t_0 are calculated, using two-body celestial mechanics. This position is called $\underline{r}_i$ and this velocity $\underline{v}_i$;

iii) the Cartesian elements of the spacecraft at the final time of the maneuver $t_f = t_0 + \Delta t$ is calculated, using two-body celestial mechanics. This position is called $\underline{r}_f$ and this velocity $\underline{v}_f$;

iv) a value for the integer number of revolutions K of the spacecraft (number of complete orbits that the spacecraft makes during the maneuver) is assumed. Then, with $\underline{r}_i$, $\underline{v}_i$, $\underline{r}_f$, $\underline{v}_f$, Δt and K all the input data to solve the Lambert Problem is available. The original Lambert Problem is one of the most important and popular topics in celestial mechanics. Several important authors worked on it, trying to find better ways to solve the numerical difficulties involved [2−7]. It can be defined as: "A Keplerian orbit, about a given gravitational center of force is to be found connecting two given points (P_1 and P_2) in a given time Δt." The solution of the Lambert Problem gives the transfer orbit, the transfer time and the ΔV required. The Lambert Problem may have none, one or two solutions;

v) then, a step of time is added to the transfer time and the algorithm goes back to the step ii, with the new transfer time Δt.

This procedure is repeated until the upper limit for the transfer time is reached. It is also assumed several values for the number K of revolutions of the spacecraft.

3 Results

To study the optimal maneuvers several simulations were performed, using the algorithm described in the last section. The initial orbit is always a circular parking orbit with semi-major axis of 7700 km. This orbit is considered equatorial to study three-dimensional maneuvers and to have the same inclination of the final orbit when a planar maneuver is considered. For the final orbit two different cases are considered. The first one is the orbit of the GPS satellites, that are circular with semi-major axis of 20160 km and inclination of 55 degrees. The second one is the orbit of the satellites that belong to the Russian constellation Molniya, that are satellites that stay in frozen orbits with

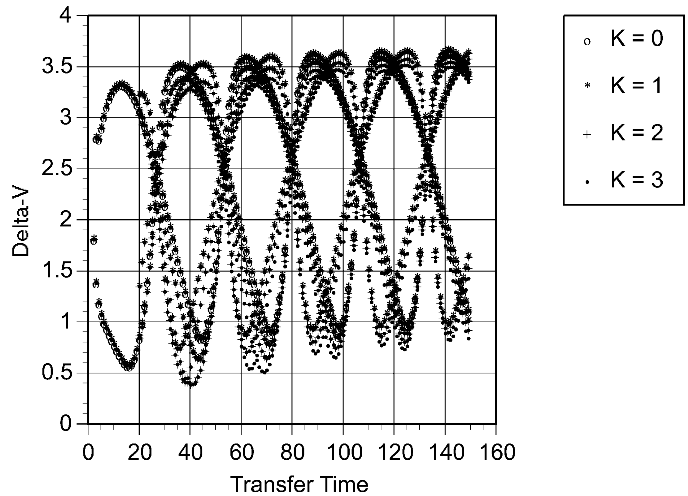

Figure 3.1. ΔV *vs.* transfer time for the first maneuver.

high eccentricity, to make them stay a long time in the apoapsis to be able to be useful satellites for communication. Their orbit have semi-major axis of 26560 km, eccentricity of 0.72 and inclination of 63.435 degrees. All the values of the variables are expressed in canonical units, except for the angles, that are expressed in degrees. The canonical units are dimensionless.

In the first case, it is simulated a very simple example, where the maneuver is planar and the two orbits are circular. Then, in the canonical system of units (using 7700 km as the unit for the measurements), the input data are:

$$a_i = 1.0; \quad \mathrm{e}_i = 0.0; \quad i_i = 55.0; \quad \Omega_i = 0.0; \quad \omega_i = 0.0; \quad T_i = 0.0;$$
$$a_f = 2.6182; \quad \mathrm{e}_f = 0.0; \quad i_f = 55.0; \quad \Omega_f = 0.0; \quad \omega_f = 0.0; \quad T_f = 0.0.$$

The nomenclature used here and in the rest of this paper is: a – semi-major axis, e – eccentricity, i – inclination of the orbit, Ω – argument of the ascending node, ω – argument of the periapsis, T – the time of the passage by the periapsis. The subscript "i" stands for the initial orbit of the spacecraft and the subscript "f" stands for the final orbit of the spacecraft.

The transfer time is constrained to the interval $0.1 \le \Delta t \le 150$ and the step of time is 1.0. It is assumed that $0 \le K \le 3$. Figure 3.1 shows the results obtained for this case: total ΔV *vs.* transfer time for each value of K.

For the second maneuver, a more generic case of a planar maneuver is used, where the final orbit is elliptic and its argument of the periapsis is also constrained. The two orbits are:

$$a_i = 1.0; \quad \mathrm{e}_i = 0.0; \quad i_i = 63.435; \quad \Omega_i = 0.0; \quad \omega_i = 0.0; \quad T_i = 0.0;$$
$$a_f = 3.4494; \quad \mathrm{e}_f = 0.72; \quad i_f = 63.435; \quad \Omega_f = 0.0; \quad \omega_f = 270.0; \quad T_f = 0.0.$$

This maneuver represents a transfer from our parking orbit to an orbit used by the Molniya constellation. The transfer time is also constrained to the interval $0.1 \le \Delta t \le$

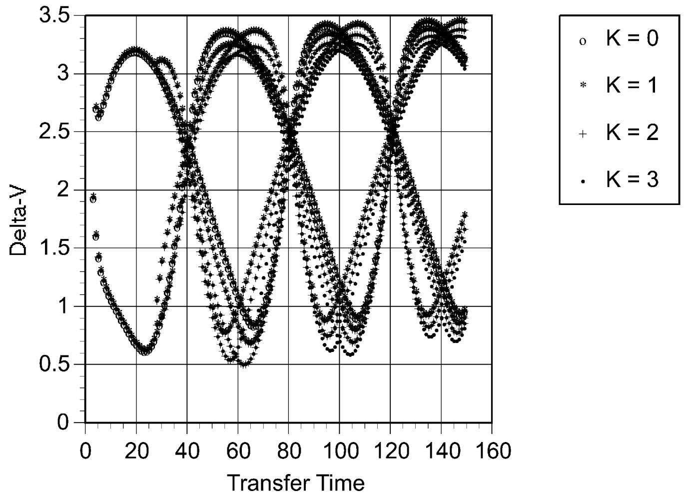

Figure 3.2. ΔV *vs.* transfer time for the second maneuver.

150 and the step of time is 1.0. It is assumed that $0 \le K \le 3$. Figure 3.2 shows the results obtained for this case: total ΔV *vs.* transfer time for each value of K.

For the third maneuver two non-coplanar orbits are used. This maneuver makes the satellite to start in an equatorial orbit and travel to the orbit of the GPS constellation. In this way, the two orbits are:

$$a_i = 1.0; \quad \mathrm{e}_i = 0.0; \quad i_i = 0.0; \quad \Omega_i = 0.0; \quad \omega_i = 0.0; \quad T_i = 0.0;$$
$$a_f = 2.6182; \quad \mathrm{e}_f = 0.0; \quad i_f = 55.0; \quad \Omega_f = 0.0; \quad \omega_f = 0.0; \quad T_f = 0.0.$$

The transfer time is also constrained to the interval $0.1 \le \Delta t \le 150$ and the step of time is 1.0. It is assumed that $0 \le K \le 3$. Figure 3.3 shows the results obtained for this case: total ΔV *vs.* transfer time for each value of K.

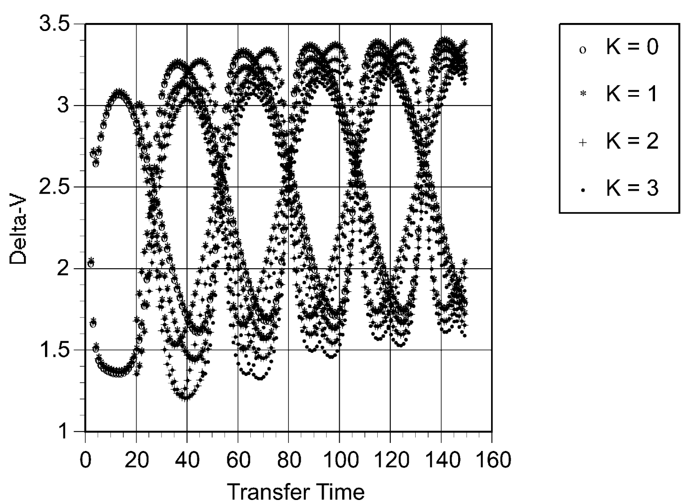

Figure 3.3. ΔV *vs.* transfer time for the third maneuver.

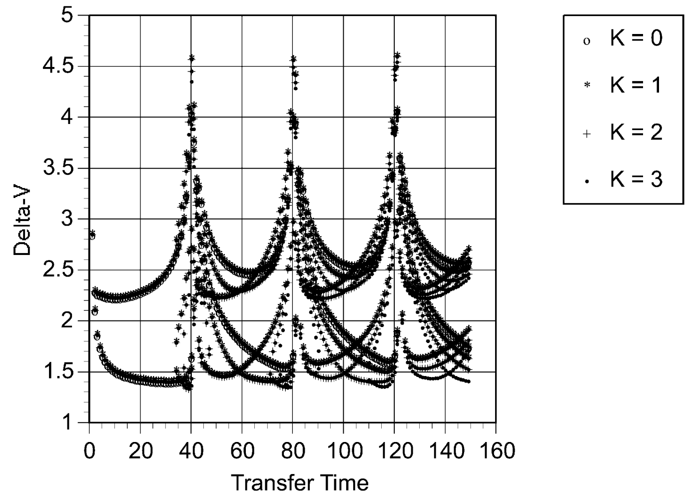

Figure 3.4. ΔV *vs.* transfer time for the fourth maneuver.

For the fourth maneuver, a three-dimensional transfer between the parking orbit and the Molniya orbit is used. The two orbits are:

$$a_i = 1.0; \quad \mathrm{e}_i = 0.0; \quad i_i = 0.0; \quad \Omega_i = 0.0; \quad \omega_i = 0.0; \quad T_i = 0.0;$$
$$a_f = 3.4494; \quad \mathrm{e}_f = 0.72; \quad i_f = 63.4350; \quad \Omega_f = 0.0; \quad \omega_f = 270.0^\circ; \quad T_f = 0.0.$$

The transfer time is also constrained to the interval $0.1 \leq \Delta t \leq 150$ and the step of time is 1.0. It is assumed that $0 \leq K \leq 3$. Figure 3.4 shows the results obtained for this case: total ΔV *vs.* transfer time for each value of K.

For the fifth maneuver, a relocation of 20 degrees for a satellite that belongs to a GPS constellation is used. It means that the satellite has to change its orbital position in 20 degrees, keeping all the other Keplerian elements constant. Then, the two orbits are:

$$a_i = 2.6182; \quad \mathrm{e}_i = 0.0; \quad i_i = 55.0; \quad \Omega_i = 0.0; \quad \omega_i = 0.0; \quad T_i = 0.0;$$
$$a_f = 2.6182; \quad \mathrm{e}_f = 0.0; \quad i_f = 55.0; \quad \Omega_f = 0.0; \quad \omega_f = 20.0; \quad T_f = 0.0.$$

The transfer time is also constrained to the interval $0.1 \leq \Delta t \leq 150$ and the step of time is 1.0. It is assumed that $0 \leq K \leq 3$. Figure 3.5 shows the results obtained for this case: total ΔV *vs.* transfer time for each value of K.

There are several characteristics that come from those simulations. First of all, it is possible to see that the solutions appear in pairs (two values of ΔV for a given transfer time). The values for the ΔV for a given family of transfer orbits oscillate with the increase of the transfer time. The first minimum of each family is usually the global minimum, what means that, after the best geometry for the maneuver is achieved, any extra time added for the maneuver does not generate a reduction in the fuel consumed. Another characteristic visible in those plots is that when K increases, the beginning of the curve shift to the right. This result is expected, because when revolutions are added to the maneuver, the minimum time required to get a solution has to increase.

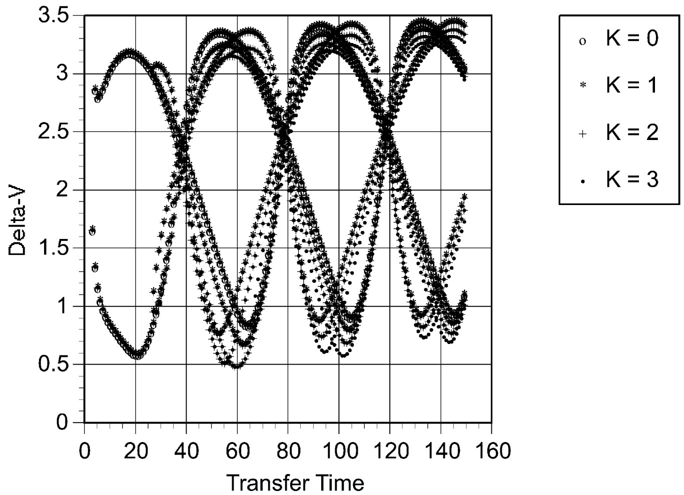

Figure 3.5. ΔV *vs.* transfer time for the fifth maneuver.

4 Conclusions

An algorithm to solve the optimal maneuver (in terms of minimum consumption of fuel) to transfer a satellite from a parking orbit to a fixed position in a final orbit with two impulses was derived and used for several maneuvers. This algorithm was explained in details and it includes single and multi-revolution transfers. Then, several cases using planar and non-planar maneuvers were solved. Several figures showed the fuel consumed *vs.* time for the transfer. Those results are useful to mission designers, because they can help to design transfer orbits to include a satellite in a given constellation.

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