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Fractional Boole Type Inequalities for Differentiable s-Convex Functions

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Abstract: We introduce a new identity involving five-point Newton-Cotes inequalities called Boole's inequalities. By employing this identity, we establish some Boole-type inequalities for functions whose first derivatives are s-convex via Riemann-Liouville fractional integral operators. The results provide a generalization of classical inequalities by using tools from fractional calculus. Additionally, we present applications that highlight the utility and relevance of the obtained inequalities in various mathematical contexts.

Keywords: *Boole's inequality, s-convex functions, Hölder inequality, Riemann-Liouville fractional integrals.*

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1 Introduction

Let $I \subset \mathbb{R}$ be an interval. A function $\Game : I \to \mathbb{R}$ is said to be convex if

$$\Game(\varrho x + (1-\varrho)y) \le \varrho \Game(x) + (1-\varrho)\Game(y)$$

holds for all $x, y \in I$ and $\varrho \in [0,1]$.

The first finding between convex functions and integrals was the Hermite–Hadamard inequality, which can be stated as follows: for every convex function $\Game$ on the interval $[\xi_1, \xi_2]$ with $\xi_1 < \xi_2$, we have

$$\Game\left(\tfrac{\xi_1+\xi_2}{2}\right) \le \tfrac{1}{\xi_2-\xi_1} \int_{\xi_1}^{\xi_2} \Game(x)\,dx \le \tfrac{\Game(\xi_1)+\Game(\xi_2)}{2}. \tag{1}$$

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