A Modified \textit{LQ}-Optimal Control Problem for Causal Functional Differential Equations

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\textbf{Abstract:} This paper continues some ideas from a preceding paper of the author, in which only point-wise initial data are considered. Here, the constraints on state variables and control involve functional initial data, leading to a modified control problem.

\textbf{Keywords:} \textit{LQ}-optimal control; modified problem; causal operators/equations.

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1 Introduction

The following problem will be considered in this paper.

Given the functional differential equations, with causal operators $A$ and $B$

$$\frac{dx}{dt} = (Ax)(t) + (Bu)(t), \quad t \in [t_0, T],$$

with $x: [t_0, T] \to \mathbb{R}^n$, $u: [t_0, T] \to \mathbb{R}^m$, $A: L^2([0, T], \mathbb{R}^n) \to L^2([t_0, T], \mathbb{R}^n)$ and $B: L^2([t_0, T], \mathbb{R}^m) \to L^2([t_0, T], \mathbb{R}^n)$, one attaches the initial value condition

$$x(t) = \varphi(t), \quad t \in [0, t_0), \quad x(t_0) = \theta,$$

and considers the minimization of the cost functional

$$C(x; \varphi, u) = \int_0^{t_0} \langle (P\varphi)(t), \varphi(t) \rangle \, dt + \int_{t_0}^T \left( \langle (Qx)(t), x(t) \rangle + \langle (Ru)(t), u(t) \rangle \right) \, dt,$$

under certain conditions to be specified below. Our main interest will be in proving the existence of an optimal triplet $(\bar{x}; \bar{\varphi}, \bar{u})$, such that

$$C(\bar{x}; \bar{\varphi}, \bar{u}) = \min C(x; \varphi, u),$$

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