Nonlinear Dynamics and Systems Theory, **26** (2) (2026) 148–161

Study of a Delay Viscoelastic Problem Involving a Generalized Fractional Proportional Derivative

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Received: February 5, 2025; Revised: March 29, 2026

Abstract: In this paper, we study a nonlinear fractional viscoelastic problem with multiple delays. We consider the fractional model of Voigt in terms of generalized fractional proportional derivative. Using the Banach contraction principle, we prove the existence and uniqueness of the solution under some assumptions, then we confirm the dependence of the latter upon the initial data. The Hyers-Ulam-Rassias stability is established and the results are illustrated by a numerical example.

Keywords: *fractional calculus; Ulam stability; Banach contraction; fractional proportional derivative; rheological models.*

Mathematics Subject Classification (2020): 74D10; 34A12; 26A33; 47H10.

Introduction

In recent years, fractional calculus has proven to be a powerful and versatile tool for modeling many physical and mechanical phenomena, despite the existence of many definitions and formulas for fractional derivatives, see [1–5]. The fractional derivative is an integral operator that has a memory term in the kernel, which is its advantage in modeling rheological phenomena. It can accurately describe the behavior of viscoelastic materials and well define the stress-strain relationships.

When we apply stress to an elastic material, the stress causes deformation, and the material will instantly return to its initial shape when this stress is removed. We say that the elastic deformation here is instantaneous and recoverable and the work is stored in the form of elastic energy.

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