Topological Sequence Entropy and Chaos of Star Maps*

J.S. Cánovas

Departamento de Matemática Aplicada y Estadística,
Universidad Politécnica de Cartagena,
Pasco de Alfonso XIII, 30203 Cartagena (Murcia), Spain

Received: October 02, 2003; Revised: May 25, 2004

Abstract: Let \( X_n = \{ z \in \mathbb{C} : z^n \in [0, 1] \} \), \( n \in \mathbb{N} \), and let \( f : X_n \to X_n \) be a continuous map such that \( f(0) = 0 \). In this paper we prove that \( f \) is chaotic in the sense of Li–Yorke iff there is a strictly increasing sequence of positive integers \( A \) such that the topological sequence entropy of \( f \) relative to \( A \) is positive.

Keywords: Star maps; Li–Yorke chaos; topological sequence entropy.

Mathematics Subject Classification (2000): 37B40, 37E25.

1 Introduction

Let \((X, d)\) be a compact metric space and let \( C(X) \) denote the set of continuous maps \( f : X \to X \). For any \( f \in C(X) \), the pair \((X, f)\) is called a discrete (semi)dynamical system. Given \( x \in X \), the sequence \((f^i(x))_{i=0}^{\infty}\) is the trajectory of \( x \) (also orbit of \( x \)). Recall that a point \( x \in X \) is periodic if \( f^i(x) = x \) for some \( i \in \mathbb{N} \). Denote by \( \text{Per}(f) \) the set of periodic points of \( f \). The map \( f \) is said to be chaotic in the sense of Li–Yorke (or simply chaotic) if there is an uncountable set \( S \subset X \setminus \text{Per}(f) \) such that for any \( x, y \in S \), \( x \neq y \), and any \( p \in \text{Per}(f) \) it holds

\[
\begin{align*}
\liminf_{n \to \infty} d(f^n(x), f^n(y)) &= 0, \\
\limsup_{n \to \infty} d(f^n(x), f^n(y)) &> 0, \\
\limsup_{n \to \infty} d(f^n(x), f^n(p)) &> 0.
\end{align*}
\]

*This paper has been partially supported by the grant PI–8/00807/FS/01 from Fundación Séneca (Comunidad Autónoma de Murcia).

© 2005 Informath Publishing Group/1562-8353 (print)/1813-7385 (online)/www.e-ndst.kiev.ua