A Duality Principle in the Theory of Dynamical Systems

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Abstract: The aim of this paper is to formulate and illustrate a duality principle for dynamical systems. There is a one-to-one correspondence between causal (nonanticipative) systems, and the anticipative ones. Several cases are dealt with, based on the nature of the functional equations describing the dynamics.

Keywords: Dynamical systems; causal; antisipative; duality principle.


1 Introduction

The dynamical systems we shall consider in this paper will be described by functional equations of various types.

The duality principle we are going to formulate and illustrate establishes a one-to-one correspondence between the class of causal systems, and the class of anticipative systems. The first class is also known as abstract Volterra systems, while the second class contains the so-called anti-Volterra systems.

The principle of duality states that: to any causal system, one can associate an anticipative system, and vice-versa.

Moreover, the mathematical treatment is basically the same for causal/anticipative couples which are in correspondence.

The idea of formulating this duality principle came from writing our joint paper [3], in which the mathematical apparatus used in dealing with anticipative systems (the corresponding describing equations are with advanced argument), has revealed a striking