Stability of the Stationary Solutions of the Differential Equations of Restricted Newtonian Problem with Incomplete Symmetry

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Received: July 10, 2002; Revised: December 23, 2002

Abstract: We investigate the Lyapunov stability of the stationary solutions of the differential equations of restricted six-body problem with the gravitational centre. The gravitational field is created by bodies \( P_0, P_1, P_2, P_3 \) and \( P_4 \) with masses \( m_0, m_1, m_2, m_3 \) and \( m_4 \), respectively. In this gravitational field the movement of a body \( P \) with zero mass \( (m = 0) \) is investigated. The bodies \( P_1, P_2, P_3 \) and \( P_4 \) form a rhombus, rotating uniformly around the centre of gravity \( P_0 \). In the article we have formulated necessary and sufficient conditions of Lyapunov stability and instability of equilibrium point of this model. All necessary analytical calculations are executed in the system of symbolical calculations (SSC) “Mathematica”.

Keywords: Hamiltonian systems; stability.


1 Introduction

It is known, that the restricted Newtonian many-body problem is very important for a wide class of applications, from theoretical physics to celestial mechanics and astrodynamics [1, 6]. It is well known [4, 5], that the differential equations of this problem are in general not integrable, therefore Poincaré considered the first problem should be the search for the exact particular solutions and the research of their stability [1]. The latter problem is the most difficult in the qualitative theory of the differential equations and can be solved within the framework of the Kolmogorov-Arnold-Mozер (KAM) theory [12, 13].

With occurrence of the systems of symbolic calculations, for example, Mathematica [10], possibilities of performance of symbolic calculations have essentially increased. Such calculations are necessary for correct application of the well known Arnold-Mozé theorem [13, 15]. Let’s consider the following restricted 6-body problem in Grebenikov-Elmabsout...