Imaginary Axis Eigenvalues of a Delay System with Applications in Stability Analysis

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Abstract: We present a matrix method for determining the imaginary axis eigenvalues of a delay differential system. Both neutral and retarded delay systems are considered. We produce a second order polynomial matrix which is singular for all imaginary axis eigenvalues of the delay system leading to the recovery of eigenvectors associated with imaginary axis eigenvalues. The use of Kronecker products is emphasized in the proofs. Examples are given to illustrate the applicability of the new results in stability analysis.

Keywords: Delay systems; stability analysis; Kronecker products; imaginary axis eigenvalues.

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1 Introduction

Consider a linear delay differential equation of the form

\[ x'(t) + Ax'(t-h) = Bx(t) + Cx(t-h), \]

where \( A, B \) and \( C \in \mathbb{R}^{n \times n} \), \( \mathbb{R} \) being the set of real numbers. When \( A = 0 \), we get a retarded delay system, otherwise the system is neutral.

The purpose of this work is to present a matrix method for determining the imaginary axis eigenvalues of the above equation. Such eigenvalues occupy a special place in the theory of delay equations. They can be used to give the frequencies of oscillating solutions, and detect the onset of Hopf bifurcations. Although the idea of the approach is taken from the theory of quadratic functionals for delay equations [8], but the proofs will be quite direct, with strong emphasis on Kronecker products. In addition, we shall look at the matrix single delay case, and we propose a \( 2n^2 \times 2n^2 \) matrix having spectrum containing all imaginary axis eigenvalues of the delay system. Our technique works equally well for both neutral and retarded delay systems. Therefore, we produce a polynomial matrix which is second order in \( s \), and is singular for all values of \( s \) which are imaginary.