Mathematical Analysis in a Model of Obligate Mutualism with Food Chain Populations

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Abstract: This paper is concerned with a three-species food chain whose populations interact with a mutualist. The mutualism is obligate for one of the predators, and is modeled by a system of autonomous ordinary differential equations. Persistence and extinction criteria are developed in the cases of trivial, periodic and almost periodic dynamics.

Keywords: Food chain; obligate mutualism; persistence; extinction; stability; periodic solutions; almost periodic solutions.

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1 Introduction

The main thrust of this paper is to model obligate mutualism with the middle and top predators of a three-species food chain. The cases of facultative mutualism with the prey and middle predator populations have been considered in [24].

Previously, models of mutualism with predator-prey systems have been considered in [2, 12, 16, 24, 27, 34]. Models of obligate mutualism have been discussed in [7, 12, 13, 14]. For general discussions of mutualism the reader is referred to [1, 7, 11, 32].

Most models of mutualism are two dimensional. There has been a fair amount of work recently on three dimensional models, where the mutualism occurs between prey

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