



Design of Stable Controllers for Takagi-Sugeno Systems with Concentric Characteristic Regions

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Abstract: The design of a fuzzy Takagi-Sugeno system with concentric regions and the use of discontinuous piecewise Lyapunov functions allows to relax stability conditions which can be expressed very easily as a set of Linear Matrix Inequalities. An adaptive algorithm allows to determine gradually the embedded sets and the corresponding local models.

Keywords: *Fuzzy control; linear matrix inequalities; Lyapunov functions; spherical coordinates.*

Mathematics Subject Classification (2000): 93D05, 93D15.

1 Introduction

The Takagi-Sugeno (TS) fuzzy model allows to represent a wide class of non-linear systems by a set of fuzzy rules for which the consequent parts are linear state models [10]. Using aggregation of rules, which induce a polyhedral partition of the state-space, a weighted sum of the linear state models is able to describe accurately the non-linear system. The so-called parallel distributed compensation (PDC) technique is an intuitive algorithm which consists of designing a fuzzy control rule according to each model rule of a TS fuzzy system. The premise part of the model rule and its corresponding control rule are identical. A sufficient condition to ensure the stability of a TS fuzzy plant model controlled with the corresponding PDC is to find a common quadratic Lyapunov function for all subsystems [11, 12]. The search of the Lyapunov function can be viewed as a convex optimization problem in terms of linear matrix inequalities (LMI) for which efficient solvers exist [1, 4]. The main drawback of this method is the conservativeness of the results which grows with the number of subsystems which must be taken into account.

The use of multiple (and in particular piecewise quadratic) Lyapunov functions is an alternative method to prove the stability of TS fuzzy controllers [6–9]. The quadratic

Lyapunov functions can be designed to ensure the continuity of the overall Lyapunov function at the boundaries of the cells which map the state space; the condition requiring the continuity of the Lyapunov function can be relaxed if the energy decreases when the trajectory moves from a cell into another [6, 7]. Another related method is to drive gradually the state space trajectory through a series of embedded sets, where an attractor of a set is included into the next set of the series [2, 3]. This algorithm reproduces the intuitive characteristic of fuzzy control for which the trajectory is smoothly driven from one region into another closer region (in terms of distance) of the origin, until it reaches the equilibrium. The attractors may be computed using comparison systems methods and vector norms, which leads, however, to conservative results [2, 3].

A TS fuzzy structure which uses generalized spherical coordinates in the premise part is proposed in this paper, for which some characteristic regions can be put more easily under the form of quadratic inequalities than the general polyhedral scheme. The design of discontinuous Lyapunov functions together with appropriate embedded sets will allow to derive relaxed stability conditions for a TS fuzzy system controlled by PDC techniques.

2 Design of Takagi-Sugeno Systems with Ellipsoidal Domains

A. Takagi-Sugeno systems with generalized spherical coordinates

1) The basic model

Let us consider the fuzzy dynamic model of the Takagi and Sugeno system described by the following IF-THEN rules R_i , $i = 1, \dots, r$:

$$\text{IF } z_1 \text{ is } M_{i,1} \text{ AND } \dots z_n \text{ is } M_{i,n} \text{ THEN } \dot{x} = A_i x + B_i u,$$

where $x = (x_1, \dots, x_n)^T$ is the state vector, $u \in \mathbb{R}$ is the control vector, $z = (z_1, \dots, z_n)^T$ are the premise variables and $M_{i,j}(\cdot)$ are the membership functions of the fuzzy sets $M_{i,j}$. We suppose that $\text{card}(z) = \text{card}(x) = n$. The state equation can be defined as follows [10]:

$$\dot{x} = \sum_{i=1}^r \lambda_i (A_i x + B_i u),$$

where $\lambda_i = \frac{\omega_i(z)}{\sum_{j=1}^r \omega_j(z)}$ with $\omega_i(z) = \prod_{j=1}^n M_{i,j}(z_j)$.

Let us introduce a basis of n -dimensional generalized coordinates which consists of one radius and $n - 1$ angles,

$$z = (\rho, \theta_1, \dots, \theta_{n-1})^T \in \mathbb{R}^n,$$

where $\rho = \sqrt{\sum_{i=1}^n \left(\frac{x_i}{\alpha_i}\right)^2}$, $\alpha_i \in \mathbb{R}$. In the case where $\alpha_i^2 = 1$ for all $i = 1, \dots, n$, $z = (\rho, \theta_1, \dots, \theta_{n-1})^T$ will correspond to the generalized spherical coordinates basis; if moreover, the dimension is 2, $z = (\rho, \theta)$ will reduce to polar coordinates, where ρ and θ are respectively the radial and the angular coordinate.

The Takagi-Sugeno system using $z = (\rho, \theta_1, \dots, \theta_{n-1})^T$ as variable for premises is described by the set of rules:

$$R_i: \text{ IF } \rho \text{ is } \rho_i \text{ AND } \theta_i \text{ is } \Theta_{i,1} \text{ AND } \dots \theta_{n-1} \text{ is } \Theta_{i,n-1} \text{ THEN } \dot{x} = A_i x + B_i u. \quad (1)$$

2) *The overlapping condition*

In most fuzzy control applications, the input membership functions $M_{i,j}(\cdot)$ and $M_{i+1,j}$ of every variable z_j overlap pairwise in an interval $[\hat{z}_{k_j,j}, \hat{z}_{k_j+1,j}]$, where the other membership functions are zero. Consider the region $\Delta_k = \bigcup_{j=1}^n [\hat{z}_{k_j,j}, \hat{z}_{k_j+1,j}]$, $k = 1, \dots, K$, where $1 \leq k_j \leq n$, n_j is the number of predicates for the variable z_j , K is the number of possible regions. Only a limited number of rules are activated in Δ_k since, for every premise z_j , only the membership functions $M_{k_j,j}$ and $M_{k_j+1,j}$ are nonzero, the rules which involve other fuzzy sets fire.

In the case where the TS system is described by equation (1), the regions Δ_k , $k = 1, \dots, K$, can be represented by the following inequalities:

$$\rho_k \leq \rho \leq \rho_{k+1}, \quad \text{or} \quad \rho_k \leq x^T P x \leq \rho_{k+1}, \quad (2)$$

where $P = \text{diag} \left(\frac{1}{\alpha_i^2} \right)_{i=1, \dots, n}$, and

$$0 \leq \Psi_k \theta_k, \quad (3)$$

where Ψ_k is a constant vector.

The set of regions where $\rho_m \leq \rho \leq \rho_{m+1}$ will be called Ω_m , $m = 1, \dots, M$. A region which encloses the origin belongs to the set Ω_1 , for which $\rho_m = 0$. From the preceding hypotheses, rules which are active in Ω_m are also active either in Ω_{m-1} or in Ω_{m+1} , and are not active elsewhere. Note that rules which are active in Ω_1 are also active in Ω_2 .

In the rest of the paper, these conditions will be referred to as the “overlapping conditions”.

B. Design of a control structure

Two kind of controllers will be examined:

- the simple linear state feedback control with regionwise valued parameters:

$$u = F_k x \quad \text{if} \quad x \in \text{region } \Delta_k; \quad (4)$$

- the Parallel Distributed Compensation controller, the most popular and natural control for TS systems, which consists of designing each control rule from the corresponding rule of a TS system, with which it shares its premise parts. In a PDC, a rule R_i of the TS system to be controlled [11, 12] corresponds to a dual regulator rule \hat{R}_i :

$$\hat{R}_i: \text{ IF } \rho \text{ is } \rho_i \text{ AND } \theta_i \text{ is } \Theta_{i,1} \text{ AND } \dots \theta_{n-1} \text{ is } \Theta_{i,n-1} \text{ THEN } u = F_i x. \quad (5)$$

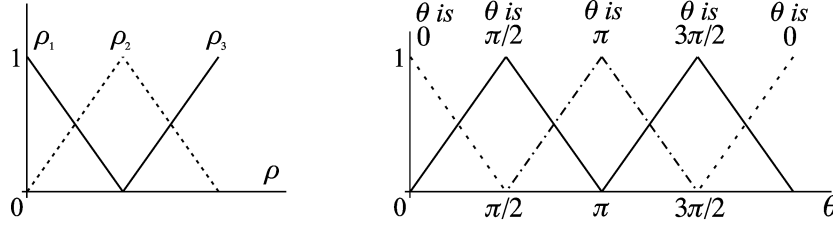


Figure 2.1. Membership functions for generalized coordinates.

C. A 2-D example

Consider a system described by the following set of rules:

$$\text{IF } \rho \text{ is } \rho_i \text{ AND } \theta \text{ is } \Theta_i \text{ THEN } \dot{x} = A_i x + B_i u,$$

where $x = (x_1, x_2)^T$ is the state vector, u is the control vector, $X_1 = x_1/a$, $X_2 = x_2/b$, $X = (X_1, X_2)^T$, $a, b \in \mathbb{R}$. ρ and θ are polar coordinates in the plane $X = (X_1, X_2)^T$, $\rho = \|X\|_2$, $\theta = \arg(X)$.

The following triangular membership functions are given on Figure 2.1.

Fuzzy sets for θ are “ θ is 0”, “ θ is $\pi/2$ ”, “ θ is π ”, “ θ is $3\pi/2$ ”. Figure 2.2 shows the regions where rules are active. For example, in region Δ_1 of Figure 2.2, only rules

$$\text{“}\rho \text{ is } \rho_1 \text{ AND } \theta \text{ is } 0\text{”}, \text{ “}\rho \text{ is } \rho_1 \text{ AND } \theta \text{ is } \pi/2\text{”},$$

$$\text{“}\rho \text{ is } \rho_2 \text{ AND } \theta \text{ is } 0\text{”}, \text{ “}\rho \text{ is } \rho_2 \text{ AND } \theta \text{ is } \pi/2\text{”}$$

are activated, the remaining rules fire. Region Δ_1 can be described by the following constraints: $x_1 \geq 0$, $x_2 \geq 0$ and $x^T P x \leq c$, where $P = \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix}$ and $c = 1$.

The regions Δ_k are a sector of a cone (when enclosing the origin) or of an annulus, for which only 4 rules are active, the other ones fire. The main differences with respect to classical TS-fuzzy controllers are now clear: the state space partition is not polyhedral, but the local models are distributed following the distance and orientation with respect to the origin, in the state-space. The notion of distance (from the equilibrium) respects the intuitive nature of fuzzy predicates such as “FAR” or “NEAR”, and some of the constraints on regions where parameters are constant can be expressed as quadratic inequalities, which shows quite useful in Lyapunov stability techniques. In the general case ($n > 2$), the characteristic regions of the TS system can be chosen as ellipsoidal.

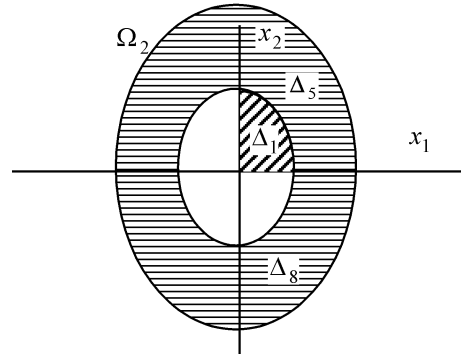


Figure 2.2. Example of regions where a limited number of rules are active.

3 LMI-based Stability Analysis of the Fuzzy Controller

A. Stability theorems based on multiple Lyapunov functions

Suppose that the original system is described by equation (1) – the premise variables are not necessarily spherical coordinates – and verifies the overlapping condition. It will thus be possible to find N disjoint regions Υ_m for which a scalar energy function V_m can be defined. Let the switching boundary Λ_{ml} for which the trajectory $x(t)$ passes from some neighboring regions Υ_m to Υ_l , i.e.

$$\Lambda_{ml} = \{x \setminus x(t^-) \in \Upsilon_m, x(t) \in \Upsilon_l\}.$$

Theorem 3.1 [6] *Suppose that there exist class K functions α and β such that, for all $m, l = 1, \dots, M$,*

- (i) $\alpha(\|x\|) \leq V_m(x) \leq \beta(\|x\|)$ for all $x \in \Upsilon_m$,
- (ii) $\dot{V}_m(x) \leq 0$ for all $x \in \Upsilon_m$,
- (iii) $V_l(x) \leq V_m(x)$ for all $x \in \Lambda_{ml}$,

then the origin is (uniformly) stable in the sense of Lyapunov.

Theorem 3.1 allows to relax the continuity condition for the Lyapunov function, and a companion theorem exists for exponential stability [7]. A corollary has been given in [6] for quadratic Lyapunov functions. We propose a simplified criterion using the special structure given in (1), which will allow the control problems to be expressed as a simple set of LMIs.

Theorem 3.2 *Consider a regionwise valued fuzzy system defined in (1). If there exists a series of positive definite matrices Z_m , $m = 1, \dots, M$, such that:*

$$\begin{aligned} x^T Z_m x &\leq 0 \quad \text{for all } x \in \Omega_m, \\ Z_{m-1} - Z_m &\leq 0 \quad \text{for all } m, \end{aligned}$$

then the origin is (uniformly) stable in the sense of Lyapunov.

Proof of the Theorem 3.1 — ??? In the domain Ω_m , for which $\rho_m \leq x^T P x \leq \rho_{m+1}$, condition (i) can always be fulfilled, since $V_m = x^T Z_m x$. Condition $x^T Z_m x \leq x^T Z_{m-1} x$ must be satisfied at the boundary $\rho_m = x^T P x$, for which the radius is fixed and the angles θ_i are any. If condition (iii) is satisfied at the boundary $\rho_m = x^T P x$, it should then also be satisfied for any $z = (\rho, \theta_1, \dots, \theta_n)^T$ and thus for any of the state space.

Remark 3.1 It possible to choose independent Lyapunov functions for every ring Ω_m , provided that these Lyapunov functions are always decreasing. The search for Lyapunov matrices should thus start from $m = M$ down to $m = 1$. If $Z = Z_m, \forall m = 1, \dots, M$, then the problem is reduced to the more general case of finding a common Lyapunov function.

B. LMI-based control of TS-systems with concentric regions

As in [6–9], control of TS-systems under a combination of piecewise-linear controls can be seen as a convex optimization problem with constraints that can be solved using powerful numerical tools, using Linear Matrix Inequalities [1, 4].

1) Application to piecewise linear control

Theorem 3.3 Consider the TS-system defined in (1) with the piecewise linear controller defined in (4). Define Ω_m , $m = 1, \dots, M$, as the set of regions Δ_k for which $\rho_m \leq x^T P x \leq \rho_{m+1}$. If there exist a series of positive-definite matrices Z_m , $m = 1, \dots, M$ and a positive constant number τ_m such that, for every region $\Delta_k \subset \Omega_m$ and for every rule R_i which is active in Δ_k :

$$A_i^T Z_m + Z_m A_i + Z_m B_i F_k + F_k^T B_i^T Z_m + \tau_m P < 0, \quad (6)$$

$$Z_{m-1} - Z_m \leq 0. \quad (7)$$

The origin is (uniformly) stable in the sense of Lyapunov.

Proof Consider the Lyapunov function $V = x^T Z_m x$.

In region Δ_k

$$\dot{V}_m = \dot{x}^T Z_m x + x^T Z_m \dot{x} = \sum_{i=1}^{\delta_k} \lambda_i (x^T (A_i^T Z_m + F_k^T B_i^T Z_m) x + x^T (Z_m A_i + Z_m B_i F_k) x).$$

$$\dot{V}_m < 0 \text{ if } \forall i, k,$$

$$x^T (A_i^T P + F_k^T B_i^T P m) x + x^T (P A_i + P B_i F_k) x < 0.$$

The LMI can be relaxed by considering the regionwise constraints, which can be written, according to the concentric nature of regions:

$$\Psi_k x < 0, \quad \rho_m - x^T P x < 0, \quad x^T P x - \rho_{m+1} < 0,$$

and, by the S -procedure [1], a sufficient condition for $\dot{V}_m < 0$ is the existence of positive constants $\tau_{1,m}$, $\tau_{2,m}$, $\tau_{3,k}$ such that:

$$\begin{aligned} & x^T (A_i^T Z_m + F_k^T B_i^T Z_m) x + x^T (Z_m A_i + Z_m B_i F_k) x \\ & - \tau_{3,k} \Psi_k x - \tau_{1,m} (\rho_m - x^T P x) - \tau_{2,m} (x^T P x - \rho_{m+1}) < 0. \end{aligned}$$

If condition $\Psi_k x < 0$ is not taken into account,

$$x^T (A_i^T Z_m + F_k^T B_i^T Z_m + \tau_{1,m} P - \tau_{2,m} P + Z_m A_i + Z_m B_i F_k) x - \tau_{1,m} \rho_m + \tau_{2,m} \rho_{m+1} < 0$$

which is satisfied if $A_i^T Z_m + F_k^T B_i^T Z_m + (\tau_{1,m} - \tau_{2,m}) P + Z_m A_i + Z_m B_i F_k < 0$ and

$$-\tau_{1,m} \rho_m + \tau_{2,m} \rho_{m+1} \leq 0.$$

Taking $\tau_{2,m} = \tau_{1,m} \frac{\rho_m}{\rho_{m+1}}$ and $\tau_m = \tau_{1,m} - \tau_{2,m} = \tau_{1,m} \left(1 - \frac{\rho_m}{\rho_{m+1}}\right)$ gives condition (6).

If the conditions in (6) are fulfilled, then $\dot{V}_m < 0$ in Ω_m . From equations (6), (7), applying Theorem 3.2, the origin is uniformly stable.

2) PDC control of TS-systems using concentric Lyapunov surfaces

Theorem 3.4 Consider the TS-system defined in (1) with the Parallel Distributed Compensation controller defined in (5). If there exist a series of positive-definite matrices Z_m , $m = 1, \dots, M$, and a positive constant number τ_m such that, for every region $\Delta_k \subset \Omega_m$, and for every rules R_i, R_j which are active in Δ_k ,

$$\begin{aligned} G_{ii}^T Z_m + Z_m G_{ii} + \tau_m P < 0, \quad \forall i = 1, \dots, \delta_k, \\ \left(\frac{G_{ij} + G_{ji}}{2} \right)^2 Z_m + Z_m \left(\frac{G_{ij} + G_{ji}}{2} \right) + \tau_m P \leq 0, \quad \forall i < j, \end{aligned} \quad (8)$$

$$Z_{m-1} - Z_m \leq 0, \quad (9)$$

where $G_{ij} = A_i + B_i F_j$, and δ_k is the number of active rules in Δ_k , the origin is (uniformly) stable in the sense of Lyapunov.

Proof The proof follows the same sketch as in [12] and in Theorem 3.3.

3) TS-system with adaptive rule selection

The algorithm in Theorem 3.1 allows to check the stability of a TS-controller with PDC with relaxed stability conditions, for which the membership functions and validity domains are defined a priori by the user. In general, there is little guideline to help to determine these crucial parameters of fuzzy controllers. As an alternative, it is proposed to build gradually the domains Ω_m (and thus the corresponding rules and local models) from the Lyapunov function found in the previous subset Ω_{m+1} .

Rules are designed in a first time only in the outer set Ω_M . The upper boundary of the new set Ω_{M-1} will be chosen as the smallest Lyapunov surface (from the common Lyapunov function which matches the stability conditions in Ω_M) which contains the lower boundary of Ω_M . The same method will apply for next subsets.

Consider a TS-system defined in (1), using $z = (\rho^M, \theta_1, \dots, \theta_{n-1})^T$ as variable for premises, for which

$$\rho_M = x^T Z_{M+1} x.$$

The set of rules, which are only active in $\Omega_M = \{\rho_M^- \leq x^T Z_{M+1} x \leq \rho_M^+\}$ is:

$$R_i: \text{IF } \rho^M \text{ is } \rho_i \text{ AND } \theta_1 \text{ is } \Theta_{i,1} \text{ AND } \dots \theta_{n-1} \text{ is } \Theta_{i,n-1} \text{ THEN } \dot{x} = A_i x + B_i u,$$

where the membership functions $\rho_{M-1}(\cdot)$ and $\rho_M(\cdot)$ fully overlap in the domain $[\rho_M^-, \rho_M^+]$. The membership functions of the other premise variables verify the overlapping condition.

Let us introduce the piecewise linear controller: $u = F_k x$ if $x \in$ region Δ_k , with $\Delta_k \subset \Omega_M$.

Theorem 3.5 If there exists a series of positive definite matrices Z_m , $m = 1, \dots, M$, and positive numbers ρ_m^- and ρ_m^+ such that:

(i) Define

$$\begin{aligned} \Omega_m &= \{\rho_m^- \leq x^T Z_{m+1} x \leq \rho_m^+\}, & \Omega_m^+ &= \{x^T Z_{m+1} x = \rho_m^+\}, \\ \Omega_m^- &= \{x^T Z_{m+1} x = \rho_m^-\}, \end{aligned}$$

where Ω_{m-1}^+ is the biggest domain that includes Ω_m^- , and Ω_{m-1}^+ is enclosed into Ω_m^+ ;

(ii) Rules in Ω_m take the form:

$$R_i: \text{ IF } \rho^m \text{ is } \rho_i \text{ AND } \theta_1 \text{ is } \Theta_{i,1} \text{ AND } \dots \theta_{n-1} \text{ is } \Theta_{i,n-1} \text{ THEN } \dot{x} = A_i x + B_i u,$$

where $\rho^m = x^T Z_{m+1} x$, and the corresponding local controller is designed in the appropriate regions Δ_k ;

(iii) $A_i^T Z_m + Z_m A_i + Z_m B_i F_k + F_k^T B_i^T Z_m + \tau_m Z_m < 0$, $\forall m = 1, \dots, M$,

then the overall system is asymptotically stable.

Proof Taking $V = x^T Z_{m+1} x$ in Ω_m , condition (iii) ensures that if a trajectory crosses a surface $x^T Z_m x = c$, where c is some constant, then the trajectory stays in the domain $x^T Z_m x \leq c$ [5]. Hence, if condition (iii) is verified, all trajectories that start in Ω_m will reach Ω_{m-1} . The trajectory converges thus towards the equilibrium (see Figure 3.3).

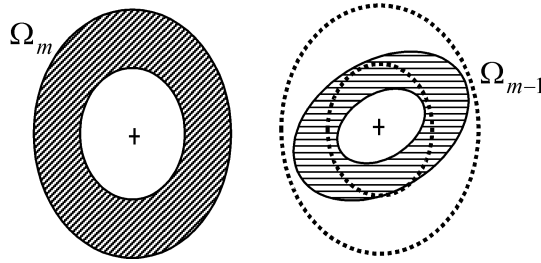


Figure 3.1. Gradual determination of domains.

Remark 3.2 The main advantage of the method is to allow a wide flexibility in the construction of regions. The original system and controllers are not “frozen”, since rules and local models are adapted from the stability conditions found for the former set. The counterpart is that, in general, a new set of local models should be determined (and identified) for every domain Ω_m .

4 Example

Consider the 2-D system described in Section 2(C) with $P = I$ (see Figure 2.1), to be controlled by the piecewise linear controller in (4). Suppose that, for $\theta \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$, the consequent part is described by:

$$\dot{x} = A_{\rho,\theta} x + Bu,$$

where

$$B = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad A_{\rho_2,\theta} = \begin{pmatrix} -1 & \cos(\theta) - 1 \\ -2 + \sin(\theta) & -1 \end{pmatrix},$$

$$A_{\rho_3,\theta} = \begin{pmatrix} -1 & 2\sin(\theta) - 1 \\ -2 + 2\cos(\theta) & -1 \end{pmatrix}, \quad A_{\rho_1,\theta} = \begin{pmatrix} -2 & -2 \\ 3 & 0 \end{pmatrix}.$$

The regionwise valued controllers for every region Δ_k are given in Table 4.1.

Region	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8
F_k^T	(-1 1)	(-0.5 0.5)	(-2 1.5)	[-0.5 1.5]	(-1 0)	(-2 2)	(-2 3)	[-3 3]

Table 4.1. Regionwise valued controllers.

It is impossible to find a common Lyapunov matrix for all controlled systems (actually 28 equations, which would be the same in a corresponding rectangular partition). However, the application of Theorem 3.3 gives

$$Z_1 = \begin{pmatrix} 11.79 & 1.64 \\ 1.64 & 5.11 \end{pmatrix}, \quad Z_2 = \begin{pmatrix} 16.05 & -4.89 \\ -4.89 & 18.34 \end{pmatrix}$$

and the overall controller is now stable.

5 Conclusion

Introducing generalized spherical coordinates in the premise part of TS fuzzy systems, it has been shown that an appropriate choice of membership functions allows to separate the state space into a number of concentric regions in which only a limited number of rules are active. PDC techniques can be used to control the TS fuzzy system. A piecewise quadratic Lyapunov function has been designed for every concentric region; the stability of the controlled system is ensured if the piecewise Lyapunov function is decreasing in the corresponding region and if it is smaller than that of the previous domain. Since the regions can be viewed as constraints which can be described with the help of quadratic inequalities, it is easy to include these into a set of inequalities which derives from the Lyapunov stability analysis, which relax LMI conditions. An adaptive algorithm has then be proposed which allows to choose the embedded sets and the corresponding local models and rules.

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