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A Quintic Nonlinear Differential System with a Non-Algebraic Limit Cycle Around a Non-Elementary Singular Point

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Abstract: In this paper, we consider a class of quintic planar polynomial differential systems with a non-elementary singular point. We establish sufficient conditions for the existence of a hyperbolic non-algebraic limit cycle surrounding this singularity. Moreover, the limit cycle is explicitly expressed in polar coordinates. To demonstrate the applicability of our results, a concrete example is provided along with its corresponding phase portrait.

Keywords: *polynomial differential system; non-algebraic limit cycle; first integral; Riccati differential equation.*

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1 Introduction

One of the main problems in the qualitative theory of differential equations is the study of limit cycles, that is, isolated periodic solutions among all periodic solutions of planar differential systems of the form

$$\begin{cases} \dot{x} = \frac{dx}{dt} = P(x, y), \\ \dot{y} = \frac{dy}{dt} = Q(x, y), \end{cases} \tag{1}$$

where P and Q are real polynomials in the variables x and y. The degree n of the polynomial system of differential equations is the maximum of the degrees of the polynomials

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