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Approximation of Invariant Solutions to the Nonlinear Filtration Equation by Modified Padé Approximants

S. Skurativskyi *, S. Mykulyak and I. Skurativska

Subbotin Institute of Geophysics of NAS of Ukraine,
Palladin av. 32, Kyiv, 03142, Ukraine

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Abstract: This paper deals with a mathematical model for oil filtration in a porous medium and its self-similar and traveling wave regimes. The model consists of the equation of mass conservation and dependencies of porosity, permeability, and oil density on pressure. The oil viscosity is considered to be the experimentally determined parabolic relationship with respect to pressure. To close the model, two types of the Darcy law are used: the classic one and the dynamic one describing the relaxation processes during filtration. In the former case, self-similar solutions are studied, while in the latter case, traveling wave solutions are the focus. Using the invariant solutions, the initial model is reduced to the nonlinear ordinary differential equations possessing the trajectories vanishing at infinity and representing the moving liquid fronts in porous media. To approximate these solutions, we elaborate the semi-analytic procedure based on modified Padé approximants. In fact, we calculate sequentially Padé approximants up to the 3-rd order for a two-point boundary value problem on the semi-infinite domain. A good agreement of evaluated Padé approximants and numerical solutions is observed. The approach provides relatively simple quasi-rational expressions of solutions and can be easily adapted for other types of model's nonlinearity.

Keywords: *nonlinear filtration; self-similar solution; relaxation; traveling wave; Padé approximant.*

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* Corresponding author: mailto:skurserg@gmail.com