



A Reliable Service Provider System-Mathematical Model and Dynamical Behavior

P. R. S. Rao¹, K. V. Ratnam^{2*} and G. Shirisha³

¹ Govt. Polytechnic, Kalidindi - 521344, Krishna District, A.P., India.

² Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad-500078, India.

³ Department of Mathematics, Stanley College of Engineering and Technology for Women, Abids, Hyderabad-500001, Telangana, India.

Received: November 3, 2021; Revised: January 13, 2026

Abstract: In this paper, a two-layered service provider system is considered in which one layer directly interacts with a client system and each member of the first layer is supported by a sub-group of members in the second layer, in completing a task put forth by the client. The mathematical model presented is a modification of an existing model studied by the authors. Since different groups are working on the task, a consensus is to be achieved by the system. In terms of mathematics, sufficient conditions are established on the system parameters so that the solutions remain asymptotically close to each other, and upon restricting the inputs, the solutions become bounded. This means that all the solutions approach a bounded solution of the system, implying that a consensus is formed. Once it is established that all members of the system are working together, it is necessary to meet the requirements of the client. Mathematically, it is achieved in terms of asymptotic stability of the desired solution through the Lyapunov functional method. For this, a set of sufficient conditions is obtained for the parameters and the functional relations of the system. Numerical examples are provided to verify the results and are supported by simulations. On the whole, our study provides a reliable service provider system that understands the requirements of the client, makes the assessment of its own capacities, gets back to the client for proper inputs and finally, delivers the output desired by the client.

Keywords: focal and non-focal parts; client and server; time delays; variable inputs; desired solution.

Mathematics Subject Classification (2020): 34A12, 34K20, 92B20, 93D20.

* Corresponding author: <mailto:vrkota@hyderabad.bits-pilani.ac.in>