Stability of an Autonomous System with Quadratic Right-Hand Side in the Critical Case

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Abstract: In this paper an autonomous system of differential equations with quadratic right-hand side is considered. In the case when the matrix of linear approximation has just one zero eigenvalue, the stability of trivial solution is investigated. System is written in the vectors-matrices form and under some additional conditions a Liapunov function of the quadratic form is constructed. A guaranteed zone of stability of trivial solution is given as well.

Keywords: Zero eigenvalue; Lyapunov stability.

Mathematics Subject Classification (2000): 34A34, 34D20, 93D30.

1 Introduction

Many problems of biological sciences, medicine sciences etc. lead to investigation of systems that are described by means of ordinary differential equations with quadratic right-hand sides (e.g. [3, 5]). Zero solution of the system with quadratic right-hand side in the case of presence of zero eigenvalue of matrix of corresponding linear part can be, in general, unstable. This effect occurs already in the scalar case. For instance, the trivial solution of simple scalar equation \( \dot{x} = -x^2 \) is unstable, since the solution of the initial

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